ILLUMINATING THE SHADOW ECONOMY:
Using lights from space to estimate a new measure of informal economic activity

Working Paper

Rawaa Harati * Morgan Hardy †

December 17, 2013

---

*Centre d’Economie de la Sorbonne, Université Paris I Panthéon-Sorbonne.
rawaa.harati(at)malix.univ-paris1.fr

†University of Brown. morgan_hardy(at)brown.edu
1 Introduction

Informal economic activity is an omnipresent feature of production and employment in developing countries and an activity that, by its very nature, is difficult to measure. The study of the informal sector, its firms and workers and its contribution to the macroeconomy, has generated a substantial literature, relatively little of which focuses on measurement (see Schneider et al., 2010, for a review). In an influential paper on the subject, Schleifer and La Porta (2008) argue that informal firms disappear as an economy grows because they are fundamentally and functionally different than formal firms of the same size. In the cross-section, informal firms are less productive, countries with larger informal shares are poorer, and total economic growth correlates with a decrease in the share of economic activity that takes place in the informal sector. Their conclusions are by no means uncontroversial, but the substantive evidence they present is an important contribution.

From the perspective of measurement, any study of the informal sector must grapple with the issue of measuring an activity that is expressly designed to go unnoticed by government tax collectors, regulators, and (consequently) census data surveyors. The data section of this paper will outline in transparent detail the most popular existing candidates for a measure of the share of total production composed of informal activity, each of which self-admittedly suffers from a one or another major issue: drastic unintuitive assumptions, crudeness, subjectivity, systematic downward bias, etc. The quest for an objective, widely available, and intuitive estimator of the informal share of a country’s economic activity remains very much open.

This paper builds a new, and arguably more intuitive, estimator of the share of economic activity that takes place in the informal sector, building on the innovative measurement work in Henderson, Storeygard and Weil (2012). Their paper proposes the use of satellite data containing observed nighttime lights to produce an alternative measure of economic growth. The authors point out that consumption of nearly all goods in the evening requires lights. Consequently, higher consumption and investment activities are correlated with more and brighter nighttime light usage. Aggregated to the national level, the paper argues that the measurement error in lights predicted economic activity is uncorrelated with that of government estimated economic activity, Gross Domestic Product (GDP). Coupling this argument with the fact that two error-prone measures with uncorrelated errors can be used to construct a single more accurate measure, observed nighttime lights improve on existing measures of total economic activity.

The model and empirics in this paper make use of an intuitive subsequent assumption that lights predicted economic activity captures a larger proportion of the informal sector than measured GDP. In fact, one common definition of informal economic activity is “market-based production of goods and services, whether legal or
illegal, that escapes detection in the official estimates of GDP.\textsuperscript{1} In contrast, any consumption or investment activity requires the same amount of light whether or not it is registered with the government. It follows then, that government measured GDP systematically undermeasures total economic activity more for countries in which the informal share of economic activity is higher while total observed lights maintain the same relationship to total economic activity, regardless of the share of an economy that is in the informal sector. Exploiting the differences between lights predicted total economic activity and measure GDP produces a new measure of the informal share of total economic activity for each country.

Formalizing this intuition, this paper proposes a model wherein the residual from the cross-country regression of log GDP on log total observed nighttime lights has a systematic and positive relationship with the log of the formal share in economic activity. This systematic relationship can be exploited to generate new estimates for the informal share in economic activity at the country level. This paper tests the model using data on total observed nighttime lights by country and two alternative measures of government measured economic activity, market exchange rate adjusted GDP and PPP adjusted GDP. The success of this approach (as measured by the new estimates’ relationship to existing estimates) is very sensitive to the choice of GDP measure. Using an existing measure that will be described below, Figure 1 shows that the relationship predicted by the model is more or less only observed for the market exchange rate adjusted GDP derived residual.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{residuals.png}
\caption{residuals as compared to an existing measure.}
\end{figure}

Regardless of the choice of government measured economic activity, the model’s initial estimates (either 192 or 177, depending on the GDP measure) for the informal share are very noisy and approximately 17% are actually negative, which is unsatisfactory for a measure of something that should be bounded between 0 and 1.

In an attempt to address some of the issues that arise with the new estimate, this paper proposes two possible explanations for the better performance of market exchange rate adjusted GDP with the model. It also explores two possible explanations for the noisiness of the model’s initial estimates. It concludes that the intuitive justification for the approach and some initial empirical evidence of its relationship to existing indicators is very compelling, but that more work is needed before this estimator could provide a usable measure of the informal share of economic activity, particularly in refining the choice of proxy for government measured economic activity.

