Money, Credit, Risk of Loss, and Limited Participation*

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Abstract

An asset market segmentation model is constructed to study the choice of credit and cash when holding money is risky due to theft and to explore the distributional effects of monetary policy across economic individuals. In equilibrium, theft distorts the intertemporal marginal rate of substitution between consumption with credit and with cash. Money is nonneutral and with anticipated monetary policy inflation decreases theft and always improves welfare of all individuals. However, with unanticipated monetary policy, there is a precautionary demand for money. Inflation decreases theft and improves welfare of some individuals. The optimal monetary policy is to minimize theft and the Friedman rule is suboptimal in general.

Key Words: money, credit, theft, limited participation, distributional effects, welfare, Friedman rule

JEL Classifications: E4; E5

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1 Introduction

This paper studies the choice of credit, which is costly to use, and money, which is risky to hold due to theft, in the asset market segmentation model and explores the asymmetric distributional effects of monetary policy across economic individuals.

Money has been one of the most preferred payment instruments in market transactions because the user does not have to reveal his/her identity. However, due to its anonymity, the user faces a risk of loss, for example, pickpocketing, purse snatching, or robbery, which results in an instant loss. Once money is lost, it is very difficult to reclaim its ownership unlike debit or credit cards. Theft increases the carrying cost of money and would distort the demand for money and the choice of credit and cash. Further, monetary policy and welfare implications with theft would be different from those without theft as well.

Recently, the safety issues of payment instruments including theft have been brought into a light by several researchers. In particular, Prescott (1987) notes that cash may not be used in order to reduce the risk of loss by theft or fire when there is an alternative means of payment. Humphrey, Pulley, and Vesala (1996) study data constructed by the Bank for International Settlements and show that as the crime rate increases the use of noncash payment instrument increases across countries. Kosse (2010) provides an empirical analysis that the safety perception is important for consumers’ preferences for payments instruments, cash and debit card. The theoretical models of multiple means of payments with theft have been recently developed by He, Huang, and Wright (2005, 2008), Bolt and Chakravorti (2008), Alvarex and Lippi (2009), Sanches and Williamson (2009), and Choi (2010a).

However, most of these studies restrict their discussion on the relationship between theft and the choice of multiple means of payments. However, Williamson and Wright (2010) notes that the safety issues would be potentially important for monetary policy analysis since they may distort the monetary policy effect. He, Huang, and Wright (2005, 2008) and Choi (2010a) study the choice of an alternative means of payments with theft and provides monetary policy implications. There are several results in common. First, in equilibrium, a buyer may choose deposit in the bank or hold nominal bonds even if the nominal interest rate is negative. Finally, due to theft, inflation may improve welfare and the Friedman rule is suboptimal.

The contribution of this paper is to explore the asymmetric effects of monetary policy across economic individuals with multiple means of payments in the existence of theft. My model where money is nonneutral provides richer implications for the choice of credit and cash across economics agents and for the relationship between theft and the distributional effects of monetary policy unlike He, Huang,

\footnote{A physical load to carry around a lot of cash would be primarily considered as the carrying cost of money. However, theft would aggravate the burden of cost.}

The source of monetary nonneutrality in the short run stems from an asset market segmentation which captures the asymmetric behaviors of economic agents on the choice of credit and cash and other macroeconomic variables in response to monetary policy. There are two types of households: traders who are connected to the asset market and receive the initial money injection from a central bank and nontraders who are unconnected to the asset market. In the short run, money is nonneutral and monetary policy results in a redistribution of wealth between traders and nontraders.\(^2\)


The model extends Ireland (1994) and Choi (2010b) by adding the risk of holding money as in Choi (2010a). In the asset market, traders exchange money and one-period government nominal bonds. In the goods market, a shopper acquires a variety of consumption goods with credit or with cash. Holding money is risky since a worker can steal cash by working less. In equilibrium, monetary policy has three distributional effects on consumption with credit and with cash. One is the direct effect via real money holding. The others are the indirect effects: one via the choice of credit and cash and the other via theft.

In the case of constant money growth, a money injection results in positive income effects on both traders and nontraders in that theft decreases and output increases. Due to lower theft, cash is preferred for a greater variety of goods. Consumption with credit and with cash all increase. Welfare increases with the money growth rate as well. However, the marginal welfare improvement of traders decreases with the money growth rate, but it is constant for nontraders. The optimal money growth rate is infinity where no theft occurs and the Friedman rule is suboptimal.

In the case of stochastic money growth, first, there is a precautionary demand for money. Next, theft has an asymmetric effect on the intertemporal marginal rate of substitution between aggregate consumption with credit in the current period and aggregate consumption with cash in the next period. For example, when the government injects money, the real money holding of traders increases for a precautionary purpose and theft decreases. Cash is preferred for a greater

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\(^2\)The idea of an asset market segmentation is supported by empirical evidence as well. For example, the asset market participation in the United States is limited. According to the Survey of Consumer Finance (2009), in 2007, 10.3% of families did not hold a checking account, only 17.9% held publicly tradeable stocks, and only 11.4% has direct ownership of pooled investment funds. Thus, a large fraction of the U.S. population does not have an initial effect from monetary policy when the Fed conducts open market operations.
variety of goods. On the other hand, inflation taxes nontraders. In order to acquire more cash for compensating consumption loss, they steal more. Also, the real money holding of nontraders decreases and credit is preferred for a greater variety of goods. Unlike the constant money growth, the stochastic money injection increases welfare of traders, but decreases that of nontraders. The optimal money growth rate minimizing theft may be positive and the Friedman rule may not be optimal in general.

The remainder of the paper is organized as follows. Section 2 describes the environment of the model and Section 3 explains the equilibrium dynamics. In Sections 4 and 5, the distributional effects of monetary policy are studied when the money growth rate is constant and stochastic. Section 6 concludes.

2 The Environment and Timing

Time is discrete and indexed by \( t = 0,1,2, ... \). There is a continuum of infinitely lived households with unit mass. Each household consists of a shopper and a worker. A fraction \( \alpha \) of the households are traders living in an island which is connected to the asset market. The rest, \( 1 - \alpha \), are nontraders living in an unconnected island and do not have access to the asset market. There is a continuum of spatially separated goods markets indexed by \( i \in [0, 1] \) in each period. The household has preferences\(^3\) given by

\[
U(\{c_t, x_t\}_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln(c_t(i)) \, di - x_t \right\},
\]

where \( E_0 \) is the expectation operator conditional on information in period 0, \( \beta \) is the discount factor, \( c_t(i) \) represents distinct and perishable consumption goods at market \( i \) in period \( t \), and \( x_t \) represents transactions costs.

At the beginning of period \( t \), traders enter the period with \( M_{r,t} \) units of currency and \( \bar{B}_t \) units of one-period nominal bonds. Nontraders enter the period with \( M_{n,t} \) units of currency. Then, while nontraders stay at their island, traders go to the asset market held within the connected island and exchange one-period government nominal bonds and money. In the asset market, the government money supply, \( M_t^s \), comes through open market operations where nominal bonds that are issued in period \( t - 1 \) and mature in period \( t \) are denoted by \( \bar{B}_t \). In period \( t \), each bond, \( \bar{B}_{t+1} \), sells for \( q_t \) units of money and is a claim to one unit of money in period \( t + 1 \). The government budget constraint is

\[
\bar{B}_t - q_t \bar{B}_{t+1} = M_{t+1}^s - M_t^s = \mu_t M_t^s
\]

where \( \mu_t > -1 \) is the net money growth rate.

\(^3\)Log preferences do not lose any key results and make the equilibrium analysis simple.
After the asset market closes, the household decides how much to work, $n_t^w$, and to steal, $n_t^s$, given one unit of time,

$$n_t^w + n_t^s = 1. \quad (1)$$

Then, a worker and a shopper leave their island for the goods market. However, before they get out of their islands, a worker steals money from shoppers of other households. A worker from a connected (unconnected) island steals cash from shoppers from a connected (unconnected) island. Thus, traders and nontraders face a spatially separated risk of holding money.\(^4\) A worker does not steal his own shopper’s money.

As in Choi (2010a), the yield from effort in stealing, $\phi(n_t^s)$, is

$$\phi(n_t^s) = \begin{cases} \pi n_t^s, & \text{if } n_t^s \in [0, \frac{1}{\pi}] \\ 1, & \text{if } n_t^s \in (\frac{1}{\pi}, 1] \end{cases} \quad (2)$$

where $\pi \in (1, \infty)$ is the degree of stealing efficiency. A worker can steal all of the other shopper’s cash by devoting more than $1/\pi$ unit of time. However, the marginal return on stealing decreases with $n_t^s$ and $\pi > 1$ implies a worker would not steal everything given one unit of time endowment and a linear stealing technology.\(^5\) The stolen money, $\phi(n_t^s)M_t$, cannot be spent within the period, but can be used for credit payoffs at the end of the period, where $M_t$ denotes the quantity of money held by other shoppers.\(^6\)

After a worker steals money, a worker and a shopper exchange consumption goods at the goods market. A worker produces consumption goods with a linear technology, $y_t = n_t^w$, and the production technology is identical for any market $i$. A shopper travels from market to market purchasing consumption good $i$.

