The co-movement of house price and stock price with shocks

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Abstract

In order to find out the exact periods of the co-movement between house price and stock price for U.S. and U.K., this paper adopted FIML Markov-Switching Model by Yoon (2006).

This paper found that house price for U.S. and U.K. showed common business cycle with stock price during the 1970s, 1990s oil shocks periods, and 1980s, IT bubble collapse, the burst housing bubble in 2008.

These results showed that international big shocks, such as oil shocks and the burst housing bubble in 2008, cause common business cycle of house price and stock price for U.S. and U.K..

JEL classification: C13; C32

Keyword: U.S., U.K., co-movement, house price, stock price, FIML Markov-Switching Model, common, business cycle, big shocks, oil shocks, burst, housing bubble

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1. Introduction

The relationship between house price and stock price has been assumed to be positive and linear in the long run. However, Okunev, J and Wilson, PJ (1997) found that real estate and equity markets are segmented with linear test, whereas the markets are fractionally integrated with nonlinear model. McMillan (2012) also found that house price and stock price are nonlinear co-integrated in the long run.

Thus the purpose of this paper is to find out whether there really are co-movements between house price and stock price in the U.S. and U.K.

To establish the nonlinear relationship between house price and stock price, we adopt Markov-Switching Model by Hamilton (1989) and FIML Markov-Switching Model by Yoon (2006).

The findings of this paper are as follows: House price and stock price are more volatile with big shock periods. Moreover, house price for U.S. and U.K. is common business cycle with stock price during 1970s’ and 1990s’ oil shocks periods, and house price is also common business cycle with stock price during 1980s, IT bubble collapse and housing bubble burst in 2008.

The paper has been divided in 4 sections. Section 2 presents FIML Markov-Switching Model. Section 3 summarizes the empirical results. Section 4 concludes this paper.

2. FIML Markov-Switching Model

In order to get the consistent estimation of the parameters of the Markov-switching model in the simultaneous equations, we consider the following FIML Markov-Switching Model.

\[ Y_{BS_t} + Z\Gamma_{S_t} = U_{S_t}, \quad U_{S_t} \sim i.i.d. N(0, \Sigma_{S_t} \otimes I_T) \]  

(1)

where

\[
Y = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1M} \\ Y_{21} & Y_{22} & \cdots & Y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{T1} & Y_{T2} & \cdots & Y_{TM} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{bmatrix}, \quad B_{S_t} = \begin{bmatrix} \beta_{11,S1} & \beta_{12,S2} & \cdots & \beta_{1M,SM} \\ \beta_{21,S1} & \beta_{22,S2} & \cdots & \beta_{2M,SM} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{M1,S1} & \beta_{M2,S2} & \cdots & \beta_{MM,SMM} \end{bmatrix}
\]
\[
Z = \begin{bmatrix}
Z_{11} & Z_{12} & \ldots & Z_{1K} \\
Z_{21} & Z_{22} & \ldots & Z_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{T1} & Z_{T2} & \ldots & Z_{TK}
\end{bmatrix} = \begin{bmatrix}
\tilde{z}_1 \\
\tilde{z}_2 \\
\vdots \\
\tilde{z}_T
\end{bmatrix}, \quad \Gamma_{S_t} = \begin{bmatrix}
\gamma_{1S_t} & \gamma_{12S_t} & \ldots & \gamma_{1M,S_t} \\
\gamma_{21S_t} & \gamma_{22S_t} & \ldots & \gamma_{2M,S_t} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{K1S_t} & \gamma_{K2S_t} & \ldots & \gamma_{KM,S_t}
\end{bmatrix}
\]

\[
U_{S_t} = \begin{bmatrix}
u_{1L,S_t} & u_{12S_t} & \ldots & u_{1M,S_t} \\
u_{21S_t} & u_{22S_t} & \ldots & u_{2M,S_t} \\
\vdots & \vdots & \ddots & \vdots \\
u_{T1,S_t} & u_{T2S_t} & \ldots & u_{TM,S_t}
\end{bmatrix} = (u_{S_{1t}} & u_{S_{2t}} & \ldots & u_{S_{M,t}})
\]