The remainder of this paper is as follows: Section 2 presents a model providing a new estimating equation for the informal share of economic activity; Section 3 describes the dataset constructed for this paper, including descriptions of the most popular existing measures for the informal share of economic activity, two alternative measures of government measured total economic activity, and observed nighttime lights; Section 4 describes the use of the new estimating equation to generate new estimates of the informal share of total economic activity in 192 countries for the market exchange rate adjusted GDP and 177 countries for the PPP adjusted GDP and discusses some possible causes for issues with these initial estimates; Section 5 presents a few comparisons of these new estimates to existing measures, highlighting the difference between the two choices of proxy for government measured economic activity; Section 6 discusses some possible explanations for why the choice of proxy for government measured economic activity has a large effect on the estimates; Section 7 develops a theory and reports results from optimally combining the new estimates with an existing measure; and Section 8 concludes.

2 The Model

2.1 Assumptions:

True economic activity in a country $i$, $GDP_i$, can be seen as actually made up of two kinds of economic activity: Formal economic activity, $FGDP_i$, and informal economic activity, $IGDP_i$. Call the informal share of total economic activity $s_i$, such that:

\begin{align*}
(1) \quad FGDP_i &= (1 - s_i) \cdot GDP_i \\
\text{and} \\
(2) \quad IGDP_i &= s_i \cdot GDP_i.
\end{align*}
Governments are unable to incorporate \( IGDP_i \) into government measured \( GDP_i \), \( MGDP_i \). Therefore:

\[
(3) \quad MGDP_i = FGDP_i \cdot e^{m_i},
\]

where \( e^{m_i} \sim \ln\mathcal{N}(0, \sigma^2_m) \) is standard measurement error.

Equations (1) and (3) imply that the natural log of measured economic activity, \( mgdp_i \), is related to the natural log of true economic activity, \( gdp_i \), in the following way:

\[
(4) \quad mgdp_i = \ln(1 - s_i) + gdp_i + m_i,
\]

where \( m_i \sim \mathcal{N}(0, \sigma^2_m) \) is standard measurement error.

Henderson, Storeyguard and Weil (2012) consider different functional forms and controls for relating total observable lights to total economic activity and claim that \( LIGHTS_i = GDP_i^\beta \) is most accurate. Because they were working in growth rather than levels, I adjust this equation to include a multiplier, \( e^\alpha \), such that:

\[
(5) \quad lights_{i,t} = \alpha + \beta gdp_{i,t} + l_i,
\]

where \( l_i \sim \mathcal{N}(0, \sigma^2_l) \) is standard measurement error.

### 2.2 Derivation of the estimator:

Equations (4) and (5) can be combined for the following specification:

\[
(6) \quad mgdp_{i,t} = \psi + [(1/\beta) lights_{i,t} + \mu_i]
\]

where:

\[
(7) \quad \psi = \ln(1 - s) - \alpha/\beta
\]

Although, in reality, there is some variation in whether or not and how countries attempt to include informal economic activity in their measure of total economic activity, this variation is not centrally reported or even reported at all for many countries. Therefore, accounting for this variation is not possible for this version of the paper. As will be discussed later, accounting for these differences in inclusion is a goal for future versions of this paper.

\( \ln(1 - s) \) is the average natural log of the formal share of economic activity across all countries.
\[
(8) \mu_i = [\ln(1 - s_i) - \ln(1 - s)] + m_i - (1/\beta)l_i.
\]

Notice that \( \mu_i \) is a noisy measure of the demeaned natural log of the formal share of economic activity in country \( i \). This means that adding an existing estimate of \( \ln(1 - s) \) to observed \( \hat{\mu}_i \) yields a measure of the natural log of the formal share of economic activity for country \( i \). Call this measure, \( \hat{x}_i \):

\[
(9) \hat{x}_i = \ln(1 - s_{i,t}) + m_i - (1/\beta)l_i,
\]

where:

\[
[m_i - (1/\beta)l_i] \sim \mathcal{N}(0, \sigma^2_x)
\]

and

\[
\sigma^2_x = \sigma^2_m + (1/\beta^2)\sigma^2_l.
\]

Using \( \hat{x}_i \) and an assumed value for \( \sigma^2_x \), an estimate of the informal share of economic activity in country \( i \) can be generated using the following formula:\footnote{\( \hat{s}_i E[s_i|\hat{x}_i] = E[1 - e^{\ln(1 - s_i)}|\hat{x}_i] = 1 - E[e^{\ln(1 - s_i)}|\hat{x}_i] = 1 - E[e^{\hat{x}_i - [m_i - (1/\beta)l_i]}|\hat{x}_i] = 1 - e^{\hat{x}_i}/e^{\sigma^2_x/2} > 1 - e^{\hat{x}_i}, \) due to Jensen’s inequality and the fact that \( x_i \) is positively correlated with \( [m_i - (1/\beta)l_i] \).}

\[
\hat{s}_i = 1 - e^{\hat{x}_i}/e^{\sigma^2_x/2}
\]