When purchasing goods, a shopper can use either non-interest bearing currency or credit as in Choi (2010a). First, the opportunity costs of money are the gross nominal interest rate and the risk of holding money. They increase with both the number of markets and the amount of consumption good purchased at market $i$ if a shopper uses cash. Next, the use of credit, which is within-period IOUs, incurs transactions costs, $\gamma(i)$, as

$$\gamma(i) = \ln \left( \frac{1}{1-i} \right) \quad (3)$$

\(^4\)The risk of holding cash differs geographically from one area to another. Especially, according to Alvarez and Lippi (2009), in Italy there is large variation in the probability of cash being stolen across regions.

\(^5\)Neither explicit punishment nor the failure rate of stealing is additionally assumed other than the decreasing marginal rate of return on stealing in order to make the model tractable for the study of the choice of credit and cash and its distributional effect of monetary policy.

\(^6\)Note that nominal bonds are safe from theft because a shopper brings only cash in the goods market to purchase consumption goods.
where $\gamma(0) = 0$ and $\lim_{i \to 1} \gamma(i) = \infty$. They increase with the number of market $i$ if a shopper uses credit.\footnote{Unlike cash, once a shopper pays the transactions costs to purchase consumption good $i$ at market $i$, the amount of consumption good $i$ purchased does not affect the costs.} Further, they infinitely increases if credit is used for every market $i$. As in Ireland (1994) and Alvarez and Lippi (2009), there is no a default risk of credit. Assuming perfect memory in credit transactions and enforcement of sellers, the credit users will payoff their debt every period. Those who default their payoffs cannot use credit or issue IOUs anymore in the future. The transactions costs appear in the form of effort as disutility,

$$x_t(i) = \int_0^1 \xi_t(i) \gamma(i) di,$$

where $\xi_t(i)$ is an indicator variable: $\xi_t(i) = 1$ if a shopper decides to use credit in market $i$ and $\xi_t(i) = 0$ if a shopper decides cash in market $i$.

The cash-in-advance constraints of traders and nontraders are

$$\int_0^1 P_t(i)(1 - \xi_{r,t}(i)) c_{r,t}(i) di \leq (1 - \pi \bar{n}_{r,t}) (M_{r,t} + B_t - q_t B_{t+1}),$$

$$\int_0^1 P_t(i)(1 - \xi_{n,t}(i)) c_{n,t}(i) di \leq (1 - \pi \bar{n}_{n,t}) M_{n,t},$$

where $c_{r,t}(i)$ is consumption of good $i$ purchased by a trader, $\bar{n}_{r,t}$ is the risk of loss for traders, $c_{n,t}(i)$ is consumption good $i$ purchased by a nontrader, $\bar{n}_{n,t}$ is the risk of loss for nontraders, and $P_t(i)$ is the price of consumption good $i$.

At the end of period $t$, traders and nontraders return to their islands and workers receive payoffs. No further trade or barter is allowed. The budget constraints of traders and nontraders are

$$\int_0^1 P_t(i)c_{r,t}(i)di + M_{r,t+1} = (1 - \pi \bar{n}_{r,t}) (M_{r,t} + B_t - q_t B_{t+1}) + P_t (1 - n_{r,t}) + \pi n_{r,t} M_{r,t},$$

$$\int_0^1 P_t(i)c_{n,t}(i)di + M_{n,t+1} = (1 - \pi \bar{n}_{n,t}) M_{n,t} + P_t (1 - n_{n,t}) + \pi n_{n,t} M_{n,t},$$

where $P_t$ is the average price level of consumption goods, $M_{r,t+1}$ is trader’s money stock for the next period, $M_{n,t+1}$ is nontrader’s money stock, $P_t n_{r,t}^w$ is trader’s income, and $P_t n_{n,t}^w$ is nontrader’s.

### 3 Equilibrium Dynamics

**Definition:** A symmetric competitive equilibrium consists of the sequences \( \{c_{j,t}(i), n_{j,t}^w, n_{j,t}^s, \xi_{j,t}, M_{j,t+1}, B_{t+1}, M_t^s\}_{t=0}^{\infty} \) and \( \{P_t(i), q_t\}_{t=0}^{\infty} \) where $i \in [0,1]$ and $j \in \{r, n\}$ which satisfy
1. \( \{c_{j,t}(i), n_{j,t}^w, n_{j,t}^s, \xi_{j,t}, M_{j,t+1}, B_{t+1}\}_{t=0}^{\infty} \) solves the household problems of traders and nontraders\(^8\) given \( \{\bar{n}_{j,t}^s, M_t^s, P_t(i), q_t\}_{t=0}^{\infty} \) for all market \( i \).

2. Markets clear in every period:

   (a) Bond Market: each trader exchanges money and bonds such as
   \[
   B_t - q_t B_{t+1} = \frac{\mu_t M_t}{\alpha}.
   \]

   (b) Money Market: for all \( t \),
   \[
   M_{t+1}^s = M_{t+1} = \alpha M_{r,t+1} + (1 - \alpha) M_{n,t+1}
   = \alpha M_{r,t} + (1 - \alpha) M_{n,t} + \alpha \left( \frac{\mu_t M_t}{\alpha} \right)
   = (1 + \mu_t) M_t,
   \]
   \[
   \bar{M}_{r,t} = M_{r,t} + \frac{\mu_t M_t}{\alpha},
   \]
   \[
   \bar{M}_{n,t} = M_{n,t}.
   \]

   (c) Goods Market: for each market \( i \),
   \[
   \alpha c_{r,t}(i) + (1 - \alpha) c_{n,t}(i) = 1 - \alpha n_{r,t}^s - (1 - \alpha) n_{n,t}^s.
   \]

3. By symmetry, \( n_{j,t}^s = \bar{n}_{j,t}^s \).

Suppose \( \lambda_{j,t}^1 \) and \( \lambda_{j,t}^2 \), where \( j \in \{ r, n \} \), denote the Lagrange multipliers associated with the cash-in-advance constraint and the budget constraint respectively for traders and nontraders. Then, the equilibrium choices of traders and nontraders for \( c_{j,t}(i), \xi_{j,t}, n_{j,t}^s, M_{j,t+1}, \) and \( B_{t+1} \) are as follows:

\[
\frac{1}{c_{j,t}(i)} - \lambda_{j,t}^1 (1 - \xi_{j,t}(i)) P_t - \lambda_{j,t}^2 P_t = 0, \tag{9}
\]

\[
\frac{1}{c_{j,t}(i)} - \lambda_{j,t}^2 P_t = 0 \quad \text{if} \ \xi_{j,t}(i) = 1, \tag{10}
\]

\[
\frac{1}{c_{j,t}(i)} - (\lambda_{j,t}^1 + \lambda_{j,t}^2) P_t = 0 \quad \text{if} \ \xi_{j,t}(i) = 0, \tag{11}
\]

\(^8\)The household optimization problem is in appendix A.
\[ \xi_{j,t}(i) = \begin{cases} 1, & \text{if } \ln(c_{1,j,t}^1(i)) - \ln \left( \frac{1}{1-i_{j,t}} \right) - \lambda_{j,t}^2 c_{1,j,t}^1(i) P_t > \ln(c_{0,j,t}^0(i)) - c_{0,j,t}^0(i)(\lambda_{j,t}^1 + \lambda_{j,t}^2) P_t, \\ 0, & \text{if } \ln(c_{1,j,t}^1(i)) - \ln \left( \frac{1}{1-i_{j,t}} \right) - \lambda_{j,t}^2 c_{1,j,t}^1(i) P_t < \ln(c_{0,j,t}^0(i)) - c_{0,j,t}^0(i)(\lambda_{j,t}^1 + \lambda_{j,t}^2) P_t. \end{cases} \]  

(12)

\[ (-P_t + \pi M_{j,t}) \lambda_{j,t}^2 = 0; \]  

(13)

\[ \beta E_t \left[ (\lambda_{j,t+1}^1 + \lambda_{j,t+1}^2) (1 - \pi n_{j,t+1}^s) \mid \mu_t \right] = \lambda_{j,t}^2; \]  

(14)

\[ \beta E_t \left[ (\lambda_{r,t+1}^1 + \lambda_{r,t+1}^2) (1 - \pi n_{r,t+1}^s) \mid \mu_t \right] = q_t (\lambda_{r,t}^1 + \lambda_{r,t}^2) (1 - \pi n_{r,t}^s), \]  

(15)

assuming the cash-in-advance constraints of traders and nontraders bind, \( \lambda_{j,t}^1 > 0 \): in equations (11) and (14),

\[ \beta E_t \left[ \frac{c_{0,j,t}^0}{c_{0,j,t+1}} \left( \frac{P_t}{P_{t+1}} \right) (1 - \pi n_{j,t+1}^s) \right] < 1. \]

**Proposition 1.** In equilibrium, \( P_t(i) = P_t \) holds for all market \( i \).

**Proof.** See appendix B.

**Lemma 1.** In equation (10), consumption with credit for traders and for non-traders are identical, \( c_{1,j,t}^1 = c_{1,j,t}^1(i) \), since the marginal value of wealth is the same across markets. In equation (11), consumption with cash is identical, \( c_{0,j,t}^0 = c_{0,j,t}^0(i) \), since the marginal value of cash is the same across markets.

**Proposition 2.** In equations (10)-(12), the cutoff of the credit-cash choice, \( i_{j,t}^* \in (0, 1) \), is determined when transactions costs equal to the marginal rate of substitution of \( c_{0,j,t}^0 \) for \( c_{1,j,t}^1 \),

\[ \frac{1}{1 - i_{j,t}^*} = \frac{c_{1,j,t}^1}{c_{0,j,t}^0} > 1. \]  

(16)
Money and credit coexist as in Ireland (1994) and Choi (2010a, 2010b). A shopper uses credit to acquire good \( i \) for \( i < i_{j,t}^* \) and cash to acquire good \( i \) for \( i > i_{j,t}^* \). Credit is used for larger purchases while cash is for smaller ones.