\[
E(U'_{S_t}U_{S_t}) = E\left(\begin{bmatrix}
u_{S_{1t}} \\
u_{S_{2t}} \\
\vdots \\
u_{S_{M,t}}
\end{bmatrix}\begin{bmatrix}
u_{S_{1t}} & u_{S_{2t}} & \ldots & u_{S_{M,t}}
\end{bmatrix}\right) = \begin{bmatrix}
\sigma_{S_{1t},S_{1t}}I_T & \sigma_{S_{1t},S_{2t}}I_T & \ldots & \sigma_{S_{1t},S_{M_t}}I_T \\
\sigma_{S_{2t},S_{1t}}I_T & \sigma_{S_{2t},S_{2t}}I_T & \ldots & \sigma_{S_{2t},S_{M_t}}I_T \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{S_{M_t},S_{1t}}I_T & \sigma_{S_{M_t},S_{2t}}I_T & \ldots & \sigma_{S_{M_t},S_{M_t}}I_T
\end{bmatrix} = \Sigma_{S_t} \otimes I_T
\]

\[Y\] is the T x M matrix of jointly dependent variables, \(B_{S_t}\) is an M x M matrix and nonsingular. \(Z\) is the T x K matrix of predetermined variables, \(\Gamma_{S_t}\) is K x M matrix and \(rank(Z) = K\). \(U_{S_t}\) is T x M matrix of the structural disturbances of the system. Thus, the model has M equations and T observations. The structural errors are assumed as a nonsingular M-variate normal (Gaussian) distribution. \(\sigma\) is the covariance of the error terms. \(\Sigma_{S_t}\) is an M x M matrix and positive definite and no restrictions are placed on it. It is assumed that all equations satisfy the rank condition for identification. Also if lagged endogenous variables are included as predetermined variables, the system is assumed to be stable. An orthogonality assumption, \(E(Z'U_{S_t})=0\), between the predetermined variables and structural errors is required and, we assume the presence of contemporaneous correlation but no intertemporal correlation in (1). If we assume that the single Markov-switching variable \(S_t\) has an N-state, first-order Markov process, then we can write the transition probability matrix in the following way:
where \( p_{ij} = \Pr(S_t = j \mid S_{t-1} = i) \) with \( \sum_{j=1}^{N} p_{ij} = 1 \) for all \( i \)

If our model involves only two unobserved two-state first order Markov-switching variables such as \( S_{1t} \) and \( S_{2t} \), the dynamics of Markov-switching variables can be represented by a single Markov-switching variable \( S_t \) in the following manner:

\[
\begin{align*}
  S_t &= 1 \quad \text{if} \quad S_{1t} = 0 \quad \text{and} \quad S_{2t} = 0 \\
  S_t &= 2 \quad \text{if} \quad S_{1t} = 0 \quad \text{and} \quad S_{2t} = 1 \\
  S_t &= 3 \quad \text{if} \quad S_{1t} = 1 \quad \text{and} \quad S_{2t} = 0 \\
  S_t &= 4 \quad \text{if} \quad S_{1t} = 1 \quad \text{and} \quad S_{2t} = 1
\end{align*}
\]

with \( p_{ij} = \Pr(S_t = j \mid S_{t-1} = i) \), \( \sum_{j=1}^{4} p_{ij} = 1 \)

To derive the FIML Markov-Switching Model in the simultaneous equations, we can obtain \( \Pr(S_t = j \mid \psi_t) \) by applying a Hamilton filter (1989) as follows:

Step 1:
At the beginning of the \( t^{th} \) iteration, \( \Pr(S_{t-1} = i \mid \psi_{t-1}), \; i = 0, 1, \ldots, N \) is given. And, we calculate

\[
\Pr(S_t = j \mid \psi_{t-1}) = \sum_{i=1}^{N} \Pr(S_{t-1} = i, S_t = j \mid \psi_{t-1})
\]

\[
= \sum_{i=1}^{N} \Pr(S_t = j \mid S_{t-1} = i) \Pr(S_{t-1} = i \mid \psi_{t-1})
\]

where \( \Pr(S_t = j \mid S_{t-1} = i), \; i = 0, 1, \ldots, N, \; j = 0, 1, \ldots, N \) are the transition probabilities.

Step 2:
Consider the joint conditional density of \( y_t \) and unobserved \( S_t = j \) variable, which is the product of the conditional and marginal densities:
\[ f(y_t, S_t = j \mid \psi_{t-1}) = f(y_t \mid S_t = j, \psi_{t-1}) \Pr(S_t = j \mid \psi_{t-1}) \]

from which the marginal density of \( y_t \) is obtained by:

\[ f(y_t \mid \psi_{t-1}) = \sum_{j=1}^{N} f(y_t, S_t = j \mid \psi_{t-1}) \Pr(S_t = j \mid \psi_{t-1}) \]

where the conditional density \( f(y_t \mid S_t = j, \psi_{t-1}) \) is obtained from (2):