3 The Data

This paper uses a dataset constructed from four sources. The first is the dataset used by Schleifer and La Porta (2008) that contains country level estimates of informal share of economic activity from 6 of the past most popular measures as well as 15 measures of country characteristics thought to effect the size of the informal share. The second dataset is used by Henderson, Storeyguard and Weil (2012) and contains country and year level data of total observed nighttime lights on a 0-63 intensity scale across all pixels within a country from 1992 to 2008. The third is the World Bank National Accounts Data, containing market exchange rate adjusted GDP expressed in US dollars from 1992 to 2009. The fourth is the IMF Data, containing PPP adjusted GDP expressed in international dollars from 1992 to 2009.

The following subsections contain an in-depth description of each important variable. These descriptions are included to increase transparency of the statistical work. They also remind the reader to consider what is actually being measured by each of these variables when interpreting the later presented statistical relationships.
3.1 Existing measures:

Subjective or survey measures:

(1) “inf-wef” - This measure is sourced from the 2006 and 2007 World Economic Forum. Schleifer and La Porta explain: “Top business leaders from 125 countries were asked to estimate the size of the informal sector using a 1-to-7 scale, where 1 indicates that more than 50% of economic activity is unrecorded and 7 that all of it is registered. For comparability with other measures, we rescaled this index on a scale from 0 to 50% of GDP. The 50% cutoff adopted by the global competitiveness report is arbitrary and introduces a downward bias in this measure.” This measure is available for 125 countries.

(2) “tax-evasion” - This measure is sourced from World Bank Enterprise Surveys in 2002 and 2006. It is the self-reported percentage of total sales that a typical establishment reports for tax purposes. The respondents in the survey are the top managers of registered businesses in (mostly) developing countries. This measure is likely to understate the size of informal economy, because entrepreneurs in the informal sector are not surveyed. This measure is available for 95 countries.

Objective measures from observable variables:

(3) “self-emp-pct” - This measure is sourced from the International Labor Organization. It is the percentage of the active labor force that is self-employed, based a definition of self-employed as “jobs where the remuneration is directly dependent upon the profits derived from the goods and services produced,” but not work by unpaid family workers. This measure is available for 133 countries.

(4) “non-agric-self” - This measure is also sourced from the International Labor Organization. It is the percentage of the active labor force that is self-employed, based on the above definition, but excluding agricultural activities. This measure is available for 100 countries.

(5) “inf-elec” - This measure is sourced from Eric Friedman and coauthors. Schleifer and La Porta explain: “For each country, the ratio of electricity consumption to overall GDP for a base period is calculated and then extrapolated to present, assuming that the elasticity of electricity consumption to overall GDP is one. The size of the informal sector is then computed as the difference between the overall and the official GDP.” Further investigation of this measure reveals that their approach is not quite so straightforward. Friedman and coauthors actually source these data from a few different previous papers and replace some countries with other existing measures where inconvenient. It is difficult to nail down the exact approach used for each country in these data. Therefore, this measure should be interpreted as loosely based
on the above quoted methodology. This measure is available for only 57 countries.

**Model approach, using several indicators and causal variables:**

(6) “inf-schneider” - This measure is sourced from Schneider and coauthors. In their paper, they use a multiple indicators, multiple causes model (MIMIC). Their indicators include: labor force participation rate amongst people aged 18-64, annual GDP growth, and change in local currency in circulation per capita. Their causes include: tax-to-GDP ratio, the heritage foundation index of economic freedom, the unemployment rate, and GDP per capita. Schleifer and La Porta state that this measure is “only as good as the model that supports it.” It is available for 145 countries.

### 3.2 Existing measures of believed determinants:

(1) **The cost of becoming formal:** includes the logarithm of the number of procedures required to legally start a business.\(^5\) This measure is available for 179 countries.

(2) **The cost of staying formal:**

(i) The cost of paying taxes: includes two measures for the cost of paying taxes, one for the monetary cost\(^6\) and another for the time cost\(^7\), available for 177 and 179 countries, respectively.

(ii) The cost of obeying government regulations: includes the cost of complying with labor laws. This is captured through an index of the difficulty of hiring a new worker, the difficulty and expense of firing a redundant worker and the non-wage labor costs such as payroll taxes and social security payments.\(^8\) These measures are available for 179 countries. Additionally, the cost of red tape is included using a measure of the percentage of senior management’s time spent in dealing with requirements imposed by government regulation.\(^9\) This measure is available for 96 countries.

(3) **The benefits of being formal:**

---


\(^6\)tax − db, from Djankov and coauthors, a measure of total taxes (except for labor taxes) payable by businesses after accounting for deductions and exemptions.