In propositions 1 and 2, the cash-in-advance constraints of traders and nontraders in equations (5) and (6) are

\[
P_t (1 - i_{r,t}^*) c_{r,t}^0 = (1 - \pi n_{r,t}^*) \left( M_{r,t} + \frac{\mu_t M_t}{\alpha} \right),
\]

(17)

\[
P_t (1 - i_{n,t}^*) c_{n,t}^0 = (1 - \pi n_{n,t}^*) M_{n,t}.
\]

(18)

Now, in equation (13), for \( j \in \{ r, n \} \), theft on traders and nontraders are determined when real money holding of other shoppers equals to the inverse of the marginal benefit of stealing, i.e. the degree of stealing efficiency,

\[
\frac{\bar{M}_{j,t}}{P_t} = \frac{1}{\pi}.
\]

(19)

In equations (17) and (18), money is nonneutral and monetary policy redistributes real money holdings between traders and nontraders. If the government injects money, then, traders initially receive the money injection, \( \mu_t M_t/\alpha \), in the asset market while nontraders do not. The price level increases in the goods market. Although inflation arises, the real money holding of traders, \( M_{r,t} + \mu_t M_t/\alpha \), increases and that of nontraders, \( M_{n,t} \), decreases.

Next, in order to control cash holdings, traders and nontraders either work more for greater revenue or steal more cash. In equation (19), for traders, the money injection increases their real money holding, \( \bar{M}_{r,t} = M_{r,t} + \mu_t M_t/\alpha \), and the marginal benefit of working becomes greater than the marginal benefit of stealing, \( \bar{M}_{r,t}/P_t > 1/\pi \). Traders steal less and work more. On the other hand, the real money holding of nontraders, \( \bar{M}_{n,t}/P_t \), decreases and the marginal benefit of stealing, \( \bar{M}_{n,t}/P_t < 1/\pi \), is greater. Nontraders steal more and work less.

In equations (7) and (8), the budget constraint is

\[
P_t i_{j,t}^* c_{j,t}^1 + M_{j,t+1} = P_t \left( 1 - n_{j,t}^s \right) + \pi n_{j,t}^s \bar{M}_{j,t}.
\]

(20)

Aggregate consumption with credit, \( i_{j,t}^* c_{j,t}^1 \), and the money stock for the next period, \( M_{j,t+1} \), depend on the revenue and the money stock from theft. Theft would distort the intertemporal marginal rate of substitution between \( i_{j,t}^* c_{j,t}^1 \) and aggregate consumption with cash in period \( t+1 \), \( (1 - i_{j,t+1}^*) c_{j,t+1}^0 \), through \( M_{j,t+1} \).

In equations (19) and (20), the sum of aggregate consumption with credit and real money holding is constant,

\[
i_{j,t}^* c_{j,t}^1 + \frac{M_{j,t+1}}{P_t} = 1,
\]

(21)
and suppose $\theta_{j,t+1} \in (0, 1)$ defines real money holding which is the fraction of the revenue. Then, in equation (21), real money holding and aggregate consumption with credit are

$$\frac{M_{j,t+1}}{P_t} = \theta_{j,t+1},$$  \hspace{1cm} (22)

$$i^*_j c^1_j = 1 - \theta_{j,t+1},$$  \hspace{1cm} (23)

where in equations (19) and (22), aggregate real money holding is

$$\frac{M_{t+1}}{P_t} = \alpha \theta_{r,t+1} + (1 - \alpha) \theta_{n,t+1} = \frac{1}{\pi}$$  \hspace{1cm} (24)

and the inflation rate is

$$\frac{P_t}{P_{t-1}} = 1 + \mu_t.$$  \hspace{1cm} (25)

In equations (16) - (25), aggregate consumption with cash for traders can be expressed in two ways. One is from equations (17) and (19) depending only on the trader’s risk of loss and the other is from equations (17), (22), (24), and (25) depending on theft, the money growth rate, and the real money holding:

$$(1 - i^*_r)c^0_r = \frac{1 - \pi n^s_r}{\pi}$$  \hspace{1cm} (26)

$$= (1 - \pi n^s_r) \left( \frac{\theta_{r,t} + \mu_t/\alpha \pi}{1 + \mu_t} \right).$$  \hspace{1cm} (27)

**Proposition 3.** In equations (11), (15), and (25), the nominal interest rate is determined by

$$q_t (1 - \pi n^s_r) = \beta E_t \left[ \left( \frac{c^0_r}{c^0_{r,t+1}} \right) \left( \frac{1 - \pi n^s_{r,t+1}}{1 + \mu_{t+1}} \right) \right] < 1.$$  \hspace{1cm} (28)

As in He, Huang, and Wright (2005, 2008) and Choi (2010a), given a positive risk of loss for traders, traders are willing to hold nominal bonds with a negative return on money, $q_t > 1$, for safekeeping purposes. In equation (28), a liquidity effect exists along with the Fisher effect because consumption with cash, $c^0_r$, increases with the money growth rate, which decreases the nominal interest rate.
Next, aggregate consumption with cash for nontraders is from equations (18) and (19) and from equations (18), (22), (24), and (25):

\[(1 - i_{n,t}^*)c_{n,t}^0 = \frac{1 - \pi n_{n,t}^s}{\pi}
= (1 - \pi n_{n,t}^s) \left( \frac{\theta_{n,t}}{1 + \mu_t} \right).
\]

Finally, the choice of credit and cash and real money holding are determined by equations (16), (23), (26), and (29),

\[\frac{i_{j,t}^*}{(1 - i_{j,t}^*)^2} = \frac{\pi (1 - \theta_{j,t+1})}{1 - \pi n_{j,t}^s},\]

and by equations (10), (11), and (14),

\[\frac{i_{j,t}^*}{1 - \theta_{j,t+1}} = \beta E_t \left[ \left( \frac{1}{1 + \mu_{j+1}} \right) \left( \frac{1 - \pi n_{j,t+1}^s}{c_{j,t+1}^0} \right) \right].\]

In equations (26) - (32), monetary policy would have three distributional effects on traders and nontraders as in Figure 1. One is the direct effect through real money holding. The others are the indirect effects; the effect via the choice of credit and cash and the effect via the change in theft. In particular, the stealing effect represents the negative income effect since theft suppresses production.

4 Constant Money Growth

This section will study the implications of the constant money growth. It will be useful to compare the distributional effects of monetary policy with those when the money growth rate is stochastic. Suppose \(\mu_t = \mu\) for all \(t\). Then, monetary policy is anticipated and the economy becomes deterministic for all periods.

4.1 Distributional Effects on Traders

In equations (22) - (27), (31), and (32), the traders choices for real money holding, \(\theta_r\), theft, \(n_{r,t}^s\), the choice of credit and cash, \(i_{r,t}^*\), and aggregate consumption with credit, \(i_{r,t}^*c_{r,t}^1\), and with cash, \((1 - i_{r,t}^*)c_{r,t}^0\), are\(^9\)

\[\theta_r = \frac{1}{\pi} - \left( \frac{1 - \alpha}{\alpha} \right) \frac{\mu}{\pi}.
\]

\(^9\)The derivation is in appendix C.
Figure 1: The Distributional Effects of Monetary Policy

Monetary Policy (Change \( \mu \))

\[ \text{Change } n^r_{rt} \text{ & } n^s_{nt} \]

Stealing Effect

Change \( \theta_{rt} \text{ & } \theta_{nt} \)

Direct Effect

Change \( C^1_{rt} \text{ & } C^1_{nt} \)

Choice Effect

Change \( i^s_{rt} \text{ & } i^s_{nt} \)

Change \( C^0_{rt} \text{ & } C^0_{nt} \)

Change \( C^0_{rt+1} \text{ & } C^0_{nt+1} \)
\( n^*_r = \frac{1}{\pi} - \frac{(1 + \mu)^2}{\beta \{1 + \beta(\pi - 1) + [1 + \beta \left(\frac{1-\alpha}{\alpha}\right)] \mu\}}, \)  

(34)

\( i^*_r = \frac{\beta(\pi - 1) + \beta \left(\frac{1-\alpha}{\alpha}\right) \mu}{1 + \beta(\pi - 1) + [1 + \beta \left(\frac{1-\alpha}{\alpha}\right)] \mu}, \)  

(35)

\( i^*_r c^1_r = 1 - \frac{1}{\pi} + \left(\frac{1 - \alpha}{\alpha}\right) \frac{\mu}{\pi}, \)  

(36)

\( (1 - i^*_r) c^0_r = \frac{(1 + \mu)^2}{\beta \{1 + \beta(\pi - 1) + [1 + \beta \left(\frac{1-\alpha}{\alpha}\right)] \mu\}}. \)  

(37)

Suppose the government increases the money growth rate permanently and the degree of stealing efficiency is greater than the ratio of nontraders to traders, \( \pi - 1 > \frac{1 - \alpha}{\alpha} \). Then, traders reduce real money holding because the money injection is permanent and they receive additional cash in the asset market every period. They steal less and the money injection results in a positive income effect unlike Choi (2010b). Because theft decreases, traders use cash for a greater variety of goods although the money injection increases nominal interest.

