Measuring Systemic Funding Liquidity Risk in the Interbank Foreign Currency Lending Market

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Abstract

This paper proposes a new framework which captures the systemic nature of funding liquidity risk. Using this framework we develop a set of indicators which measure different aspects of the systemic funding liquidity risk in the interbank foreign currency lending market: (i) systemic funding liquidity needs, (ii) systemic vulnerability, (iii) systemic importance and (iv) systemic liquidity shortages. We also analyze the systemic funding liquidity risk of the Korean banking system under the new framework. The Korean banking system has become more vulnerable to the systemic funding liquidity risk of foreign currency debt since 2006. The systemic importance of foreign bank branches and the systemic vulnerability of domestic banks have simultaneously increased as the domestic banks have relied heavily on FX swap transactions with foreign bank branches to raise foreign currency funds.

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1 Introduction

Emerging markets suffered from massive capital outflows during the 2008 global financial crisis. The spillover of the global financial crisis to emerging markets has varied according to the degree of their openness to the international capital markets and the structure of the financial system. Indeed, emerging countries with large exposure to international banks and capital markets faced challenges in rolling over their foreign currency debt. Among the emerging markets, in particular, the Korean financial markets were suffered much more damaged from captial outflows associated with financial deleveraging by international banks and foreign investors. Especially after the failure of Lehman Brothers, the rising concern about counterparty risk in addition to funding liquidity risk led to further dislocations in the Korean financial market.

Why has the Korean financial system been so vulnerable to the rollover risk of foreign currency debt? An immediate answer to this question is the high proportion of short-term foreign debt. For the past decade, the proportion of short-term foreign debt had sharply increased. The recent rise of short-term debt was mainly attributable to branches of foreign banks, but domestic banks faced the most severe shortages in the foreign currency liquidity. Thus, the high proportion of short-term foreign debt cannot fully account for Korea's exceptionally vulnerable situation during the recent global financial crisis. Another possible answer may be the increasing interconectedness between banks which causes difficulties in rolling over liabilities spill over to the financial system as a whole. In order to detect the systemic linkages through which financial spillovers occur, it is necessary to look into the interbank foreign currency lending market in detail.

There has been a substantial body of work that examines interbank exposures as a possible source of financial spillover, see Allaen and Babus(2009) for a detailed survey. Allen and Gale (2000) model financial contagion via interbank exposures and investigate how the banking system responds to liquidity preference shocks when banks are connected under different

interbank market structures.[1] They show that better connected networks are more robust since the proportion of the losses in one bank's portfolio is spread to more banks. Freixas et al. (2000) and Dasgupta (2004) also explore how linkages between banks, represented by interbank credit lines or crossholding of deposits, can lead to contagious effects. Minguez-Afonso and Shin (2007) use lattice-theoretic methods to study liquidity and systemic risk in high-value payment systems and interbank payment systems. Gai and Kapadia (2007) applies statistical techniques from network theory to develop a general model of contagion in complex financial systems. Much of this literature finds that greater connectivity reduces the likelihood of widespread default as with Allen and Gale (2000).

Another strand of the literature focuses on the impact of indirect linkages on financial spillover. Lagunoff and Schreft (2001) construct a dynamic, stochastic game-theoretic model where agents hold diversified portfolios that link their financial positions to those of other agents. De Vries (2005) shows that there is dependency between banks' portfolios, given the fat tail property of the underlying assets, and this carries the potential for systemic breakdown. Cifuentes et al. (2005) incorporate two channels of contagion - direct balance sheet interconnections among financial institutions and contagion via changes in asset prices which may interact with externally imposed solvency requirements to generate amplified endogenous responses. These studies share the same finding that financial systems are inherently fragile.

Besides the theoretical studies, there is a growing empirical literature which analyzes contagion risk through interbank connections for particular countries. Most of these papers use balance sheet information to model the interbank linkages in the form of an interbank lendig matrix. The contagion effect arising from interbank lending is assessed by simulating the breakdown of a single bank. Recent examples are Sheldon and Maurer (1998) for Switzerland, Furfine (2003) for the United States, Wells (2002) for the United Kingdom, Boss et al. (2004) for Austria, Upper and Worms (2004) for Germany, Degryse and Nguyen (2007) for Belgium,

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[1]Contagion means that the financial distress of one bank affects another bank that is financially sound quite apart from its exposure to the bank in stress.

Mistrulli (2007) for Italia, Iyer and Peydro-Alcalde (2007) for India, and Cocco et al. (2009) for Portugal. These papers find that the banking systems demonstrate high resilience, even to large shocks. Most of them, however, focus on the credit or solvency risk of a bank failure and usually do not consider the funding liquidity risk, such as the drying up of credit lines.

This paper, departing from the previous studies, focuses on systemic funding liquidity risk in the interbank foreign currency lending market. Foreign funding involves higher risk than funding from other sources because international banks may react collectively towards domestic banks to a greater extent than ordinary domestic depositors. When foreign funding dries up, this represents a type of currency crisis which will emerge as a pure liquidity crisis without accompanying solvency problems. Banks' foreign funding, therefore, is of particular interest to central banks as lenders of last resort.

Borio (2000), Strahan (2008) and Brunnermeier and Pedersen (2009) define funding liquidity as the ability to raise cash at short notice either via asset sales or new borrowing. Since bank loans usually have a longer maturity than bank borrowings, banks may face the risk that funding liquidity in the foreign currency will dry up. The potential shortage of funding liquidity is often called rollover risk, because it may be difficult or impossible for a bank to "roll over" its short-term borrowings.[2] If a bank cannot roll over its borrowings in a foreign currency, it must either sell the foreign assets or accept increased currency risk by borrowing domestically. The funding liquidity risk has a systemic nature due to balance sheet interconnections among banks. If a bank face a liquidity shortage, it tries to call in loans to other banks, thus spreading its liquidity problems throughout the financial system. This is why we need to measure the funding liquidity risk at the level of financial system as a whole.

A funding liquidity shock has a different knock-on process from a default shock in two re-

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[2]There are three forms of funding liquidity risk-rollover risk, margin funding risk and redemption risk-which are different incarnations of the same funding liquidity risk and arise primarily from a maturity mismatch between the purchased asset and its funding. In our study, we focus on rollover risk. For more details, see Brunnermeier (2008).

spects. Firstly, the default losses are spread from debtors to lenders, whereas the funding liquidity needs are transferred from lenders to debtors. Secondly, default contagion may occur only if there is a bank whose loss from the initial shock excceds its capital, so that the default contagion stops if no additional banks fail. On the other hand, a funding liquidity shock could continue to spread throughout the interbank market until the inital funding liquidity needs are met by liquidating external assets since the interbank market cannot create liquidity. The knock-on effects of a liquidity crunch are key processes in the recent deleveraging of the global financial system that is taking place with financial institutions all contracting their balance sheets at the same time.