\(^7\)time − tax, also sourced from Djankov and coauthors, measures the hours per year it takes to prepare, file and pay (or withhold) corporate income tax, value-added tax, and social security contributions.

\(^8\)hiringindex, firingindex, nonwagecost, all sourced from Juan Botero and coauthors and DoingBusiness2008.

\(^9\)mgmt − time, sourced from the World Bank’s Enterprise Surveys.
(i) The efficiency of courts: includes the logged number of steps required to collect on a bounced check, available for 109 countries, and a measure for the efficiency of the bankruptcy procedure, available for 88 countries.

(ii) The quality of property rights: includes an index for corruption and one for the rule of law. These indices are available for 205 and 209 countries, respectively.

(iii) The scope of domestic markets: includes the density of the paved road network, which is admittedly rough but available for 198 countries.

(iv) The benefits from access to finance: includes two objective measures of the size of financial markets, private credit and market capitalization of domestic firms, both as a ratio of GDP. These measures are available for 176 and 117 countries, respectively. One additional indicator is the ease of access to credit.

3.3 Observed nighttime lights:

The measures of total nighttime lights observed from outer space and government measured economic activity are sourced from the exact dataset used in Henderson, Storeygaurd and Weil’s paper. The measure of total nighttime lights observed was generated using data originally collected from several US Airforce weather satellites. The cleaned grid cells produced from this data contain an integer between 0 and 63, representing an estimate of the intensity of human produced light at night in that location on the planet. Henderson, Storeyguard and Weil categorize these grid cells into countries and then total their values to the country and year level, producing an estimate of the total nighttime lights observed from space for a country and year from 1992 to 2008.

3.4 Government measured economic activity:

Both the World Bank’s measure of GDP and the IMF’s measure of GDP are described as follows: “GDP at purchaser’s prices is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural

---

10 formalism – c, sourced from Djankov and coauthors and DoingBusiness2008.
11 easeefficiency, also sourced from a recent paper by Djankov and coauthors.
12 corrupt2004 and rule2004, both sourced from Daniel Kaufmann, Art Kraay and Massimo Mastruzzi.
13 road – density, sourced from the World Development Indicators
14 priv – credit and mcap, both sourced from World Development Indicators.
resources.” The difference between these two measures comes from their alternative conversion from domestic currency to either U.S. dollars using single year official exchange rates (World Bank measure)\textsuperscript{16}, or international dollars, using PPP exchange rates (IMF measure). PPP exchange rates are derived by constructing a sample bundle of staple goods from one country and comparing the relative price of that bundle to the price of a similar bundle in other countries.\textsuperscript{17}

4 The New Measure

In order to generate new estimates of the informal share using the model presented in this paper, values for $\ln(1 - s)$ and $\sigma^2_x$ must be assumed. This paper assumes a value of $-0.376$ for $\ln(1 - s)$.\textsuperscript{18} For $\sigma^2_x$, it assumes different values for the two different proxies of government measured economic activity: .7 for the market exchange rate adjusted GDP and .5 for the PPP adjusted GDP.\textsuperscript{19} Using $\hat{\mu}_i$, the residual from the linear regression of the natural log of the chosen proxy for government measured economic activity on the natural log of total observed lights, an estimate of $\hat{s}_i$, $inf - lights_i$, is generated for each country using the following formulas:

For market exchange rate adjusted GDP: $inf - lights_i = 1 - e^{\hat{\mu}_i - 0.376}/e^{0.7/2}$

For PPP adjusted GDP: $inf - lights_i = 1 - e^{\hat{\mu}_i - 0.376}/e^{0.5/2}$

The estimator generates values for 192 countries, using the market exchange rate adjusted GDP, and 177 values for countries using the PPP adjusted GDP, with Singapore at the lowest and Turkmenistan at the highest in rank for both. The below histogram displays the distribution of both sets of estimates:

\textsuperscript{16}“For a few countries where the official exchange rate does not reflect the rate effectively applied to actual foreign exchange transactions, an alternative conversion factor is used.”

\textsuperscript{17}For a comparison of these two approaches see the article at http://www.imf.org/external/pubs/ft/fandd/2007/03/basics.htm on the IMF website.

\textsuperscript{18}This value is the average across all country-level averages of any existing measure.

\textsuperscript{19}These values are approximately equal to the difference in observed variance of $x$ and the variance of average existing measures of $ln(1 - s)$. 
Although the true share is obviously not negative, the negative values shown above do not mean this entire approach is fruitless. Rather, these issues are most likely a result of deviations in assumptions of the model from reality causing noisiness in the estimates. The model assumes that any difference in lights predicted economic activity and government measured economic activity is either due to its informal share or random noise of mean 0. This implies that when a country’s government measured economic activity exceeds its lights predicted economic activity, it is assumed to have a lower informal share than when the inverse is true. However, there are other reasons that lights may underpredict government measured economic activity.