**Proposition 4.** Unlike Choi (2010b), in equations (33) - (37), a money injection decreases real money holding and theft. The choice of credit and cash decreases. Aggregate consumption with credit and with cash increase with the money growth rate.

*Proof.* See appendix D. \( \Box \)

**Corollary 1.** Due to a positive income effect, consumption with credit, \( c^1_r \), and with cash, \( c^0_r \), increases with the money growth rate.

*Proof.* See appendix E. \( \Box \)

**Proposition 5.** In equation (28), a permanent change in the money growth rate results in the Fisherian effect without the liquidity effect, \( q = \frac{\beta}{1 + \mu} < 1 \).
4.2 Distributional Effects on Nontraders

In equations (22) - (25), (29), (30), (31), and (32), the nontraders choices for real money holding, $\theta_n$, theft, $n_n^*$, the choice of credit and cash, $i_n^*$, and aggregate consumption with credit, $i_n^*c_n^1$, and with cash, $(1 - i_n^*)c_n^0$, are\(^\text{10}\)

$$\theta_n = \frac{1 + \mu}{\pi}$$ \hfill (38)

$$n_n^* = \frac{1}{\pi} - \left(\frac{1 + \mu}{\beta \pi}\right) \left[ (1 - \beta) + \frac{\beta (1 + \mu)}{\pi} \right],$$ \hfill (39)

$$i_n^* = \frac{\beta (\pi - 1 - \mu)}{\pi},$$ \hfill (40)

$$i_n^* c_n^1 = \frac{\pi - 1 - \mu}{\pi},$$ \hfill (41)

$$(1 - i_n^*)c_n^0 = \left(\frac{1 + \mu}{\beta \pi}\right) \left[ (1 - \beta) + \frac{\beta (1 + \mu)}{\pi} \right].$$ \hfill (42)

When the money injection is permanent, nontraders do not receive the money injection in the first place whereas the price level increases. In equation (19), they need to increase their money holding permanently as well. Thus, first they increase real money holding and steal less unlike Choi (2010b) because the marginal benefit of working is greater than the marginal benefit of stealing. There is a positive income effect. Cash is used for a greater variety of goods.

**Proposition 6.** Unlike Choi (2010b), in equations (38) - (42), a money injection increases real money holding and decreases theft. The choice of credit and cash decreases. Aggregate consumption with credit decreases with the money growth rate, but aggregate consumption with cash increases.

*Proof.* See appendix G. \hfill \Box

**Corollary 2.** Due to a positive income effect, consumption with credit, $c_n^1$, and with cash, $c_n^1$, increase with the money growth rate.

*Proof.* See appendix H. \hfill \Box

\(^{10}\)The derivation is in appendix F.
4.3 Optimal Money Growth and Welfare

In propositions 4 and 6, theft decreases with the money growth rate and output increases. Thus, the money injection has positive effects on production for both traders and nontraders.

**Proposition 7.** Both welfare of traders and nontraders increase with the money growth rate. As inflation gets higher, the marginal welfare improvement decreases for traders, but it is constant, $\beta$, for nontraders. The optimal money growth rate is infinity where theft is driven to zero. The Friedman rule does not hold.

*Proof.* See appendix I.

In the coexistence of credit and cash with theft, inflation always improves the welfare of traders and nontraders until there occurs no theft as in Figure 2. It is a very interesting result in that inflation does not tax nontraders even in a high inflation unlike Choi (2010b) and there is no welfare cost of inflation. As of He, Huang, and Wright (2005, 2008), the Friedman rule is not optimal.

5 Stochastic Money Growth

Now, suppose the money growth rate, $\mu_t$, is independent and identically distributed and monetary policy is not anticipated anymore. Then, traders and nontraders can demand cash for precautionary purposes. Next, unlike the constant money growth rate, the effects on theft of traders and nontraders are asymmetric. Further, a change in theft also affects the intertemporal marginal rate of substitution between $i^*_j, c^1_{jt}$ and $(1 - i^*_j, c^0_{jt+1})$ as discussed in equation (20).

5.1 Distributional Effects on Traders

In equations (22) - (27), (31), and (32), the traders choices are\(^\text{11}\)

\[
\theta_{r,t+1} = \left[ \frac{1 - \left( \frac{1 - \alpha}{\alpha} \right) E_t [\mu_{t+1}]}{1 - \left( \frac{1 - \alpha}{\alpha} \right) \mu_t} \right] \theta_{r,t}, \tag{43}
\]

\[
i^*_{r,t} = \beta \Psi_r \left\{ 1 - \left[ \frac{1 - \left( \frac{1 - \alpha}{\alpha} \right) E_t [\mu_{t+1}]}{1 - \left( \frac{1 - \alpha}{\alpha} \right) \mu_t} \right] \theta_{r,t} \right\}, \tag{44}
\]

\(^{11}\)The derivation is in appendix J.1 and J.2.
Figure 2: Welfare of traders and nontraders
\[
n_{r,t}^s = \frac{1}{\pi} - \frac{1}{\beta \Psi_r} \left\{ 1 - \beta \Psi_r + \beta \Psi_r \left[ \frac{1 - \left( \frac{1-\alpha}{\alpha} \right) E_t [\mu_{t+1}]}{1 - \left( \frac{1-\alpha}{\alpha} \right) \mu_t} \right] \theta_{r,t} \right\}^2, \tag{45}
\]

\[
i_{r,t}^* c_{r,t}^1 = 1 - \left[ \frac{1 - \left( \frac{1-\alpha}{\alpha} \right) E_t [\mu_{t+1}]}{1 - \left( \frac{1-\alpha}{\alpha} \right) \mu_t} \right] \theta_{r,t}, \tag{46}
\]

\[
(1 - i_{r,t}^*) c_{r,t}^0 = \frac{1}{\beta \Psi_r} \left\{ 1 - \beta \Psi_r + \beta \Psi_r \left[ \frac{1 - \left( \frac{1-\alpha}{\alpha} \right) E_t [\mu_{t+1}]}{1 - \left( \frac{1-\alpha}{\alpha} \right) \mu_t} \right] \theta_{r,t} \right\}^2, \tag{47}
\]

where

\[
\Psi_r = E_t \left[ \left( \frac{1}{1 + \mu_{t+1}} \right) \frac{1 - \pi n_{r,t+1}^s}{c_{r,t+1}^0} \right].
\]

Suppose there is an unanticipated money injection into the asset market and the expected money growth rate is smaller than the ratio of traders to nontraders,

\[
E_t [\mu_{t+1}] < \frac{\alpha}{1 - \alpha}.
\]

Then, although traders receive the money injection in the asset market, they increase their real money holding for precautionary purposes. In other words, the precautionary demand for money implies a lower amount of cash for current credit payoffs. In order to reserve aggregate consumption with credit, traders work more and steal less as in equation (19). Due to the money injection and lower theft, cash is used for a greater variety of goods.

**Proposition 8.** In equations (43) - (47), the money injection increases real money holding and decreases theft. The choice of credit and cash, \(i_{r,t}^*\), decreases as of Choi (2010b). Aggregate consumption with credit decreases with the money growth rate and that with cash increases.

*Proof.* See appendix K.

**Corollary 3.** Consumption with credit, \(c_{r,t}^1\), does not change with the money growth rate and consumption with cash, \(c_{r,t}^0\), increases.

*Proof.* See appendix L.
In proposition 8 and corollary 3, the effects of monetary policy on \(c_{r,t}^1\) in equation (46) consist of the direct and choice effects.\(^{12}\) Holding less cash for credit payments due to a greater real money holding decreases \(c_{r,t}^1\), the direct effect. However, spending on credit for less variety of goods increases \(c_{r,t}^1\), the choice effect. The net effect would be ambiguous in general. In this model, the two effects cancel out and \(c_{r,t}^1\) does not change. Next, in equation (47), \(c_{r,t}^0\) is affected by the stealing and choice effect.\(^{13}\) Due to lower theft, there is a positive income effect and \(c_{r,t}^0\) increases.

### 5.2 Distributional Effects on Nontraders

In equations (22) - (25), (29), (30), (31), and (32), the nontraders choices are\(^{14}\)

\[
\theta_{n,t+1} = \left[ \frac{1 + E_t[\mu_{t+1}]}{1 + \mu_t} \right] \theta_{n,t}, \tag{48}
\]

\[
i_{n,t}^* = \beta \Psi_n \left\{ 1 - \left[ \frac{1 + E_t[\mu_{t+1}]}{1 + \mu_t} \right] \theta_{n,t} \right\}, \tag{49}
\]

\[
n_{n,t}^s = \frac{1}{\pi} - \frac{1}{\beta \Psi_n} \left\{ 1 - \beta \Psi_n + \beta \Psi_n \left[ \frac{1 + E_t[\mu_{t+1}]}{1 + \mu_t} \right] \theta_{n,t} \right\}^2, \tag{50}
\]

\[
i_{n,t}^*c_{n,t}^1 = 1 - \left[ \frac{1 + E_t[\mu_{t+1}]}{1 + \mu_t} \right] \theta_{n,t}, \tag{51}
\]

\[
(1 - i_{n,t}^*)c_{n,t}^0 = \frac{1}{\beta \Psi_n} \left\{ 1 - \beta \Psi_n + \beta \Psi_n \left[ \frac{1 + E_t[\mu_{t+1}]}{1 + \mu_t} \right] \theta_{n,t} \right\}^2, \tag{52}
\]

where

\[
\Psi_n = E_t \left[ \left( \frac{1}{1 + \mu_{t+1}} \right) \frac{1 - \pi n_{n,t+1}^s}{c_{n,t+1}^0} \right].
\]

Suppose the expected money growth rate is greater than a negative one,

\[
E_t[\mu_{t+1}] < -1.
\]

---

\(^{12}\)See appendix L.