In order to capture the systemic nature of funding liquidity risk, we propose a framework for measuring the systemic funding liquidity risk of the banking system by applying input-output analysis methodologies. Our framework differs from the existing literature in three ways. Firstly, most previous studies focus on solvency concerns, while we focus on funding liquidity risk in a foreign currency. The global financial crisis reminds us of the relevance of liquidity spillovers, specifically that interconnectedness means difficulties in rolling over liabilities may spill over to the financial system as a whole. Secondly, most empirical studies limit their analysis to within the interbank market. In our study, we model non-financial sectors and foreign financial institutions as external sectors outside the domestic banking system, so that we provide a framework for measuring the impact of the global deleveraging on the domestic banking system.[3] Lastly, to estimate bank-to-bank exposures, it is generally assumed that banks spread their lending as evenly as possible (i.e. the maximum entropy method). This assumption might bias the results, in the light of the theoretical findings that better connected networks are more resilient to the propagation of shocks. As Upper and Worms (2004) point out, interbank lending is relatively concentrated. To cope with this problem, we develop a new algorithm which estimates a matrix of interbank loans by limiting the number of counterparties on the basis of the size of bank.

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[3]We recognize domestic branches of foreign banks as constituent parts of the domestic banking system to analyze their interactions with domestic banks.

Furthermore, our framework provides four measures of systemic funding liquidity risk: (i) the overall systemic funding liquidity indicator for the banking system, (ii) the systemic vulnerability indicator, (iii) the systemic importance indicator for an individual bank and (iv) the systemic liquidity shortage indicator. The systemic funding liquidty indicator measures the amount of assets directly or indirectly liquidated in the banking system when the system as a whole is unable to roll over external borrowings. The systemic vulnerability indicator, which is for finding out which banks are most exposed in the case of a systemic funding liquidity crisis, measures the amount of a bank's assets directly or indirectly liquidated when other banks are unable to roll over their external borrowings respectively. The systemic importance indicator, which is useful in finding out systemically important financial institutions, computes the amount of assets directly or indirectly liquidated in the banking system when a bank is unable to roll over its external borrowings. To evaluate the liquidity shortage due to a systemic funding liquidity shock, we calcualte the amount of the bank's liquidity needs when all banks are unable to roll over their external borrowings.

We apply the new framework to analyze the systemic foreign currency funding liquidity risk of the Korean banking system. The Korean banking system has been vulnerable to the systemic funding liquidity risk of foreign currency debt since 2006. We find that the systemic importance of branches of foreign banks and the systemic vulnerability of domestic banks have simultaneously increased as the domestic banks relied heavily on FX swap transactions with foreign bank branches to raise dollars.

The rest of the paper is organized as follows. Section 2 introduces a new framework for analysing the interbank foreign currency lending market. Section 3 provides methodologies for measuring systemic funding liquidity risk in the banking system. In Section 4, we analyze the systemic foreign currency funding liquidity risk in the Korean banking system. Section 5 concludes.

Table 1: foreign currency denominated balance Sheet for bank i

foreign currency Assets	foreign currency Liabilities
$\sum_{j=1}^{N} b_{ji}$	$\sum_{j=1}^{N} b_{ij}$
$\sum_{k=1}^{M} u_{ki}$	$\sum_{k=1}^{M} v_{ik}$
s_i	

2 Framework

Consider a banking system which consists of N banks. Outside the banking system, there exist M external sectors which include nonbank financial institutions, non-financial corporations, households and so on. Let b_{ij} denote the foreign currency borrowings of bank i from bank j. Similarly, let v_{ik} be the foreign currency borrowings of bank i from external sector k and let u_{ki} be the foreign currency loans of bank i to external sector k. The foreign currency short (long) position of bank i is denoted by $s_i > 0$ ($s_i < 0$). Table 1 illustrates the foreign currency denominated balance sheet of bank i. The balance sheet identity implies that the total uses of foreign currency funds must be equal to the total sources of foreign currency funds for each bank:

$$\sum_{j=1}^{N} b_{ji} + \sum_{k=1}^{M} u_{ki} + s_i = \sum_{j=1}^{N} b_{ij} + \sum_{k=1}^{M} v_{ik}, \quad \text{for all } i = 1, \ldots, N. \tag{1}$$

In order to model the knock-on effects of a foreign currency funding liquidity shock, we assume that a bank meets its foreign currency liquidity needs by liquidating foreign currency assets in a given pecking order when it is unable to roll over external foreign currency borrowings. Since premature liquidation of long-term assets is costly, we set the following pecking order: (i) short-term foreign currency assets vis-à-vis financial institutions and marketable securities; (ii) short-term foreign currency loans to non-financial corporations and households; (iii) long-term assets. If a bank meet its foreign currency liquidity needs by new external fund-

raising or a spot market FX purchase, no further knock-on effects occur. Since we aim to measure the systemic funding liquidity risk under global deleveraging with foreign currency market tumoil, we exclude new foreign currency borrowings and FX purchases from the pecking order. To separate funding liquidity risk from solvency issues, we also assume that each bank retains enough domestic currency assets that can be used to prevent insolvency.

Let p_j^b, p_k^u be the pecking order indicatiors for assets vis-à-vis banks and assets vis-à-vis external sectors, respectively:[4]

$$p_j^b = \begin{cases} 1 & \text{if foreign currency loans to bank } j \text{ are liquid assets.} \\ 0 & \text{if foreign currency loans to bank } j \text{ are illiquid assets.} \end{cases}$$

$$p_k^u = \begin{cases} 1 & \text{if foreign currency loans to external sector } k \text{ are liquid assets.} \\ 0 & \text{if foreign currency loans to external sector } k \text{ are illiquid assets.} \end{cases}$$

Let x_i^p be the amount of bank i's total liquid foreign currency assets:

$$x_i^p = \sum_{j=1}^{N} p_j^b b_{ji} + \sum_{k=1}^{M} p_k^u u_{ki}.$$

If bank i is unable to roll over Δv_{ik} of borrowing from external sector k, it must liquidate the same amount of foreign currency assets. It is also assumed that banks liquidate their liquid

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[4]We assume that the liquidation value of a liquid asset is equal to its book value. If asset j is sold at q_j of its book value, then the pecking order indicatior may be expressed as

$$p_j = \begin{cases} q_j & \text{if } j \text{ is a liquid asset,} \\ 0 & \text{otherwise.} \end{cases}$$

foreign currency assets proportionally to their weight:

$$\Delta v_{ik} = \Delta v_{ik} \sum_{j=1}^{N} \frac{p_j^b b_{ji}}{x_i^p} + \Delta v_{ik} \sum_{k=1}^{M} \frac{p_k^u u_{ki}}{x_i^p}.$$

Thus, the foreign currency funding liquidity shock of bank i, Δv_{ik}, induces the foreign currency liquidity need of bank j by $\Delta v_{ki} \frac{p_j^b b_{ji}}{x_i^p}$. Bank j's foreign currency liquidity need, in turn, induces other banks' liquidity needs and this knock-on process continues systemwidely until the external foreign currency assets of the banking system decrease by the same amount of initial shock. This is because the interbank market can only redistribute liquidity but not create liquidity. So when there is a external foreign currency liquidity shock, the external foreign currency assets of the banking system are liquidated ultimately by the amount of the initial shock, i.e. $\sum_{k=1}^{M} \sum_{i=1}^{N} \Delta u_{ki} = \Delta v_{ik}$. The total amount of liquidation of foreign currency assets, however, is much greater than the initial shock due to the knock-on effects in the interbank market, i.e. $\sum_{i=1}^{N} \sum_{j=1}^{N} \Delta b_{ij} + \sum_{k=1}^{M} \sum_{i=1}^{N} \Delta u_{ki} > \Delta v_{ik}$. Consequently, even a small external shock can make a huge impact on the banking system as a whole.