One reason lights may underpredict government measured economic activity is top coding. Because of satellite limitations, the maximum value any grid square can take is 63. This will cause countries like Singapore and Japan to have a total lights understating total GDP for reasons other than a negative informal share. Top coding is an issue in any country where a lot of economic activity is centralized into just a few grid squares. Additionally, there is a difficulty in categorizing inclusion of grid squares for island nations. Under categorization of grid squares can cause a similar problem to that of Singapore and Japan while over-categorizing can overstate the informal share of economic activity. This further complicates this top coding issue, because many of the countries that suffer from the downward bias from top coding would be islands or small nations, making the net bias in estimates for these countries ambiguous. Additionally, because the informal share is calculated based on deviations from the average country, these measurement issues increase overall noise in all estimates.

Due to an experiment carried out for 18 days during the winters of 1996 and 1997 where the settings of one of the satellites were altered so that a true radiance measure could be calculated, it is possible to explore this issue using an alternative
measure of total observed nighttime lights from 2006. The resulting distribution has a higher mean and a smaller standard deviation, consistent with the story that top coding introduces a downward bias in the informal share estimates for some counties and contributes to overall noise. The country with the lowest informal share in this distribution is now Japan, while Terkmenistan remains with the highest. The histogram of this distribution is reported below:

Another reason that a country’s lights may underpredict government measured economic activity is a violation of the assumption that the government does not include an attempted measure of $IGDP$ in $MGDP$. In reality, there is a lot of heterogeneity of governments’ approaches to calculating $MGDP$. Some governments attempt to include $IGDP$ more that others. Countries that attempt to include $IGDP$ would thus be assigned lower estimates of informal share than they should under the current assumptions of this model. Additionally, they will push down the average, overstating the size of the informal share in those counties that don’t attempt this inclusion.

This issue is more difficult to immediately examine empirically, due to the fact that the varying extent and approaches to the inclusion of $IGDP$ are not centrally reported or coded. A thorough treatment of this issue would require many days of research into and coding of each country’s unique approach to $GDP$ estimation. However, a crude test of this issue can be administered using information from Jacques Charmes, a professor at the Institute of Research for Development (IRD). According to Charmes, many African and a few Asian countries regularly attempt to include estimates of the informal sector into their National Accounts, but Latin American countries are data scarce in this area, because national accounts in these countries
are compiled by Central Banks which are "less concerned by the identification of this component of the national economy."  

Using the information from Charmes that Latin American countries do not incorporate any IGDP measurement attempts into MGDP, one crude empirical test of whether issues with this estimator stem from heterogeneity in countries’ inclusion of IGDP into MGDP is to apply the estimator only to the subset of countries that are in Latin America. Some empirical evidence for this interpretation is that the lights model estimates countries in Latin America to have a 24% more informal share on average than countries not in Latin America, while the average of past measures estimates less than 1% more, on average. 

Applying the model only to Latin America seems to decrease noise greatly: 

Both of these tests support the theory that this estimator is promising and could be used for research or policy analysis with further tweaking of the model’s assumptions to include a treatment of the top coding and heterogeneous MGDP calculations. Future research on this area should focus on correcting these issues.

5 Comparison with Existing Measures

It is clear that the estimator or data will need refinement. However, comparisons with existing measures can provide evidence of its potential usefulness, once cleaned.

\[20\text{http://mospi.nic.in/informal_paper01.htm}]}
5.1 Correlations with existing measures:

The below correlations tables show the relationships of existing measures to each other and to the new measure derived from market exchange rate adjusted GDP and from PPP adjusted GDP. They contain pair wise correlations and numbers of common observations, meaning that each correlation is calculated with any country for which the two measures exist.

### Pair Wise Correlations Between Existing Measures and the New Measure from market exchange rate adjusted GDP

<table>
<thead>
<tr>
<th>Measures</th>
<th>inf_lights</th>
<th>inf_wef</th>
<th>tax_evasion</th>
<th>self_emp_pct</th>
<th>non_agric_self</th>
<th>inf_schneider</th>
<th>inf_elec</th>
</tr>
</thead>
<tbody>
<tr>
<td>inf_lights</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inf_wef</td>
<td>0.4007</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tax_evasion</td>
<td>0.0439</td>
<td>0.2484</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>self_emp_pct</td>
<td>0.1608</td>
<td>0.7015</td>
<td>0.2483</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>non_agric_self</td>
<td>0.2226</td>
<td>0.6760</td>
<td>0.3832</td>
<td>0.8063</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inf_schneider</td>
<td>0.3829</td>
<td>0.6953</td>
<td>0.1706</td>
<td>0.4653</td>
<td>0.6233</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>inf_elec</td>
<td>0.3205</td>
<td>0.6114</td>
<td>0.3485</td>
<td>0.6099</td>
<td>0.5367</td>
<td>0.7098</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