\(^{13}\)See appendix L.

\(^{14}\)The derivation is in appendix J.1 and J.3.
Then, because nontraders do not receive the money injection in the asset market, their real money holding decreases\(^{15}\) and they need more cash in order to compensate consumption loss due to inflation. Nontraders steal more and work less as in equation (19). Further, due to lower real money holding and higher theft, credit is used for a greater variety of goods.

**Proposition 9.** In equations \((48) - (52)\), the money injection decreases real money holding and increases theft. The choice of credit and cash, \(i_{n,t}^*\), increases as of Choi (2010b). Aggregate consumption with credit increases with the money growth rate, but aggregate consumption with cash decreases.

*Proof.* See appendix M. \(\square\)

**Corollary 4.** Consumption with credit, \(c_{n,t}^1\), does not change with the money growth rate and consumption with cash, \(c_{n,t}^0\), decreases.

*Proof.* See appendix N. \(\square\)

In proposition 9 and corollary 4, the effects on \(c_{n,t}^1\) in equation (51) consist of the direct and choice effects.\(^{16}\) Increasing real money holding decreases \(c_{n,t}^1\), the direct effect. However, spending on credit for less variety of goods increases \(c_{n,t}^1\), the choice effect. The net effect would be ambiguous in general. In this model, the two effects cancel out and \(c_{n,t}^1\) does not change. Next, in equation (52), due to higher theft, inflation taxes nontraders. There is a negative income effect and \(c_{n,t}^0\) decreases.\(^{17}\)

### 5.3 Optimal Money Growth and Welfare

Unlike the case of constant money growth, in propositions 8 and 9, traders steal less and nontraders steal more given the unanticipated money injection. In other words, inflation increases traders production and suppresses nontraders.

**Proposition 10.** Welfare of traders increases with the money growth rate while that of nontraders decreases. The optimal money growth rate, \(\mu_t^*\), may be positive or negative to minimize theft. The Friedman rule is not optimal in general,

\[
\mu_t^* = \frac{-1 \pm A^\frac{1}{2}}{1 \pm \left(\frac{1-\alpha}{\alpha}\right) A^\frac{1}{2}},
\]

\(^{15}\)Nontraders would demand more money for precautionary purposes if the government extracts money.

\(^{16}\)See the appendix N.

\(^{17}\)See the appendix N.
where
\[ A = \frac{A_{n,t}}{A_{r,t}} \left[ 1 + E_t[\mu_{t+1}] \right] \left[ 1 - \left( \frac{1 - \alpha}{\alpha} \right) E_t[\mu_{t+1}] \right]. \]

**Proof.** See appendix O.

In equation (53), the optimal money growth rate is positive if \( A > 1 \) and \((1 - \alpha)A^{1/2}/\alpha < 1\). Unlike the constant money growth, inflation taxes nontraders. Welfare may increase or decrease with the stochastic money growth rate. The optimal money growth rate will minimize theft, but theft does not disappear. The Friedman rule is not optimal as of He, Huang, and Wright (2005, 2008) and Choi (2010b).

### 6 Conclusion

An asset market segmentation model is constructed to study the choice of credit, which is costly to use, and money, which is risky to hold due to theft, and to explore the asymmetric distributional effects of monetary policy on the choice of credit and cash and welfare across economic individuals.

In equilibrium, theft distorts the intertemporal marginal rate of substitution between consumption with credit and with cash. Money is nonneutral and monetary policy has the distributional effects. With anticipated monetary policy inflation decreases theft and always improves welfare of all individuals. However, with unanticipated monetary policy, there is a precautionary demand for money. Inflation may decrease theft and improve welfare for some individuals. The optimal monetary policy is to minimize theft and the Friedman rule is suboptimal in general.

### Appendix

#### A The Household Optimization

Given equations (1) - (7), traders solve the following optimization problem,

\[
\max_{\{c_{r,t,n_{r,t},\xi_{r,t},M_{r,t+1},B_{t+1}}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln(c_{r,t}(i)) \, di - \int_0^1 \xi_{r,t}(i) \ln \left( \frac{1}{1 - i} \right) \, di \right\}
\]

subject to

\[
\int_0^1 P_t(i)(1 - \xi_{r,t}(i))c_{r,t}(i) \, di \leq (1 - \pi \bar{n}_{t+1})(M_{r,t} + B_t - q_t B_{t+1})
\]
\[
\int_0^1 P_t(i)c_{r,t}(i)di + M_{r,t+1} = (1 - \pi \bar{n}_{r,t})(M_{r,t} + B_t - q_i B_{t+1}) + P_t \left(1 - n_{r,t}^s\right) + \pi n_{r,t}^s \bar{M}_{r,t}
\]

where inequality (54) is the nonnegativity constraints and the no Ponzi-scheme constraint. Now, given equations (1) - (4), (6), and (8), nontraders solve the optimization problem,

\[
\max_{\{c_{n,t}, n_{n,t}, \xi_{n,t}, M_{n,t+1}\}} \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln(c_{n,t}(i))di - \int_0^1 \xi_t(i)\ln\left(\frac{1}{1-i}\right)di \right\}
\]

subject to

\[
\int_0^1 P_t(i)(1 - \xi_{n,t}(i))c_{n,t}(i)di \leq (1 - \pi \bar{n}_{n,t}^s)M_{n,t}
\]

\[
\int_0^1 P_t(i)c_{n,t}(i)di + M_{n,t+1} = (1 - \pi \bar{n}_{n,t}^s)M_{n,t} + P_t \left(1 - n_{n,t}^s\right) + \pi n_{n,t}^s \bar{M}_{n,t}
\]

where inequality (55) is the nonnegativity constraints.

**B  Proof of** \(P_t(i) = P_t\)

Assume that every worker has same technology to produce good \(i\) for every market \(i\) and that perfect competition prevails in all goods markets as in Ireland (1994) and Choi(2010a, 2010b). Then, in equation (9), the marginal utility of consumption is identical: \(c_{j,t}(i)P_t(i) = c_{j,t}(k)P_t(k)\), for any market \(i, k \in [0, 1]\).

In equilibrium, the goods market clearing condition implies \(c_{j,t}(i) = n_{t}^w\) for all \(i\) and each market \(i\) sells consumption goods at the price of \(P_t(i) = P_t\).

In other words, when a shopper chooses either cash or credit to purchase goods, he bears the opportunity costs of cash and credit. In other words, for a worker, it is indifferent between accepting cash or credit since a worker does not take any of the opportunity costs of cash or credit.

**C  Derivation of Equations (33) - (37)**

First, given the constant money growth, equations (22), (23), (24), (26), (27), (31), and (32) imply

\[
\frac{M_r}{P} = \theta_r, \tag{56}
\]

21
\[ i_r^* c_1^r = 1 - \theta_r, \quad (57) \]

\[ (1 - i_r^*) c_0^r = \frac{1 - \pi n_r^s}{\pi} = (1 - \pi n_r^s) \left( \frac{\theta_r + \mu/\alpha \pi}{1 + \mu} \right), \quad (58) \]

\[ \frac{i_r^*}{(1 - i_r^*)^2} = \frac{\pi (1 - \theta_r)}{1 - \pi n_r^s}, \quad (59) \]

\[ \frac{i_r^*}{1 - \theta_r} = \left( \frac{\beta}{1 + \mu} \right) \left( \frac{1 - \pi n_r^s}{c_0^r} \right). \quad (60) \]

In equation (58), the real money holding of traders to the next period is

\[ \theta_r = \frac{1}{\pi} - \left( \frac{1 - \alpha}{\alpha} \right) \frac{\mu}{\pi}, \quad (61) \]

and equations (57) and (61) imply

\[ i_r^* c_1^r = 1 - \frac{1}{\pi} + \left( \frac{1 - \alpha}{\alpha} \right) \frac{\mu}{\pi}. \quad (62) \]

In equations (58), (60), and (61),

\[ \frac{i_r^*}{1 - i_r^*} = \frac{\beta \left[ \pi - 1 + \left( \frac{1 - \alpha}{\alpha} \right) \mu \right]}{1 + \mu}. \]

which implies

\[ i_r^* = \frac{\beta (\pi - 1) + \beta \left( \frac{1 - \alpha}{\alpha} \right) \mu}{1 + \beta (\pi - 1) + \left[ 1 + \beta \left( \frac{1 - \alpha}{\alpha} \right) \right] \mu}. \quad (63) \]

Now, equations (58) and (60) imply

\[ \frac{i_r^*}{1 - i_r^*} = \frac{\beta \pi (1 - \theta_r)}{1 + \mu}. \quad (64) \]

Thus, in equations (59), (63), and (64), theft on traders is

\[ n_r^s = \frac{1}{\pi} - \frac{(1 + \mu)^2}{\beta \left[ 1 + \beta (\pi - 1) + \left[ 1 + \beta \left( \frac{1 - \alpha}{\alpha} \right) \right] \mu \right]} \]

which implies

\[ (1 - i_r^*) c_0^r = \frac{(1 + \mu)^2}{\beta \left[ 1 + \beta (\pi - 1) + \left[ 1 + \beta \left( \frac{1 - \alpha}{\alpha} \right) \right] \mu \right]} . \]
D Proof of Proposition 4