In order to measure the systemwide knock-on effects, it is necessary to combine balance sheet information of individual banks and different sources of data. The structure of foreign currency lending exposures in the banking system can be represented in matrix form. Table 2 illustrates a systemwide matrix which shows uses and sources of foreign currency funds for each individual bank.[5] Since banks do not lend to themselves, all diagonal elements of the interbank matrix are zero.

Most data are extracted from banks' balance sheets, which can provide information on the aggregate foreign currency lending exposure of the reporting bank vis-à-vis all other banks.

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[5]The foreign currency exposures between foreign financial institutions are not modeled, i.e. the lower-right submatrix of Table 2 is vacant. If foreign financial institutions are included in the banking system, we can analyze the process of deleveraging at the level of global financial system. We can also extend our framework by modeling the response of non-financial sector to a liquidity squeeze, which is another spillover channel.

Table 2: Souces and Uses of Foreign Currency Funds in the Banking System

Uses ↓	Sorces→	Banks					External Sectors				
		1	...	i	...	N	1	...	k	...	M
Banks	1	0	...	b_{1i}	...	b_{1N}	v_{11}	...	v_{1k}	...	v_{1M}
	⋮	⋮	⋱	⋮	⋱	⋮	⋮	⋱	⋮	⋱	⋮
	i	b_{11}	...	0	...	b_{iN}	v_{i1}	...	v_{ik}	...	v_{iM}
	⋮	⋮	⋱	⋮	⋱	⋮	⋮	⋱	⋮	⋱	⋮
	N	b_{N1}	...	b_{Ni}	...	0	v_{N1}	...	v_{Nk}	...	v_{NM}
External Sectors	1	u_{11}	...	u_{1i}	...	u_{1N}					
	⋮	⋮	⋱	⋮	⋱	⋮					
	k	u_{k1}	...	u_{ki}	...	u_{kN}					
	⋮	⋮	⋱	⋮	⋱	⋮					
	M	u_{M1}	...	u_{Mi}	...	u_{MN}					
foreign currency short position		s_1	...	s_i	...	s_N					

To estimate bank-to-bank exposures, it is generally assumed that banks spread their foreign currency lending as evenly as possible given the asset and liability positions reported in the balance sheets of all other banks (i.e. the maximum entropy method).[6] Despite the maximum entrophy method(ME) being intuitively appealing and well-founded in information theory, it has many shortcomings as Upper (2007) pointed out. Firstly, ME always returns positive b_{ij} as long as both bank i's borrowing and bank j's lending are non-zero. Fixed screening and monitoring costs of lending, however, may make small exposures unprofitable. Secondly, relationship lending may limit the number of counterparties of any one bank and could thus lead to a higher degree of market concentration than suggested by ME. Cocco, Gomes and Martins (2009) shows that banks want to establish relationships with banks whose liquidity shocks are less correlatated with their own. Thirdly, ME results in all banks holding essentially the same portfolio of interbank assets and liabilities, differing only by size and by the fact that no bank has any claims on itself. Furthermore, ME might bias the results since better connected networks are more resilient to the propagation of shocks as shown in Allen and Gale (2000).[7]

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[6]The concept of entropy originated from physics and was introduced into the contagion literature by Sheldon and Maurer (1998). Since this method assumes that all lending and borrowing is as dispersed as possible, it rules out the possibility of relationship banking.

[7]This is confirmed by Mistrulli (2007), who analyzes how contagion propagates within the Italian interbank market using actual bilateral exposures. He also applies the maximum entropy method to the same dataset, and

To address these problems, we develop a new algorithm which estimate a matrix of interbank foreign currency lending exposures by limiting the number of counterparties on the basis of bank size. Since our algorithm ties the number of non-zero elements of the row (or column) to the size of the row (or column) sum, the resulting matrix may be a better description of the reality that the number of counterparties of big banks is much greater than that of small banks.[8] The details of the algorithm are given in the Appendix.

3   Measuring Systemic Funding Liquidity Risk

In this section, we propose new systemic funding liquidity risk indicatiors which are able to capture the knock-on effects through the interbank foreign currency lending exposures. The systemic funding liquidity indicators are calculated from the systemwide multipliers that show how an inital liquidity shock is amplified throughout the banking system.

3.1   Systemwide Multiplier

The sources/uses matrix of foreign currency funds may be divided into four parts as follows:

(i) Interbank foreign currency lending exposures

$$
\mathbf{B} = \begin{bmatrix} 0 & \dots & b_{1j} & \dots & b_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & \dots & 0 & \dots & b_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{N1} & \dots & b_{Nj} & \dots & 0 \end{bmatrix}
$$

finds that it tends to generally underrate the extent of contagion.

[8]Memmel and Stein (2008) find that large banks have a well diversified interbank portfolio whereas small banks are highly concentrated in their interbank exposures.

(ii) Banks' borrowings from the external sectors

$$
\mathbf{V} = \begin{bmatrix}
v_{11} & \cdots & v_{1k} & \cdots & v_{1M} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
v_{i1} & \cdots & v_{ik} & \cdots & v_{iM} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
v_{N1} & \cdots & v_{Nk} & \cdots & v_{NM}
\end{bmatrix}
$$

(iii) Banks' lending to the external sectors

$$
\mathbf{U} = \begin{bmatrix}
u_{11} & \cdots & u_{1j} & \cdots & u_{1N} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
u_{h1} & \cdots & u_{hj} & \cdots & u_{hN} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
u_{M1} & \cdots & u_{Mj} & \cdots & u_{MN}
\end{bmatrix}
$$

(iv) Foreign currency short position

$$
\mathbf{S} = \begin{bmatrix} s_1 & \cdots & s_j & \cdots & s_N \end{bmatrix}
$$

Let $R_i \in \{1, 2, \ldots, N + M\}$ denote the liquidity ranking of asset i within the foreign currency asset portfolio, $i = 0, 1, 2, \ldots, N + M$:

$$
\mathbf{R} = \begin{bmatrix}
R_1 \\
\vdots \\
R_i \\
\vdots \\
R_{N+M}
\end{bmatrix}.
$$

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Suppose liquid foreign currency assets are restricted within hth liquidity ranking. The pecking order vector may be written as

$$\mathbf{p} = \mathbf{1}_{[R \leq h]} = \begin{bmatrix} \mathbf{p}_b \\ \mathbf{p}_u \end{bmatrix}$$

where $\mathbf{1}_{[R \leq h]}$ is a vector whose ith element is 1 if $R_i \leq h$ and 0 otherwise. Let the total foreign currency asset vector be denoted by $\mathbf{x}$. From the balance sheet identity (1), the fundamental equation of foreign currency funding in the banking system can be expressed in the matrix formula:

$$
\begin{aligned}
\mathbf{x} &= \mathbf{B}^\top \mathbf{i}_b + \mathbf{U}^\top \mathbf{i}_u + \mathbf{S}^\top \\
&= \underbrace{\mathbf{B}^\top \mathbf{p}_b + \mathbf{U}^\top \mathbf{p}_u}_{\equiv \mathbf{x}_p} + \underbrace{\mathbf{B}^\top (\mathbf{i}_b - \mathbf{p}_b) + \mathbf{U}^\top (\mathbf{i}_u - \mathbf{p}_u) + \mathbf{S}^\top}_{\equiv \mathbf{x}_n} \\
\mathbf{x} &= \mathbf{B}\,\mathbf{i}_b + \mathbf{V}\,\mathbf{i}_v \\
&= \underbrace{diag(\mathbf{p}_b)\,\mathbf{B}\,diag(\mathbf{x}_p)^{-1}}_{\equiv \Lambda}\,\mathbf{x}_p + \underbrace{(\mathbf{I} - diag(\mathbf{p}_b))\,\mathbf{B}\,\mathbf{i}_b}_{\equiv \mathbf{b}_n} + \underbrace{\mathbf{V}\,\mathbf{i}_v}_{\equiv \mathbf{v}} \\
\Rightarrow \mathbf{x} &= \Lambda(\mathbf{x} - \mathbf{x}_n) + \mathbf{b}_n + \mathbf{v}
\end{aligned}
\tag{2}
$$

where $\mathbf{i}$ denotes a vector that contains a column of ones, $\mathbf{x}_p$ is the total liquid asset vector, $\mathbf{x}_n$ is the total illiquid asset vector, $\mathbf{b}_n$ is the illiquid interbank asset vector, and $\mathbf{v}$ is the total external asset vector.

Solving equation (2) for $\mathbf{x}$ yields

$$\mathbf{x} = (\mathbf{I} - \Lambda)^{-1}[\mathbf{v} + \mathbf{b}_n - \Lambda \mathbf{x}_n]. \tag{3}$$

From equation (3) we have

$$\Delta\mathbf{x} \;=\; (\mathbf{I} - \mathbf{\Lambda})^{-1}\Delta\mathbf{v}. \tag{4}$$

Equation (4) implies that an external foreign currency funding liquidity shock, $\Delta\mathbf{v}$, induces systemwide foreign currency liquidity needs by $(\mathbf{I} - \mathbf{\Lambda})^{-1}\Delta\mathbf{v}$. The systemwide multiplier $(I - \mathbf{\Lambda})^{-1}$ provides the basis for measuring the systemic funding liquidity risk:

$$\mathbf{\Gamma} \equiv (\mathbf{I} - \mathbf{\Lambda})^{-1} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & \gamma_{NN} \end{bmatrix}.$$

The element γ_{ij} indicates the total amount of bank i's foreign currency assets directly or indirectly liquidated when bank j is unable to roll over one unit of its external foreign currency borrowings. The column sums $\sum_{i=1}^{N} \gamma_{ij}$ measure the amount of foreign currency assets directly or indirectly liquidated in the entire banking system when bank j is unable to roll over one unit of its external borrowings. The row sums $\sum_{j=1}^{N} \gamma_{ij}$ calculate the amount of bank i's foreign currency assets directly or indirectly liquidated when all banks are each unable to roll over one unit of their external borrowings.

3.2   Systemic Funding Liquidity Risk Indicators

We propose four indicators which measure different aspects of systemic funding liquidity risk by using the systemwide multiplier matrix $\mathbf{\Gamma} \equiv (\mathbf{I} - \mathbf{\Lambda})^{-1}$.

3.2.1 Overall systemic funding liquidity risk indicator

The indicator of overall systemic foreign currency funding liquidity risk, denoted by **SRI**, guages the extent of amplification of the external liquidity shock throughout the banking system due to interconnectedness. **SRI** is defined as[9]

$$\mathbf{SRI} = \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{\gamma}_{ij}$$

where

$$\bar{\mathbf{\Gamma}} \equiv \mathbf{\Gamma} \cdot diag(\mathbf{\delta}) = \begin{bmatrix} \bar{\gamma}_{11} & \bar{\gamma}_{12} & \cdots & \bar{\gamma}_{1N} \\ \bar{\gamma}_{21} & \bar{\gamma}_{22} & \cdots & \bar{\gamma}_{1N} \\ \vdots & \cdots & \ddots & \vdots \\ \bar{\gamma}_{N1} & \bar{\gamma}_{N2} & \cdots & \bar{\gamma}_{NN} \end{bmatrix} \text{ and}$$

$$\mathbf{\delta} = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_i \\ \vdots \\ \delta_N \end{bmatrix}, \qquad \delta_i = \frac{v_i}{\sum_{j=1}^{N} v_j}, \qquad v_i \text{ is the bank } i\text{'s external foreign currency borrowings.}$$

SRI indicates the systemwide amount of foreign currency assets directly or indirectly liquidated when the system as a whole cannot roll over one unit of external borrowings.[10] Here, the one unit of external borrowings comprises each bank's external borrowing share δ_i, $i = 1, \ldots, N$. However, **SRI** does not consider the size of external shock, but only captures the structure of interbank lending exposures. In order to reflect the potential size of liquidity

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[9]Time subscripts have been suppresed in order to alleviate the notation.

[10]In our study we assume that the external foreign currency liquidity shocks are exogenous. Thus our systemic indicators only capture the knock-on effects through the banking system, but do not count knock-on effects through the external sectors. To estimate these external knock-on effects, we need information on foreign currency lending exposures between external sectors.

shocks, **SRI** may be adjusted by the magnitude of foreign borrowings:

$$\mathbf{SRI}^* = \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \bar{\gamma}_{ij} \right) \frac{v_t}{v_0}$$

where v_t, v_0 are total foreign borrowings in the current year and the base year, respectively.

3.2.2 Systemic Vulnerability Indicator

The liability structure and counterparties of individual banks may affect its systemic vulnerability to external liquidity shocks. A bank could face a liquidity shortage without a direct external liquidity shock if its borrowings rely heavily on the banks which are largely exposed exteral shocks. To evaluate the vulnerability of an individual bank to systemic funding liquidity shocks, we calculate the amount of bank i's foreign currency assets indirectly liquidated when other banks as a whole are unable to roll over one unit of external foreign currency borrowings. The one unit of shock consists of ζ_{ij}, which is the foreign currency borrowing share of bank j calculated with the exception of bank i. The systemic vulnerability indicator for bank i, **SVI**$_i$, is defined as

$$\mathbf{SVI}_i = \sum_{j=1}^{N} \gamma_{ij} \cdot \zeta_{ij}$$

where

$$\zeta_{ij} = \begin{cases} \frac{v_j}{\sum_{h=1}^{N} v_h - v_i}, & \text{if i} \neq \text{j} \\ 0, & \text{if i} = \text{j} \end{cases}$$

SVI may tell us which banks are most exposed in the systemic funding liquidity crisis.

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3.2.3 Systemic Importance Indicator

A bank might be considered systemically important with respect to funding liquidty risk if its liquidity shortage would have significant spillover effects which could destabilize the banking system.[11] To measure the bank's funding liquidity spillover effects through interbank lending, we calculate the amount of foreign currency assets directly or indirectly liquidated by other banks when bank i is unable to roll over δ_i unit of its external foreign currency borrowings. Here δ_i is the bank i's foreign currency borrowing share in the banking system. The systemic inportance indicator for bank i, $\mathbf{SII}_i$, is defined as

$$\mathbf{SII}_i = \left(\sum_{j=1}^{N} \gamma_{ji} - \gamma_{ii} \right) \cdot \delta_i$$

where

$$\delta_i = \frac{v_i}{\sum_{j=1}^{N} v_j}, \qquad v_i \text{ is the bank } i\text{'s external foreign currency borrowing.}$$

$\mathbf{SII}$ is useful in finding out which financial institutions are systemically important with respect to the foreign currency funding liquidity risk.[12]

3.2.4 Liquidity Shortage Indicator

Liquidity shortages arise when banks face difficulties in securing short-term funding. A bank is said to face a foreign curruncy liquidity shortage if its total foreign currency needs exceeds its total foreign currency denominated liquid assets. As assumed in Section 2, the foreign currency needs must be met by liquidating the foreign currency denominated assets in the

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[11]Because a variety of factors could make a firm systemically important, there are several criteria for classifying firms as systemically important: size, contagion, concentration, correlation, and conditions. See Thomson (2009) for discussion.