### Pair Wise Correlations Between Existing Measures and the New Measure from PPP adjusted GDP

<table>
<thead>
<tr>
<th>Measures</th>
<th>inf_lights</th>
<th>inf_wef</th>
<th>tax_evasion</th>
<th>self_emp_pct</th>
<th>non_agric_self</th>
<th>inf_schneider</th>
<th>inf_elec</th>
</tr>
</thead>
<tbody>
<tr>
<td>inf_lights</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inf_wef</td>
<td>0.1821</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tax_evasion</td>
<td>-0.1280</td>
<td>0.2484</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>self_emp_pct</td>
<td>-0.0284</td>
<td>0.7015</td>
<td>0.2483</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>non_agric_self</td>
<td>0.0084</td>
<td>0.6760</td>
<td>0.3832</td>
<td>0.8063</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inf_schneider</td>
<td>0.2066</td>
<td>0.6953</td>
<td>0.1706</td>
<td>0.4653</td>
<td>0.6233</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>inf_elec</td>
<td>0.1661</td>
<td>0.6114</td>
<td>0.3485</td>
<td>0.6099</td>
<td>0.5367</td>
<td>0.7098</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Note that both lights measures have their strongest relationship with \( inf – wef \), \( inf – schneider \), \( inf – elec \) and \( non – agric – self \) and their least strong relationships with \( tax – evasion \) and \( self – emp – pct \). However, these differing correlations are difficult to interpret, because they could be a result of the various subsamples used to produce them rather than a result of differences in the underlying nature of their estimators. If an existing measure happens to exist for countries whose \( MGDP \) approach is a better match for this paper’s model, then it would appear that those measures are more related, even though it is an artifact of the sample selection.

The possibilities for various comparisons and various subsamples are somewhat endless, but two facts should be noted: (1) the market exchange rate adjusted GDP estimates appear to have much more of a relationship to existing estimates than PPP adjusted GDP estimates. (2) either proxy produces new estimates that exhibit a lower correlation with existing measures than existing measures do with each other. Therefore, if this new approach yields a believable estimate of the informal share, it is an ideal measure for use in combining with existing estimates in the manner later outlined in this paper.

5.2 “Testing” the model with existing measures:

Consider the following specification:

\[
lights_i = \alpha + \beta \text{mgdp}_i - \beta \ln(1 - s_i) + l_i - \beta m_i
\]

According to the model, if a past estimate of \( \ln(1 - s_i) \) is a good one, then it should improve the fit of the above regression, the coefficient on \( \text{mgdp}_i \) should increase and the coefficients on \( \text{mgdp}_i \) and \( \ln(1 - s_i) \) should be the same absolute value, with \( \text{mgdp}_i \)’s coefficient positive and \( \ln(1 - s_i) \)’s coefficient negative.

The below table presents the results from this exercise for the market exchange rate adjusted GDP estimates. For each existing measure, the table first reports the specification without the existing measure, but limiting the sample to only those countries where that measure exists, for comparison. It also includes the first and last column pertaining to the lights measure, even though the results of which are tautological, as an example of what should occur if an existing measure is in agreement with this model.
For the market exchange rate adjusted GDP estimates, the measures $inf - wef$ and $inf - schneider$ follow the predictions of the model exactly, while $tax - evasion$ deviates the most. This is encouraging, seeing as $inf - wef$, although crude, is the most believable existing measure of the informal share while $tax - evasion$ is arguably the least believable. A similar table for PPP adjusted GDP displays much messier results for this test. However, $inf - wef$ and $inf - schneider$ still deviate the least from the predictions of the model.
Existing Measures in the New Model - PPP Adjusted GDP