In equations (33) - (37),

\[
\frac{\partial \theta_r}{\partial \mu} = -\frac{1 - \alpha}{\alpha\pi} < 0. \tag{65}
\]

\[
\frac{\partial n^*_r}{\partial \mu} = -\left(1 + \mu\right) \left\{1 + \beta \left[2(\pi - 1) - \left(\frac{1}{\alpha}\right)\right] + \left[1 + \beta \left(\frac{1}{\alpha}\right)\right] \mu\right\} \beta\pi \left\{1 + \beta(\pi - 1) + \left[1 + \beta \left(\frac{1}{\alpha}\right)\right] \mu\right\}^2 < 0, \tag{66}
\]

\[
\frac{\partial i^*_r}{\partial \mu} = \frac{\beta \left[\frac{1}{\alpha} - (\pi - 1)\right]}{1 + \beta(\pi - 1) + \left[1 + \beta \left(\frac{1}{\alpha}\right)\right] \mu} < 0, \tag{67}
\]

\[
\frac{\partial i^*_r c^1_r}{\partial \mu} = -\frac{\partial \theta_r}{\partial \mu} > 0, \tag{68}
\]

\[
\frac{\partial (1 - i^*_r)c^0_r}{\partial \mu} = -\frac{\partial n^*_r}{\partial \mu} > 0. \tag{69}
\]

E Proof of Corollary 1

The effect of monetary policy on consumption with credit is

\[
\frac{\partial c^1_r}{\partial \mu} = \left(\frac{1}{i^*_r}\right) \frac{\partial i^*_r c^1_r}{\partial \mu} - \left(\frac{c^1_r}{i^*_r}\right) \frac{\partial i^*_r}{\partial \mu} = -\left(\frac{1}{i^*_r}\right) \frac{\partial \theta_r}{\partial \mu} - \left(\frac{c^1_r}{i^*_r}\right) \frac{\partial i^*_r}{\partial \mu} > 0 \tag{70}
\]

Direct Effect Choice Effect

The effect of monetary policy on consumption with cash is

\[
\frac{\partial c^0_r}{\partial \mu} = \left(\frac{1}{1 - i^*_r}\right) \frac{\partial (1 - i^*_r)c^0_r}{\partial \mu} + \left(\frac{c^0_r}{1 - i^*_r}\right) \frac{\partial i^*_r}{\partial \mu} = -\left(\frac{1}{1 - i^*_r}\right) \frac{\partial n^*_r}{\partial \mu} + \left(\frac{c^0_r}{1 - i^*_r}\right) \frac{\partial i^*_r}{\partial \mu} = 1 > 0. \tag{71}
\]

Stealing Effect Choice Effect
F Derivation of Equations (38) - (42)

Given constant money growth, equations (22), (23), (24), (29), (30), (31), and (32) imply

\[ \frac{M_n}{P} = \theta_n, \]  
(72)

\[ i_n^* c_n^1 = 1 - \theta_n, \]  
(73)

\[ (1 - i_n^*)c_n^0 = \frac{1 - \pi n_n^s}{\pi} = (1 - \pi n_n^s) \left( \frac{\theta_n}{1 + \mu} \right), \]  
(74)

\[ \frac{i_n^*}{(1 - i_n^*)^2} = \frac{\pi (1 - \theta_n)}{1 - \pi n_n^s}, \]  
(75)

\[ \frac{i_n^*}{1 - \theta_n} = \left( \frac{\beta}{1 + \mu} \right) \left( \frac{1 - \pi n_n^s}{c_n^0} \right). \]  
(76)

In equation (74), the real money holding of nontraders to the next period is

\[ \theta_n = \frac{1 + \mu}{\pi}, \]  
(77)

and equations (73) and (77) imply

\[ i_n^* c_n^1 = \frac{\pi - 1 - \mu}{\pi}. \]  
(78)

In equations (74), (76), and (77),

\[ \frac{i_n^*}{1 - i_n^*} = \frac{\beta (\pi - 1 - \mu)}{1 + \mu}, \]  

which implies

\[ i_n^* = \frac{\beta (\pi - 1 - \mu)}{\pi}. \]  
(79)

Now, equations (74) and (76) imply

\[ \frac{i_n^*}{1 - i_n^*} = \frac{\beta \pi (1 - \theta_n)}{1 + \mu}. \]  
(80)
Thus, in equations (75), (79), and (80), theft on nontraders is

\[ n_n^* = \frac{1}{\pi} - \left( \frac{1 + \mu}{\beta \pi} \right) \left[ (1 - \beta) + \frac{\beta(1 + \mu)}{\pi} \right] \]

which implies

\[ (1 - i_n^*)c_n^0 = \left( \frac{1 + \mu}{\beta \pi} \right) \left[ (1 - \beta) + \frac{\beta(1 + \mu)}{\pi} \right]. \]

G Proof of Proposition 6

In equations (38) - (42),

\[ \frac{\partial \theta_n}{\partial \mu} = \frac{1}{\pi} > 0. \tag{81} \]

\[ \frac{\partial n_n^*}{\partial \mu} = - \left[ \frac{1 - \beta}{\beta \pi} + \frac{2(1 + \mu)}{\pi^2} \right] < 0, \tag{82} \]

\[ \frac{\partial i_n^*}{\partial \mu} = - \frac{\beta}{\pi} < 0, \tag{83} \]

\[ \frac{\partial i_n^* c_n^1}{\partial \mu} = - \frac{\partial \theta_n}{\partial \mu} < 0, \tag{84} \]

\[ \frac{\partial (1 - i_n^*)c_n^0}{\partial \mu} = - \frac{\partial n_n^*}{\partial \mu} > 0. \tag{85} \]

H Proof of Corollary 2

The effect of monetary policy on consumption with credit is

\[ \frac{\partial c_n^1}{\partial \mu} = \left( \frac{1}{i_n^*} \right) \frac{\partial i_n^* c_n^1}{\partial \mu} \left( i_n^* \right) \frac{\partial i_n^*}{\partial \mu} = - \left( \frac{1}{i_n^*} \right) \frac{\partial \theta_n}{\partial \mu} - \left( \frac{c_n^1}{i_n^*} \right) \frac{\partial i_n^*}{\partial \mu} \]

\[ = \frac{1}{\beta(\pi - 1 - \mu)} \left\{ \frac{1 - \alpha}{\alpha} + \left[ \frac{\beta(\pi - 1 - \mu)}{\pi} \right]^2 \right\} > 0. \tag{86} \]
The effect of monetary policy on consumption with cash is
\[
\frac{\partial c^0_n}{\partial \mu} = \left( \frac{1}{1-i^*_n} \right) \frac{\partial (1-i^*_n) c^0_n}{\partial \mu} + \left( \frac{c^0_n}{1-i^*_n} \right) \frac{\partial i^*_n}{\partial \mu} = -\left( \frac{1}{1-i^*_n} \right) \frac{\partial n^*_n}{\partial \mu} + \left( \frac{c^0_n}{1-i^*_n} \right) \frac{\partial i^*_n}{\partial \mu}
\]
Stealing Effect
Choice Effect
\[
= \left( \frac{1}{1-i^*_n} \right) \left[ \frac{1-\beta}{\pi \beta} + \frac{2(1+\pi)}{\pi^2} + \frac{\beta(\pi-1-\mu)(1+\mu)}{\pi^4} \right] > 0.
\] (87)

I Proof of Proposition 7

With equation (16), welfare can be defined as, for \( j \in \{r, n\} \),
\[
W_j = i^*_j ln \left( c^1_j \right) + (1 - i^*_j) ln \left( c^0_j \right) - \int_0^{i^*_j} \gamma (i) \, di
\]
\[
= ln \left( c^1_j \right) - (1 - i^*_j) ln \left( \frac{1}{1-i^*_j} \right) - \int_0^{i^*_j} ln \left( \frac{1}{1-i} \right) \, di
\]
\[
= ln \left( c^1_j \right) + (1 - i^*_j) ln \left( 1 - i^*_j \right) - (1 - i^*_j) ln \left( 1 - i^*_j \right) + (1 - i^*_j)
\]
\[
= ln \left( c^1_j \right) + (1 - i^*_j).
\]

Now, the effect of monetary policy on welfare is
\[
\frac{\partial W_j}{\partial \mu} = \frac{1}{c^1_j} \frac{\partial c^1_j}{\partial \mu} - \frac{\partial i^*_j}{\partial \mu} = \frac{1}{i^*_j c^1_j} \frac{\partial i^*_j c^1_j}{\partial \mu} - \left( \frac{1}{i^*_j} + 1 \right) \frac{\partial i^*_j}{\partial \mu},
\]
where
\[
\frac{\partial c^1_j}{\partial \mu} = \left( \frac{1}{i^*_j} \right) \frac{\partial i^*_j c^1_j}{\partial \mu} - \left( \frac{c^1_j}{i^*_j} \right) \frac{\partial i^*_j}{\partial \mu}.
\]