[12]The systemic vulnerability and systemic importance might have different implications for defualt risk since loan losses would be trasferred from borrowing banks to lending banks, while liquidity shocks transferred in the opposite direction.

given pecking order. Once a bank face a liquidity shortage, it must sell illiquid assets.[13] To evaluate liquidity shortages due to systemic funding liquidity shocks, we calculate the amount of the bank's liquidity shortage when each bank cannot roll over some portion of its external foreign currency borrowings Δv_j. The liquidity shortage indicator for bank i associated with external liquidity shocks $\Delta \mathbf{v}$, $\mathbf{LSI}_i$, is defined as

$$\mathbf{LSI}_i = 1 - \frac{\sum_{j=1}^{N} \theta_{ij} + \eta_i}{x_{pi}}$$

where

$$\boldsymbol{\Theta} = \begin{bmatrix} \theta_{11} & \theta_{12} & \dots & \theta_{1N} \\ \theta_{21} & \theta_{22} & \dots & \theta_{1N} \\ \vdots & \dots & \ddots & \vdots \\ \theta_{N1} & \theta_{N2} & \dots & \theta_{NN} \end{bmatrix} = \sum_{k=0}^{\infty} \boldsymbol{\Phi}_k, \qquad \boldsymbol{\eta} = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_i \\ \vdots \\ \eta_N \end{bmatrix} = \sum_{k=0}^{\infty} \boldsymbol{\omega}_k,$$

$$\begin{aligned}
\boldsymbol{\Phi}_0 &= diag([z_{0,1}, \dots, z_{0,j}, \dots, z_{0,N}]), \\
\boldsymbol{\Phi}_k &= diag([z_{k,1}, \dots, z_{k,j}, \dots, z_{k,N}]) \boldsymbol{\Lambda} \, \boldsymbol{\Phi}_{k-1}, \\
z_{k,j} &= \begin{cases} \min(\Delta v_j, x_{p,j}), & \text{if } k = 0 \\ \frac{\mu_{k,j} - \omega_{k,j}}{\mu_{k,j}}, & \text{if } k \neq 0 \end{cases} \\
\boldsymbol{\mu}_k &= \boldsymbol{\Lambda} \, \boldsymbol{\Phi}_{k-1} \mathbf{i} - \boldsymbol{\tau}_k, \\
\boldsymbol{\tau}_k &= \begin{cases} \mathbf{x}_p, & \text{if } k = 0 \\ \boldsymbol{\tau}_{k-1} - \boldsymbol{\Phi}_{k-1} \, \mathbf{i}, & \text{if } k \neq 0 \end{cases} \\
\boldsymbol{\omega}_k &= [\omega_{k,1}, \dots, \omega_{k,j}, \dots, \omega_{k,N}]^{\top}, \\
\omega_{k,j} &= \begin{cases} \max(0, \, \Delta v_j - x_{p,j}), & \text{if } k = 0 \\ \max(0, \, \mu_{k,j}) \quad, & \text{if } k \neq 0 \end{cases}
\end{aligned}$$

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[13]To meet the foreign currency needs, the bank can also sell domestic currency denominated assets or borrow in domestic currency, and convert it in a straight FX spot transaction.

The liquidity shortage indicator could not be directly derived from the systemwide multiplier Γ since the exhaustion of interbank assets could lead to selling illiquid external assets without further knock-on effects.[14] In order to consider the broken knock-on process due to selling external illiquid assets, it is necessary to compute its remaining liquid assets $\tau_{k,i}$ and additional liquidity shortage $\omega_{k,i}$ in each round k. If there are no remaining liquid assets, i.e. $\tau_{k,i} = 0$, then the knock-on process through bank i is stopped, i.e. $z_{k,j} = 0$, whereas the liquidity shortages $\omega_{k,i}$ continue to accumulate. Here, $\sum_{j=1}^{N} \theta_{ij}$ and η_i indicate the foreign currencies raised by selling liquid assets and illiquid assets, respectively, to meet the foreign currency liquidity needs. Thus η_i represents the total amount of liquidity shortages of bank i. Once a bank faces liquidity shortages, i.e. $\eta_i > 0$, all its liquid assessts are already exhausted, i.e. $\sum_{j=1}^{N} \theta_{ij} = x_{pi}$, resulting in $\mathbf{LSI}_i = -\frac{\eta_i}{x_{pi}} < 0$. Negative $\mathbf{LSI}$ implies that bank i faces a liquidity shortage by $\mathbf{LSI}_i\%$ of toal liquatatable assets x_{pi} when each bank j is unable to roll over Δv_j of its external foreign currency borrowings.

4 Systemic foreign currency Funding Liquidity Risk in the Korean Banking System

In this section, we analyze the systemic foreign currency funding liquidity risk of the Korean banking system by using the four indicators-SRI, SVI, SII and LSI-introduced in the previous section.

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[14]If no banks face liquidity shortages after being hit by the systemic liquidity shocks $\Delta\mathbf{v}$, i.e. $\eta_i = 0 \; \forall i$, the following equation holds.

$$
\begin{aligned}
\boldsymbol{\Theta} &= \boldsymbol{\Gamma} \Delta\mathbf{v} \quad \text{hence} \\
\mathbf{LSI}_i &= 1 - \frac{\sum_{j=1}^{N} \gamma_{ij} \Delta v_j}{x_{pi}}.
\end{aligned}
$$

4.1 Data

We first construct a systemwide foreign currency asset/liability matrix which is illustrated in Table 2 by using data from the foreign currency denominated balance sheets of domestic banks including doemstic branches of foreign banks.[15] Since the business stratagies and behaviors of the domestic branches of foreign banks are very similar, we treat them as a single bank.[16] This would not underrate the systemic risk since the foreign currency lending exposures between foreign bank branches are negligible. The external sectors are classified into four parts: the domestic non-bank financial sector, the domestic non-financial sector, the foreign financial sector and the foreign non-financial sector.[17] We also place long-term assets and liabilities in the external sector as they are not relevant to the liquidity risk. The classification of the external sectors is illustrated in Table 3. We use quarterly data of 15 banks from the third quarter of 2002 to the second quarter of 2009, which yields a time span of 27 quarters.

Unfortunately, we could not identify bank-to-bank exposures, i.e. individual elements of Matrix **B**, since banks' balance sheets do not provide information on counterparts.[18] The balance sheets only contain information on total interbank foreign currency lending and borrowing from which we have to estimate Matrix **B**. As mentioned in Section 2, we develop a new algorithm for estimating bank-to-bank exposures, which results a higher degree of market concentration than the maximum entropy method. This algorithm limits the number of counterparties on the basis of the size of total exposures and results "incomplete structure of claims" which is a better description of reality. See Appendix for more details on the algorithm.