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(mgdp)</td>
<td>1.099***</td>
<td>1.000***</td>
<td>0.993***</td>
<td>1.064***</td>
<td>1.040***</td>
<td>1.028***</td>
<td>1.007***</td>
</tr>
<tr>
<td></td>
<td>(0.0276)</td>
<td>(0.0368)</td>
<td>(0.0390)</td>
<td>(0.0427)</td>
<td>(0.0431)</td>
<td>(0.0341)</td>
<td>(0.0346)</td>
</tr>
<tr>
<td>ln(1-inf_wef)</td>
<td></td>
<td></td>
<td></td>
<td>0.313</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.566)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(1-tax_evasion)</td>
<td></td>
<td>0.694**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.308)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(1-self_emp_pct)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.590**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.252)</td>
</tr>
<tr>
<td>Constant</td>
<td>-14.65***</td>
<td>-12.17***</td>
<td>-11.89***</td>
<td>-13.70***</td>
<td>-12.92***</td>
<td>-12.88***</td>
<td>-12.15***</td>
</tr>
<tr>
<td></td>
<td>(0.666)</td>
<td>(0.917)</td>
<td>(1.047)</td>
<td>(1.043)</td>
<td>(1.076)</td>
<td>(0.840)</td>
<td>(0.881)</td>
</tr>
<tr>
<td>Observations</td>
<td>177</td>
<td>121</td>
<td>121</td>
<td>90</td>
<td>90</td>
<td>121</td>
<td>121</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.901</td>
<td>0.861</td>
<td>0.861</td>
<td>0.876</td>
<td>0.883</td>
<td>0.884</td>
<td>0.890</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
<th>(12)</th>
<th>(13)</th>
<th>-14</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(mgdp)</td>
<td>1.126***</td>
<td>1.069***</td>
<td>1.101***</td>
<td>1.113***</td>
<td>0.869***</td>
<td>0.867***</td>
<td>1.220***</td>
</tr>
<tr>
<td></td>
<td>(0.0353)</td>
<td>(0.0388)</td>
<td>(0.0343)</td>
<td>(0.0376)</td>
<td>(0.0624)</td>
<td>(0.0633)</td>
<td>(2.42e-08)</td>
</tr>
<tr>
<td>ln(1-non_agric_self)</td>
<td>0.547***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.182)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(1-inf_sch)</td>
<td>-0.290</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0910</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.347)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.346)</td>
<td></td>
</tr>
<tr>
<td>ln(1-inf_elec)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.220***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(7.69e-08)</td>
</tr>
<tr>
<td></td>
<td>(1.021)</td>
<td>(0.846)</td>
<td>(1.002)</td>
<td>(1.604)</td>
<td>(1.646)</td>
<td>(1.629)</td>
<td>(6.02e-07)</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1.220***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(7.69e-08)</td>
</tr>
<tr>
<td>Observations</td>
<td>96</td>
<td>137</td>
<td>137</td>
<td>55</td>
<td>55</td>
<td>192</td>
<td>177</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.923</td>
<td>0.884</td>
<td>0.885</td>
<td>0.785</td>
<td>0.786</td>
<td>0.877</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

5.3 New estimates relationship to believed causes:

An exercise from Schleifer and La Porta’s paper was replicated examining the relationship between the new estimates and alleged causes of the informal share of economic activity. Schleifer and La Porta find that these indicators predict a few of the existing measures, with $inf - wef$ being the best predicted, however after they control for income all significance of predictors disappears with the exception of $inf - wef$. This suggests that much of the results in the specification without the income control are due to direct correlations between existing indicators with total economic activity, which is itself related to the causal indicators.

The new estimates from market exchange rate adjusted GDP are negatively and significantly related to measures of efficiency of the courts, quality of property rights and benefits from access to finance. They are positively and significantly related to measures for the cost of becoming formal, the time it takes to pay taxes and the diffi-
culty of hiring a new worker. The negative and significant relationship to measures of quality of property rights and benefits from access to finance are more or less robust to controlling for income (with the relationship with corruption actually increasing in absolute value) while the other relationships are not. The new estimates from PPP adjusted GDP are much less related to believed causes, but maintain a significant relationship with benefits from access to finance even after controlling for income.

6 The importance of proxy choice

The major take-away of this paper so far is that the initial results of this model and data are very sensitive to the choice of proxy for government measured economic activity. Because the model attributes all deviations of the government measure from lights predicted economic activity to differences in the size of the informal sector, violations of the model’s assumptions about these measures are lethal to the production of reliable estimates. The assumption that the government does not observe informal economic activity is currently creating the most problems. This can be seen in section 4, where fixing the top coding issue in the lights data only improves the estimator slightly, but limiting the sample to Latin America only (where this assumption is believed to be less violated) greatly decreases the noise of the estimates.

It is with the importance of not violating this assumption in mind that the market exchange rate adjusted GDP may be the correct measure for use with this model. The major intuitive difference between PPP adjusted GDP and market exchange rate adjusted GDP is that PPP adjusted GDP trades accuracy in measurement of tradable GDP for that of non-tradable GDP. The informal sector mainly produces non-tradable GDP. Therefore, the PPP adjusted GDP is trading accuracy in government measurement of formal economic activity for increased accuracy in any measurement of informal economic activity, which this model assumes does not exist. It seems reasonable to believe that part of the reason market exchange rate adjusted estimates work ”better” with this model is due to the fact it violates less the assumption shown to be so problematic in section 4.