For traders, welfare is
\[
W_r = -ln(\pi \beta) + ln \left( 1 + \mu + \beta \left[ (\pi - 1) + \left( \frac{1-\alpha}{\alpha} \right) \mu \right] \right) + \frac{1+\mu}{1+\mu + \beta \left[ (\pi - 1) + \left( \frac{1-\alpha}{\alpha} \right) \mu \right]}
\] (88)
and in equations (35) and (36) and in inequalities (67) and (68), the effect of monetary policy on welfare is positive, given \( \pi - 1 > (1-\alpha)/\alpha \),
\[
\frac{\partial W_r}{\partial \mu} = \frac{1 + \beta \left( \frac{1-\alpha}{\alpha} \right)}{1 + \mu + \beta \left[ (\pi - 1) + \left( \frac{1-\alpha}{\alpha} \right) \mu \right] + \left[ 1 + \mu + \beta \left[ (\pi - 1) + \left( \frac{1-\alpha}{\alpha} \right) \mu \right] \right]^2} > 0,
\] (89)
and the marginal rate of welfare improvement decreases and gets to zero if the money growth rate is too high,

\[
\frac{\partial^2 W_r}{\partial \mu^2} = \frac{-1 - \frac{1-\alpha}{\alpha}}{\{1 + \mu + \beta [(\pi - 1) + \frac{1-\alpha}{\alpha} \mu]\}^2} - \frac{2 \{1 + \mu + \beta [(\pi - 1) + \frac{1-\alpha}{\alpha} \mu]\}^2 (1 + \frac{1-\alpha}{\alpha})}{\{1 + \mu + \beta [(\pi - 1) + \frac{1-\alpha}{\alpha} \mu]\}^4} < 0,
\]

(90)

\[
\left. \frac{\partial W_r}{\partial \mu} \right|_{\mu \to \infty} = 0.
\]

(91)

Next, for nontraders, welfare is

\[
W_n = -\ln(\beta) + 1 - \frac{\beta(\pi - 1 - \mu)}{\pi}
\]

(92)

and in equations (40) and (41) and in inequalities (83) and (84), the effect of monetary policy on welfare is also positive,

\[
\frac{\partial W_n}{\partial \mu} = \beta > 0.
\]

(93)

Overall, in inequalities (89) and (93), the optimal money growth rate of the economy is positive,

\[
\frac{\partial W}{\partial \mu} = \alpha \frac{\partial W_r}{\partial \mu} + (1 - \alpha) \frac{\partial W_n}{\partial \mu} > 0.
\]

(94)

**J Derivation of Equations (43) - (52)**

**J.1 Derivation of Equations (43) and (48)**

In equations (26) - (30), for all \( t \), the following relation holds

\[
\frac{\theta_{r,t}}{\theta_{n,t}} = \frac{1 - \frac{(1-\alpha)}{\alpha} \mu_t}{1 + \mu_t}
\]

(95)

where in equations (26) and (29),

\[
\frac{1}{\pi} = \frac{\theta_{r,t} + \frac{1}{\pi} \frac{(\mu_t)}{\alpha}}{1 + \mu_t}
\]
and in equations (27) and (30),
\[
\frac{1}{\pi} = \frac{\theta_{n,t}}{1 + \mu_t}.
\]
Now, in equation (24), for all \( t \),
\[
\alpha \theta_{r,t+1} + (1 - \alpha) \theta_{n,t+1} = \frac{1}{\pi}
\]
which implies
\[
\alpha \theta_{r,t+1} + (1 - \alpha) \theta_{n,t+1} = \alpha \theta_{r,t} + (1 - \alpha) \theta_{n,t}.
\] (96)

In equation (96), the following relations hold
\[
\theta_{r,t+1} \left[ \alpha + (1 - \alpha) \left( \frac{\theta_{n,t+1}}{\theta_{r,t+1}} \right) \right] = \theta_{r,t} \left[ \alpha + (1 - \alpha) \left( \frac{\theta_{n,t}}{\theta_{r,t}} \right) \right]
\] (97)
and
\[
\theta_{n,t+1} \left[ \alpha \left( \frac{\theta_{r,t+1}}{\theta_{n,t+1}} \right) + (1 - \alpha) \right] = \theta_{n,t} \left[ \alpha \left( \frac{\theta_{r,t}}{\theta_{n,t}} \right) + (1 - \alpha) \right].
\] (98)

Thus, in equations (95), (97), and (98), the real money holding of traders and nontraders in equations (43) and (48) is as follows:
\[
\theta_{r,t+1} = \left[ \frac{1 - \left( \frac{1 - \alpha}{\alpha} \right) E_t [\mu_{t+1}] }{1 - \left( \frac{1 - \alpha}{\alpha} \right) \mu_t} \right] \theta_{r,t}
\] (99)
and
\[
\theta_{n,t+1} = \left[ \frac{1 + E_t [\mu_{t+1}] }{1 + \mu_t} \right] \theta_{n,t}.
\] (100)

**J.2 Derivation of Equations (44) - (47)**

Given the real money holding of traders in equation (43), first, traders aggregate consumption with credit in equation (23) is
\[
i^*_{r,t} c^1_{r,t} = 1 - \theta_{r,t+1}.
\] (101)
Next, in equation (32), the traders choice of credit and cash is
\[
i^*_{r,t} = \beta \Psi_r (1 - \theta_{r,t+1}) \in (0, 1),
\] (102)
where
\[ \Psi_r = E_t \left[ \frac{1}{1 + \mu_{t+1}} \frac{1 - \pi n^s_{r,t+1}}{c^0_{r,t+1}} \right]. \]

Third, by inserting equation (102) into equation (31), the traders risk of holding money is
\[ n^s_{r,t} = \frac{1}{\pi} - \frac{[1 - \beta \Psi_r (1 - \theta_{r,t+1})]^2}{\beta \Psi_r}. \] (103)

Fourth, in equations (26) and (103), trader’s aggregate consumption with cash is
\[ (1 - \bar{i}^*_{r,t}) c^0_{r,t} = \frac{[1 - \beta \Psi_r (1 - \theta_{r,t+1})]^2}{\beta \Psi_r}. \] (104)

Thus, by inserting equation (99) into equations (101) - (104), the traders choices are derived as in equations (44) - (47).

### J.3 Derivation of Equations (49) - (52)

Given the real money holding of traders in equation (48), first, nontraders aggregate consumption with credit in equation (23) is
\[ i^*_{n,t} c^1_{n,t} = 1 - \theta_{n,t+1}. \] (105)

Next, in equation (32), the nontrader’s choice of credit and cash is
\[ i^*_{n,t} = \beta \Psi_n (1 - \theta_{n,t+1}) \in (0, 1), \] (106)

where
\[ \Psi_n = E_t \left[ \frac{1}{1 + \mu_{t+1}} \frac{1 - \pi n^s_{n,t+1}}{c^0_{n,t+1}} \right]. \]

Third, by inserting equation (106) into equation (31), the nontraders risk of holding money is
\[ n^s_{n,t} = \frac{1}{\pi} - \frac{[1 - \beta \Psi_n (1 - \theta_{n,t+1})]^2}{\beta \Psi_n}. \] (107)

Fourth, in equations (27) and (107), nontraders aggregate consumption with cash is
\[ (1 - \bar{i}^*_{n,t}) c^0_{n,t} = \frac{[1 - \beta \Psi_n (1 - \theta_{n,t+1})]^2}{\beta \Psi_n}. \] (108)

Thus, by inserting equation (100) into equations (105) - (108), the nontraders choices are derived as in equations (49) - (52).
K Derivation of Proposition 8

In equations \((43) - (47),\)

\[
\frac{\partial \theta_{r,t+1}}{\partial \mu_t} = \frac{(1-\alpha)}{\alpha} \left[ 1 - \frac{(1-\alpha)}{\alpha} E_t [\mu_{t+1}] \right] \theta_{r,t} > 0. \tag{109}
\]

\[
\frac{\partial n^s_{r,t}}{\partial \mu_t} = -2 [1 - \beta \Psi_r (1 - \theta_{r,t+1})] \frac{\partial \theta_{r,t+1}}{\partial \mu_t} < 0, \tag{110}
\]

\[
\frac{\partial i^*_{r,t}}{\partial \mu_t} = -\beta \Psi_r \frac{\partial \theta_{r,t+1}}{\partial \mu_t} < 0, \tag{111}
\]

\[
\frac{\partial i^*_{r,t} c^1_{r,t}}{\partial \mu_t} = -\frac{\partial \theta_{r,t+1}}{\partial \mu_t} < 0, \tag{112}
\]

\[
\frac{\partial (1 - i^*_{r,t}) c^0_{r,t}}{\partial \mu_t} = -\frac{\partial n^s_{r,t}}{\partial \mu_t} > 0. \tag{113}
\]

L Derivation of Corollary 3

Inequalities \((111)\) and \((112)\) and in equations \((101)\) and \((102)\) imply

\[
\frac{\partial c^1_{r,t}}{\partial \mu_t} = \left( \frac{1}{i^*_{r,t}} \right) \frac{\partial i^*_{r,t} c^1_{r,t}}{\partial \mu_t} - \left( \frac{c^1_{r,t}}{i^*_{r,t}} \right) \frac{\partial i^*_{r,t}}{\partial \mu_t} = \left( \frac{1}{i^*_{r,t}} \right) \left( \frac{\partial \theta_{r,t+1}}{\partial \mu_t} (\beta \Psi_r c^1_{r,t} - 1) \right) = \left( \frac{1}{i^*_{r,t}} \right) \left( \frac{\partial \theta_{r,t+1}}{\partial \mu_t} \right) \beta \Psi_r (1 - \theta_{r,t+1}) - 1 = 0. \]