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[15]To capture off-balance sheet exposures, we also treat interbank FX swap transactions as a foreign currency short-term lending.

[16]With the similar reasons we treat regional banks as a single bank as well.

[17]To construct pecking order vector **p**, we assumed that short-term foreign currency assets vis-à-vis financial institutions and marketable securities are repaid or sold at their book values and other assets are illiquid.

[18]In some countries (e.g. in Italy, Hungary, and Mexico), information on interbank bilateral exposures is available from credit registers and supervisory reports.

Table 3: Souces and Uses of Foreign Currency Funds in the Banking System

Uses ↓ \ Sources →			Short Term						Long Term	
			Residents				Non-Residents		Residents	Non-Residents
			$Bank_1$	$\cdots$	$Bank_N$	Non-Banks	Others	Fin.Corps	Others	
Short Term	Residents	$Bank_1$ / $\cdots$ / $Bank_N$	$\mathbf{B}_{[N\times N]}$					$\mathbf{V}_{[N\times M]}$			
		Non-Banks									
		Others									
	Non-Residents	Fin.Corps	$\mathbf{U}_{[M\times N]}$								
		Others									
Long Term	Residents										
	Non-Residents										
Foreign currency short position			$\mathbf{S}_{[1\times N]}$								

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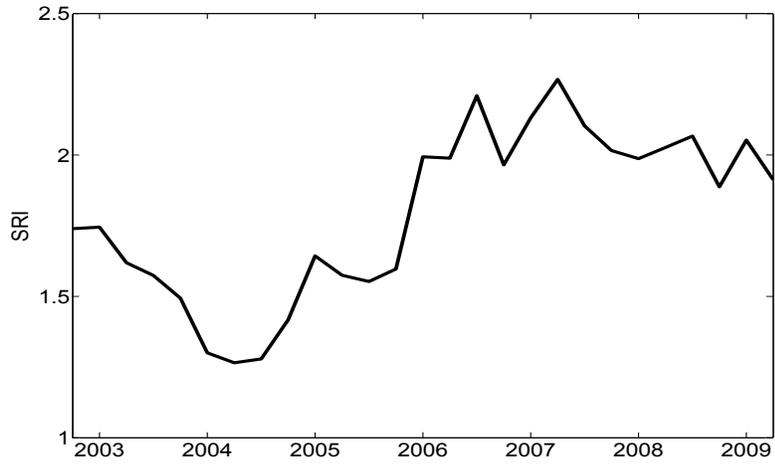
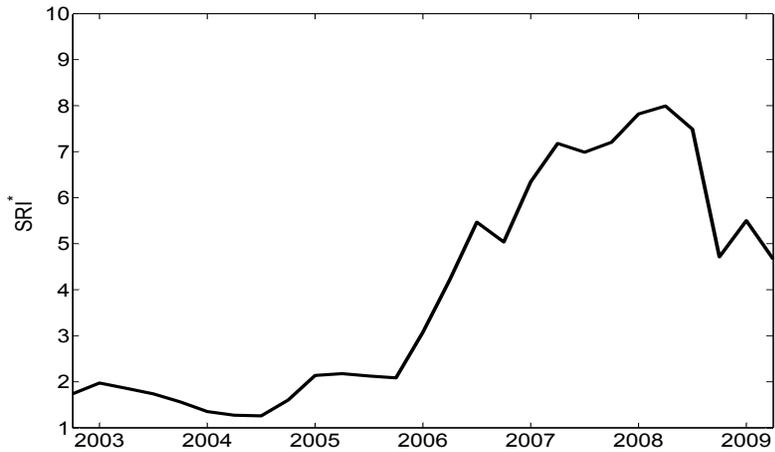Figure 1: Systemic Foreign Currency Funding Liquidity Risk in the Korean Banking System

Figure 2: Systemic Foreign Currency Funding Liquidity Risk in the Korean Banking System When the Size of Liquidity Shocks Are Adjusted

4.2 Results

4.2.1 Overall Systemic Foreign Currency Funding Liquidity Risk

The overall systemic foreign currency funding liquidity risk rose rapidly in 2006 as shown in Figure 1, reflecting the increasing interconnectedness in the banking system. This increase was mainly due to the expansion of interbank FX swap transactions associated with a sharp rise in foreign bank branches' overseas short-term borrowing. The branches of foreign banks have played a pivotal role in foreign currency financial intermediation since 2006.[19] The domestic banks increased foreign currency fund-raising by conducting FX swap transactions with the branches of foreign banks, which can finance in U.S. dollars from their head offices at low cost.[20] Foreign bank branches ordinarily perform interest rate arbitrage transactions in which they first borrow foreign currency and convert it into domestic currency in an FX swap transaction and then purchase Korean government bonds.[21]

The overall systemic funding liquidity risk indicator (**SRI**) is calculated under the assumption that the banking system is hit by unitary foreign currency liquidity shocks in each time period. If we adjust the size of liquidity shocks by the magnitude of foreign debts, the adjusted **SRI** rose to its highest level in the second quarter of 2008 as shown in Figure 2.

4.2.2 Systemic Vulnerability

Domestic banks are more vulnerable to global financial deleveraging than foreign bank branches as Figure 3 shows. In particular, the systemic vulnerability indicator (**SVI**) of domestic banks

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[19]Short-term borrowing mediated by domestic branches of foreign banks more than doubled from end of 2005 to end 2006-$23.3 billion to $51.8 billion-and the gross amount of short-term foreign borrowings through the branches of foreign banks started to exceed that of domestic banks beginning in 2006.

[20]From 2006 banks' overseas borrowing was stimulated in Korea in part by rising interest rate differentials relative to other countries, notably Japan. In addition, forward sales of dollars to local banks by Korean shipbuilders with long-term contracts reportedly prompted overseas borrowing by those banks to cover their exchange rate risk.

[21]An interest rate arbitrage transaction is a type of what is termed the dollar carry trade, which refers to borrowing in U.S. dollars at low short-term interest rates to make investments in the stock, bond, and real estate markets of fast-growing economies with an aim of generating profit from the rate differential.
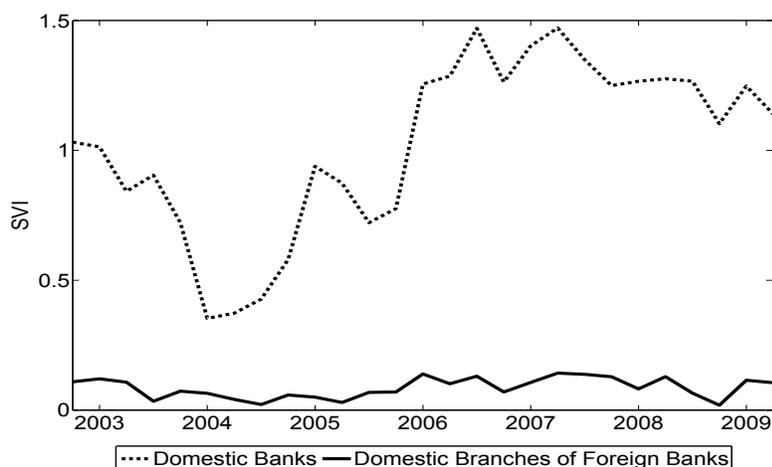
Note: We aggregate the domestic banks after computing the systemwide mulitpliers in order to capture the knock-on effects between domestic banks. To aggregate the systemwide multipliers, the matrix **Γ** must be summed up column-wise and averaged row-wise.