However, there is also an important alternative hypothesis that the difference in effectiveness of these proxies is merely a result of the Belassa-Samuelson effect, a known bias in the market exchange rate adjusted GDP that understates GDP in countries with cheaper non-tradables and overstates it in countries with more expensive non-tradables. This would mean that the residual from the regression of log government measured economic activity on log total observed nighttime lights would be systematically and positively related to this bias for the market exchange rate adjusted GDP and thus the model’s estimates of the informal share using this proxy would be negatively related to this bias. Because the relative price of non-tradables is arguably negatively correlated with the informal share of economic activity, a relationship would be produced between the estimates of the model using market exchange rate adjusted GDP and existing measures of the informal share purely due to this
bias and not through the channels outlined in the model.

It is likely that both of these proxy options are not ideal for the reasons explained above. These initial results and their analysis suggest that any productive future work in this area will require a much more careful choice of proxy for government measured economic activity. This proxy would need to adhere to the assumption that it does not include the informal sector and not contain any bias that systematically relates the resulting estimates of informal share to existing measures of informal share. Alternatively, future work could relax the assumptions of the model in some way that does not destroy the possibility for identification of the informal share.

7 Optimally Combining Estimates

Ala Henderson, Storeyguard and Weil (2012), as long as the correlation between a past estimate and this new estimate is not perfect, these past estimates can be combined with this new estimate via an optimally weighted average to produce a better estimate of $\ln(1 - s_i)$. For example, let $z_i$ be a past estimate of $\ln(1 - s_i)$. Then:

$$z_i = \ln(1 - s_i) + \psi_i, \text{ where } \psi_i \sim \mathcal{N}(0, \sigma^2_\psi).$$

Combine this measure with the lights data measure as follows:

$$\ln(1 - s_i) = (\lambda)z_i + (1 - \lambda)x_i$$

Assuming $\text{cov}([m_i - (1/\beta)l_i], \psi_i) = 0$,\(^{21}\) $\text{var}(\ln(1 - s_i) - \ln(1 - s_i))$ can be derived as follows:

$$\text{var}(\ln(1 - s_i) - \ln(1 - s_i))$$

$$= \text{var}((\lambda)(z_i - \ln(1 - s_i)) + (1 - \lambda)(x_i - \ln(1 - s_i)))$$

$$= (\lambda^2)\sigma^2_\psi + (1 - \lambda)^2\sigma^2_x$$

Therefore, the lambda that minimizes $\text{var}(\ln(1 - s_i) - \ln(1 - s_i))$ is:

$$\lambda^* = (\sigma^2_\psi)/(\sigma^2_\psi + \sigma^2_x)$$

The observed data provides:

$$\text{var}(z) = \sigma^2_s + \sigma^2_\psi$$

\(^{21}\)An extreme assumption, but one also made admittedly extreme for the purpose of example in Henderson, Storeyguard and Weil (2012).
\[ var(x) = \sigma_s^2 + \sigma_x^2 \]
\[ cov(x, z) = \sigma_s^2 \]

So then:

\[ \hat{\sigma}_x^2 = var(x) - cov(x, z) \]
\[ \hat{\sigma}_\psi^2 = var(z) - cov(x, z) \]

So, the optimal weight is defined as follows:

\[ \hat{\lambda}^* = \frac{var(x) - cov(x, z)}{(var(x) + var(z) - 2cov(x, z))} \]

Using the above methodology to generate an optimal weight using the current estimates gives the new estimates a weight of 0 for the market exchange rate adjusted GDP estimates and a weight of 0.02 for the PPP adjusted GDP estimates. This is due to the incredible amount of noise in these initial estimates. Future versions of this paper may find a higher optimal weight. Another thing to consider is that the above optimal weighting derivation assumes that both measures are unbiased, while the World Economic Forum measure, as well as most existing measures, are self-admittedly biased downward. So it is possible that this optimal weighting estimator is biased downward as well.

8 Conclusion

This paper suggests a new approach to addressing an important measurement issue. Informal economic activity is known to be an important part of many countries’ total economic activity, particularly in the developing world. To date, there does not exist a truly satisfying estimator or measure of the size of the informal share in total economic activity. Development of such an estimator is of extreme importance to scholars and policymakers interested in informal economic activity and its significance.

This paper investigates the possibility that differences in lights predicted economic activity and government estimated economic activity can be exploited to estimate the informal share. This paper finds that the lights data approach to measurement of the informal share in economic activity is a promising new direction for future research in this area, but that more work is needed before the estimator is truly useful, particularly in the area of choosing the correct proxy for government measured economic activity. It suggests that possible next steps to be taken in this area include relaxing the strong assumption of the initial model, and creating a dataset
that accounts for country-level GDP-measurement methodologies which differentially attempt to incorporate estimates of informal activity into \textit{GDP} measurement.
Bibliography


Feld, L. and Schneider, F. (2010). Survey on the Shadow Economy and Undeclared Earnings in OECD Countries, Invited Paper written for publication in the German Economic Review, Department of Economics, University of Linz, Linz, Austria