Inequalities \((111)\) and \((113)\) and in equations \((102)\) and \((104)\) imply

\[
\frac{\partial c^0_{r,t}}{\partial \mu_t} = \left( \frac{1}{1 - i^*_{r,t}} \right) \frac{\partial (1 - i^*_{r,t}) c^0_{r,t}}{\partial \mu_t} + \left( \frac{c^0_{r,t}}{1 - i^*_{r,t}} \right) \frac{\partial i^*_{r,t}}{\partial \mu_t} = -\left( \frac{1}{1 - i^*_{r,t}} \right) \frac{\partial n^s_{r,t}}{\partial \mu_t} + \left( \frac{c^0_{r,t}}{1 - i^*_{r,t}} \right) \frac{\partial i^*_{r,t}}{\partial \mu_t} \]

\[
= \left( \frac{1}{1 - i^*_{r,t}} \right) \frac{\partial \theta_{r,t+1}}{\partial \mu_t} \left( 2 [1 - \beta \Psi_r (1 - \theta_{r,t+1})] - c^0_{r,t} \beta \Psi_r \right) \]

\[
= \left[ 1 - \beta \Psi_r (1 - \theta_{r,t+1}) \right] \frac{\partial \theta_{r,t+1}}{\partial \mu_t} \left( 2 \frac{1 - \beta \Psi_r (1 - \theta_{r,t+1})}{1 - i^*_{r,t}} \right) = \frac{\partial \theta_{r,t}}{\partial \mu_t} > 0. \]
M  Derivation of Proposition 9

In equations (48) - (52),
\[
\frac{\partial \theta_{n,t+1}}{\partial \mu_t} = -\frac{(1 + E_t[\mu_{t+1}]) \theta_{n,t}}{(1 + \mu_t)^2} < 0. \tag{114}
\]
\[
\frac{\partial n^s_{n,t}}{\partial \mu_t} = -2 [1 - \beta \Psi_n (1 - \theta_{n,t+1})] \frac{\partial \theta_{n,t+1}}{\partial \mu_t} > 0. \tag{115}
\]
\[
\frac{\partial i^*_{n,t}}{\partial \mu_t} = -\beta \Psi_n \frac{\partial \theta_{n,t+1}}{\partial \mu_t} > 0, \tag{116}
\]
\[
\frac{\partial i^*_{n,t}c^1_{n,t}}{\partial \mu_t} = -\frac{\partial \theta_{n,t+1}}{\partial \mu_t} > 0, \tag{117}
\]
\[
\frac{\partial (1 - i^*_{n,t})c^0_{n,t}}{\partial \mu_t} = -\frac{\partial n^s_{n,t}}{\partial \mu_t} < 0. \tag{118}
\]

N  Derivation of Corollary 4

Inequalities (116) and (117) and equations (105) and (106) imply
\[
\frac{\partial c^1_{n,t}}{\partial \mu_t} = \left(\frac{1}{i^*_{n,t}}\right) \frac{\partial i^*_{n,t}c^1_{n,t}}{\partial \mu_t} - \left(\frac{c^1_{n,t}}{i^*_{n,t}}\right) \frac{\partial i^*_{n,t}}{\partial \mu_t} = -\left(\frac{1}{i^*_{n,t}}\right) \frac{\partial \theta_{n,t+1}}{\partial \mu_t} - \left(\frac{c^1_{n,t}}{i^*_{n,t}}\right) \frac{\partial i^*_{n,t}}{\partial \mu_t}
\]
\[
\frac{\partial n^s_{n,t}}{\partial \mu_t} = -\beta \Psi_n \frac{\partial \theta_{n,t+1}}{\partial \mu_t} > 0,
\]
\[
\frac{\partial i^*_{n,t}c^1_{n,t}}{\partial \mu_t} = -\frac{\partial \theta_{n,t+1}}{\partial \mu_t} > 0,
\]
\[
\frac{\partial (1 - i^*_{n,t})c^0_{n,t}}{\partial \mu_t} = -\frac{\partial n^s_{n,t}}{\partial \mu_t} < 0.
\]

Inequalities (116) and (118) and equations (106) and (108) imply
\[
\frac{\partial c^0_{n,t}}{\partial \mu_t} = \left(\frac{1}{1 - i^*_{n,t}}\right) \frac{\partial (1 - i^*_{n,t})c^0_{n,t}}{\partial \mu_t} + \left(\frac{c^0_{n,t}}{1 - i^*_{n,t}}\right) \frac{\partial i^*_{n,t}}{\partial \mu_t} = -\left(\frac{1}{1 - i^*_{n,t}}\right) \frac{\partial \theta_{n,t+1}}{\partial \mu_t} + \left(\frac{c^0_{n,t}}{1 - i^*_{n,t}}\right) \frac{\partial i^*_{n,t}}{\partial \mu_t}
\]
\[
\frac{\partial n^s_{n,t}}{\partial \mu_t} = \left(\frac{1}{1 - i^*_{n,t}}\right) \frac{\partial \theta_{n,t+1}}{\partial \mu_t} \left\{2 [1 - \beta \Psi_n (1 - \theta_{n,t+1})] - c^0_{n,t} \beta \Psi_n\right\}
\]
\[
= \left[\frac{1 - \beta \Psi_n (1 - \theta_{n,t+1})}{1 - i^*_{n,t}}\right] \frac{\partial \theta_{n,t+1}}{\partial \mu_t} \left\{2 - \frac{1 - \beta \Psi_n (1 - \theta_{n,t+1})}{1 - i^*_{n,t}}\right\} = \frac{\partial \theta_{n,t}}{\partial \mu_t} > 0.
\]
O Proof of Proposition 10

With equation (16), welfare can be defined as, for \( j \in \{ r, n \} \),

\[
W_{j,t} = i_{j,t}^* \ln (c_{j,t}^1) + (1 - i_{j,t}^*) \ln (c_{j,t}^0) - \int_0^{i_{j,t}^*} \gamma (i) \, di
\]

\[
= \ln (c_{j,t}^1) - (1 - i_{j,t}^*) \ln \left( \frac{1}{1 - i_{j,t}^*} \right) - \int_0^{i_{j,t}^*} \ln \left( \frac{1}{1 - i} \right) \, di
\]

\[
= \ln (c_{j,t}^1) + (1 - i_{j,t}^*) \ln \left( 1 - i_{j,t}^* \right) - (1 - i_{j,t}^*) \ln \left( 1 - i_{j,t}^* \right) + (1 - i_{j,t}^*)
\]

\[
= \ln (c_{j,t}^1) + (1 - i_{j,t}^*).
\]

Now, the effect of monetary policy on welfare is

\[
\frac{\partial W_{j,t}}{\partial \mu_t} = \frac{1}{c_{j,t}^1} \frac{\partial c_{j,t}^1}{\partial \mu_t} - \frac{1}{i_{j,t}^*} \frac{\partial i_{j,t}^*}{\partial \mu_t} = \frac{1}{i_{j,t}^*} \frac{\partial i_{j,t}^* c_{j,t}^1}{\partial \mu_t} - \left( \frac{1}{i_{j,t}^*} + 1 \right) \frac{\partial i_{j,t}^*}{\partial \mu_t},
\]

where

\[
\frac{\partial c_{j,t}^1}{\partial \mu_t} = \left( \frac{1}{i_{j,t}^*} \right) \frac{\partial i_{j,t}^* c_{j,t}^1}{\partial \mu_t} - \left( \frac{c_{j,t}^1}{i_{j,t}^*} \right) \frac{\partial i_{j,t}^*}{\partial \mu_t}.
\]

For traders, since the money injection creates a positive income effect, the effect of monetary policy on trader’s welfare is positive from equations (44) and (46) and from inequalities (111) and (112):

\[
\frac{\partial W_{r,t}}{\partial \mu_t} = \beta \Psi_r \frac{\partial \theta_{r,t+1}}{\partial \mu_t} > 0. \tag{119}
\]

For nontraders, since the money injection creates a negative income effect, the effect of monetary policy on welfare is negative from equations (49) and (51) and from inequalities (116) and (117):

\[
\frac{\partial W_{n,t}}{\partial \mu_t} = \beta \Psi_n \frac{\partial \theta_{n,t+1}}{\partial \mu_t} < 0. \tag{120}
\]

Now, in inequalities (109), (114), (119), and (120), the effect of monetary policy on welfare is

\[
\frac{\partial W_t}{\partial \mu_t} = \alpha \frac{\partial W_{r,t}}{\partial \mu_t} + (1 - \alpha) \frac{\partial W_{n,t}}{\partial \mu_t} = \alpha \beta \Psi_r \frac{\partial \theta_{r,t+1}}{\partial \mu_t} + (1 - \alpha) \beta \Psi_n \frac{\partial \theta_{n,t+1}}{\partial \mu_t}
\]

\[
= \beta (1 - \alpha) \left\{ \frac{\Psi_r \theta_{r,t} \left[ 1 - (1 - \alpha) E_t[\mu_{t+1}] \right]}{\left[ 1 - (1 - \alpha) \mu_t \right]^2} - \Psi_n \theta_{n,t} \left[ 1 + E_t[\mu_{t+1}] \right] \right\} = 0.
\]
Thus, the following condition,

\[
\frac{\Psi_r \theta_{r,t} \left[ 1 - \left( \frac{1-\alpha}{\alpha} \right) E_t[\mu_{t+1}] \right]}{\left[ 1 - \left( \frac{1-\alpha}{\alpha} \right) \mu_t \right]^2} = \frac{\Psi_n \theta_{n,t} \left[ 1 + E_t[\mu_{t+1}] \right]}{(1 + \mu_t)^2},
\]

implies the optimal money growth rate in equation (53).

References