Figure 3: Systemic Vulnerability Indicators for the Domestic Banks and the Branches of Foreign Banks

has risen above one since 2006. This implies that domestic banks need to liquidtate more than one unit of foreign currency assets, as a whole, when foreign bank branches cannot roll over one unit of their foreign currency borrowings. The domestic banks rely on the foreign bank branches to raise short-term foreign currency funds through FX swaps. So once the foreign bank branches face a liquidity shortage, domestic banks may be unable to roll over their FX swap transactions. Accordingly, even though there may be no direct foreign currency liquidity shocks to domestic banks, they may face a foreign currency liquidity shortage due to the indirect knock-on effects. The branches of foreign banks, by contrast, are much less exposed to systemic foreign currency funding liquidity risk than the domestic banks. This is because they rarely rely on domestic banks in their foreign currency borrowing.

4.2.3 Systemic Importance

The foreign bank branches can have a huge impact on the banking system when they face a foreign currency funding liquidiy shock as shown in Figure 4. If the foreign bank branches unwind their interest rate arbitrage positions, i.e. sell domestic currency bonds and close out
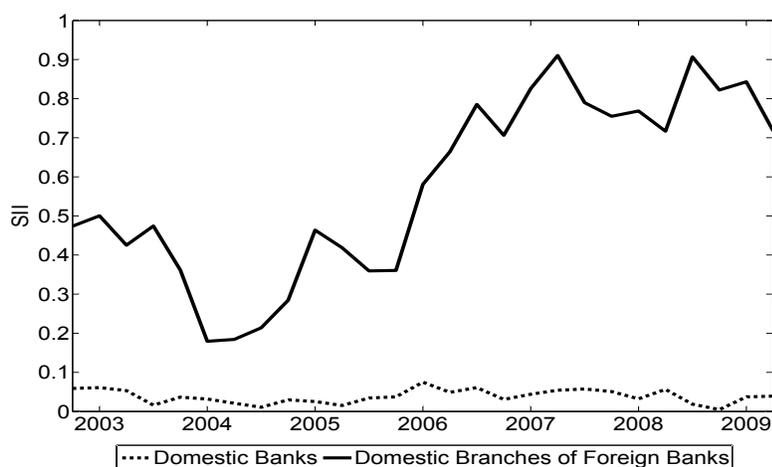
Figure 4: Systemic Importance Indicators for Domestic Banks and Branches of Foreign Banks

their FX swap positions, domestic banks could encounter foreign currency liquidity shortages. By contrast, the domestic banks have litte impact on the banking system when they are hit by external liquidity shocks. Even though the the domestic banks' impact on the banking system in this event is rather small, its impact on non-financial sectors could be immense since it would be forced to reduce foreign currency loans to non-financial corporations.

4.2.4 Systemic Liquidity Shortage

The liquidity shortage indicator is a useful tool for stress testing. Given a foreign currency liqidity shock scenario, it is possible to measure how much liquidity shortage each bank faces. To do this, we first set up a scenario in which each bank fails to roll over 20% of its foreign currency borrowings from foreign financial institutions. Under this scenario, we measure direct or indirect foreign currency liquidity needs for each bank. If the total foreign currency liquidity needs of a bank exceed its total liquid foreign currency assets, the bank is said to face a liquidity shortage (i.e. **LSI** < 0).[22] Figure 5 shows the trend of **LSI** for domestic banks and foreign bank branches under the scenario of liquidity shocks. The foreign bank branches do not face liquidity shortages for the whole period, but the domestic banks face liquidity shortages

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[22]A foreign currency liquidity shortage does not imply insolvency because banks can purchase foreign currency in the FX spot market if they have enough local currency assets.
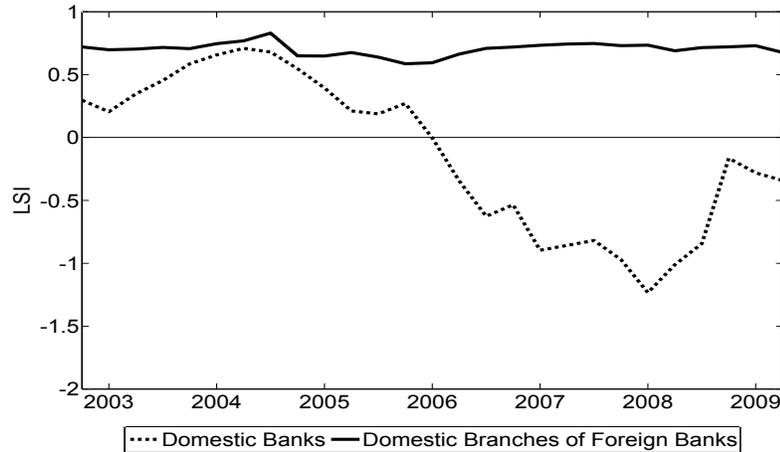
Figure 5: Liquidity Shortage Indicators for Domestic Banks and Branches of Foreign Banks during 2006-2009 under the scenario.

As shown in Figure 6, if we count only direct foreign currency liquidity needs, there are no liquidity shortages at domestic banks either. This implies that the conventional liquidity risk measure, which only count the direct liquidity needs, may underestimate the funding liquidity risk of the banking system. Figure 7 gives the number of banks which face a liquidity shortage under the given scenario. The number of banks increased drastically from 2006 and rose to its highest level in the first quarter of 2007.

5 Conclusions

In this paper, we propose a framework which captures the systemic nature of funding liquidity risk in the interbank market and develop four indicators of systemic funding liquidity risk: (i) an overall systemic funding liquidity indicator for the banking system, (ii) a systemic vulnerability indicator and (iii) a systemic importance indicator for individual banks, and (iv) a systemic liquidity shortage indicator which is a good tool for stress testing. We also develop a new algorithm for estimating bank-to-bank exposures. This algorithm limits the number of counterparties on the basis of the size of total exposures and results in an "incomplete struc-
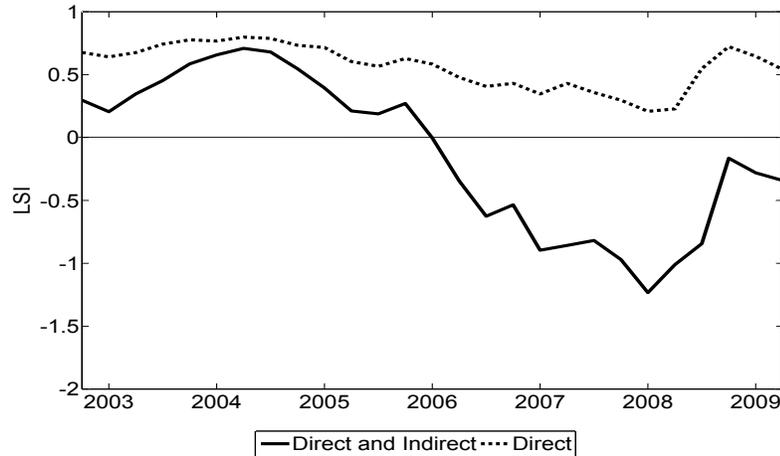
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Figure 6: Direct or Indirect Liquidity Shortage Indicator of Domestic Banks.

ture" of claims which is a better description of reality.

We analyze the systemic foreign currency funding liquidity risk in the Korean banking system under the framework by using the data from the foreign currency denominated balance sheets. The Korean banking system has been vulnerable to systemic funding liquidity risk of foreign currency debt since 2006 due to the increasing interconnectedness between domestic banks and branches of foreign banks. We also find that the systemic importance of branches of foreign banks and the systemic vulnerability of domestic banks have simultaneously increased as the domestic banks came to rely heavily on FX swap transactions with foreign bank branches to raise dollars.

This paper is only a first step in the development of a framework for assessing systemic risk. Our framework can be extended in many directions which fall into broad categories: (i) expanding the system by including some external sectors to analyze their interactions with the domestic banking system, (ii) modifying the pecking order indicator to incorporate the amplification of asset price shocks through illiquid asset sales, and (iii) modeling the probability and size of the external funding liquidity shocks.
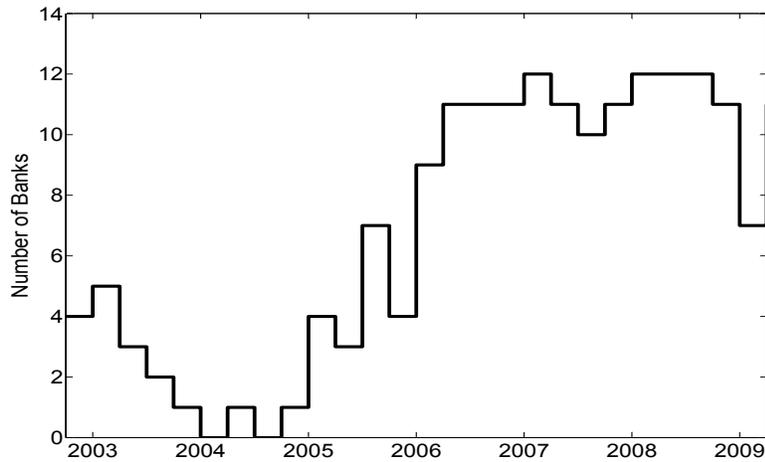
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Figure 7: Number of Banks Facing a Liquidity Shortage in the Korean Banking System when Each Bank cannot Roll Over 20% of Its Short-term Financial Borrowings

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Appendix: Algorithm for Estimating the Interbank foreign currency lending exposures

To construct the matrix of bilateral foreign currency lending exposures, we develop a new algorithm by modifying the RAS method. The RAS is basically an iterative scaling method whereby a non-negative matrix is adjusted until its column sums and row sums equal given vectors x^* and y^* (Schneider and Zenios, 1990). The RAS method always returns positive b_{ij} as long as both x_j^* and bank y_i^* are non-zero. The resulting matrix gives Allen and Gale's (2000) complete structure of claims, in which banks symmetrically hold claims on all other banks in the economy, conditioned on the size-structure of the banks. The complete structure of claims is a rather poor description of reality because it is irrelevant to relationship lending and the fixed costs of screening and monitoring.

To address these problems, we limit the number of positive elements by eliminating the element which is a minimum value row-wise and column-wise at the same time after applying the RAS method. This procedure iterates $N - n$ times, where N is the number of rows or columns and n is the minimum number of non-zero elements of each row and column. The resulting matrix has at least n non-zero elements for each row and column. Since the number of non-zero elements of the row (or column) depends on the size of the row (or column) sum, the matrix **B** may be a better description of the reality that large banks have a well diversified interbank portfolio whereas small banks have highly concentrated interbank exposures.

Suppose there are N banks that may lend to each other. In this case, the interbank market

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can be represented as an $N \times N$ matrix

$$\mathbf{B} = \begin{bmatrix} 0 & \cdots & b_{1j} & \cdots & b_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & \cdots & 0 & \cdots & b_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{N1} & \cdots & b_{Nj} & \cdots & 0 \end{bmatrix}$$

In this matrix, the columns represent banks' lending, while the rows represent banks' borrowings. All the diagnoal elements are zero due to the fact that banks do not lend to themselves. The sum of each bank's interbank lending and borrowing, $x_j^* = \sum_i b_{ij}$ and $y_i^* = \sum_j b_{ij}$, are known, while the distributions of these exposures over the banking system, i.e. the elements of matrix $\mathbf{B}$, are unknown. The objective of the algorithm is to estimate the matrix $\mathbf{B}$ only by using the row sums y_i^* and column sums x_j^*.

Algorithm

(i) Construct the inital matrix $\mathbf{B}^{(0,0)}$

$$\begin{aligned} \mathbf{M}^0 &= \frac{\mathbf{y}^* \cdot \mathbf{x}^*}{\mathbf{x}^*\mathbf{i}} \\ \mathbf{M}^1 &= \mathbf{M}^0 - diag(\check{\mathbf{M}}^0) \\ \mathbf{B}^{(0,0)} &= \mathbf{M}^1 \, diag(\mathbf{M}_1\mathbf{i})^{-1} \, diag(\mathbf{y}^*) \end{aligned}$$

where $\check{\mathbf{M}}^0$ is a vector with the diagonal elements of $\mathbf{M}^0$.

(ii) Perform an iteration $h = 1, 2, \ldots$ where in each step we:

a. Eliminate the trivial elements of $\mathbf{B}^{(h-1,0)}$

$$\mathbf{b}_{ij}^{(h-1,0)} \quad = \quad 0 \quad \text{if } \mathbf{b}_{ij}^{(h-1,0)} \leq \min(h^{th} \text{ low value of } i\text{th row, } h^{th} \text{ low value of } j\text{th column}),$$

$$i = 1, \ldots, N \text{ and } j = 1, \ldots, N.$$

b. Apply the RAS algorithm by doing an iteration k = 1, 2, ... where in each step we:

(b-1) construct $\mathbf{R}^{(h,k)}$ and $\mathbf{S}^{(h,k)}$ matrix

$$\mathbf{R}^{(h,k)} \quad = \quad diag(\mathbf{y}^*)\, diag(\mathbf{y}^{(h,k)})^{-1}$$
$$\mathbf{S}^{(h,k)} \quad = \quad diag(\mathbf{x}^*)\, diag(\mathbf{x}^{(h,k)})^{-1}$$

where

$$\mathbf{x}^{(h,k)} \quad = \quad \mathbf{i}^{\top} \mathbf{B}^{(h-1,k-1)}$$
$$\mathbf{y}^{(h,k)} \quad = \quad \mathbf{B}^{(h-1,k-1)} \mathbf{i}$$

(b-2) Adjust $\mathbf{B}^{(h-1,k-1)}$ by using $\mathbf{R}^{(h,k)}$ and $\mathbf{S}^{(h,k)}$

$$\mathbf{B}^{(h-1,k)} \quad = \quad \mathbf{R}^{(h,k)} \mathbf{B}^{(h-1,k-1)} \mathbf{S}^{(h,k)}$$

(b-3) Stop when $\mathbf{x}^{(h,k)}$ and $\mathbf{y}^{(h,k)}$ are sufficiently close to $\mathbf{x}^*$ and $\mathbf{y}^*$ in a least-squares sense.

$$\mathbf{B}^{(h,0)} \quad = \quad \mathbf{B}^{(h-1,k)}$$

c. Stop if $h = N - n$, where n is the minimum number of non-zero elements of each row and column.