\[
\begin{align*}
 f(y_t \mid S_t = j, \psi_{t-1}) &= (2\pi)^{-M/2} \det(\Sigma_{S_t})^{-1/2} \left| \det(B_{S_t}) \right| \cdot \exp\left( -\frac{1}{2} (y_t B_{S_t} + z_t \Gamma_{S_t}) \Sigma_{S_t}^{-1} (y_t B_{S_t} + z_t \Gamma_{S_t})^T \right) \\
 &= \frac{1}{T} (Y_{B_{S_t}} + Z_{\Gamma_{S_t}})^T (Y_{B_{S_t}} + Z_{\Gamma_{S_t}}), \quad y_t \text{ is the } t^{th} \text{ row of the } Y \text{ matrix. } z_t \text{ is the } t^{th} \text{ row of the } Z \text{ matrix. } B_{S_t} \text{ and } \Gamma_{S_t} \text{ is obtained from (1).}
\end{align*}
\]

Step 3:
Once \( y_t \) is observed at the end of time \( t \), we update the probability terms:

\[
\begin{align*}
 \Pr(S_t = j \mid \psi_t) &= \Pr(S_t = j \mid \psi_{t-1}, y_t) \\
 &= \frac{f(S_t = j, y_t \mid \psi_{t-1})}{f(y_t \mid \psi_{t-1})} \\
 &= \frac{f(y_t \mid S_t = j, \psi_{t-1}) \Pr(S_t = j \mid \psi_{t-1})}{f(y_t \mid \psi_{t-1})}
\end{align*}
\]

As a byproduct of the above filter in Step 2, we obtain the log likelihood function:

\[ \ln L = \sum_{i=1}^{T} \ln f(y_i \mid \psi_{t-1}) \]

which can be maximized in respect to the parameters of the model.
3. Empirical Results

Let’s consider the Housing Price Index\(^2\) and the Dow Jones Industrial Index\(^3\) in the U.S. An OLS regression for \(t=1970:\text{II}\) to \(2015:\text{II}\) using quarterly data is given by equation (3)

\[
\Delta Y_t = \alpha + \beta \Delta H_t + e_t \tag{3}
\]

where, \(\Delta Y_t\) is the log difference of Dow Jones Industrial Index; \(\Delta H_t\) is the log difference of Housing Price Index in the U.S.

In equation (3), we try to find out whether \(\beta\) is really constant before and after bubble burst periods. To do this, we adopt simple two-state Markov-switching parameters in order to incorporate structural breaks.

\[
\Delta Y_t = \alpha_{S_t} + \beta_{S_t} \Delta H_t + e_t \tag{4}
\]

where \(\alpha_{S_t} = \alpha_0 S_t + \alpha_1 (1-S_t)\), \(\beta_{S_t} = \beta_0 S_t + \beta_1 (1-S_t)\),

\[
\Pr(S_t = 0 \mid S_{t-1} = 0) = q, \Pr(S_t = 1 \mid S_{t-1} = 1) = p, \\
p = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix},
\]

(Table 1) reports estimation results in the Hamilton’s Markov-Switching Model.

\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
\hline
\(\alpha_0\) & 4.8893 (0.7871) \\
\(\alpha_1\) & -1.6905 (1.8843) \\
\(\beta_0\) & -1.3092 (0.3290) \\
\(\beta_1\) & 1.4291 (0.7274) \\
\(\sigma_0\) & 4.6698 (0.4774) \\
\(\sigma_1\) & 10.6487 (1.0580) \\
\(q\) & 0.8700 (0.0573) \\
\(p\) & 0.8023 (0.1022) \\
\textbf{Log Likelihood} & -618.47 \\
\hline
\end{tabular}
\end{table}

Standard errors of the parameters estimates are reported in the parentheses

The coefficient \(\beta_0\) is significant and negative correlated during the regime 0 periods.


\(^3\) Source: Federal Reserve Bank of St. Louis (https://research.stlouisfed.org/fred2/)

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The coefficient $\beta_1$ is also significant and positive correlated during the regime 1 periods. The positive coefficient $\beta_1$ showed the co-movement between house price and stock price during the regime 1 periods. These findings offer the evidence that the coefficient $\beta$ is not stable and linear. The yield of house market $\Delta H_t$ is more stable than the yield of stock market $\Delta Y_t$ because absolute values of ($\beta_0=1.3092$, $\beta_1=1.4291$) are larger than 1.

$(\sigma_1=10.6487) > (\sigma_0=4.6698)$ showed the uncertainty of regime 1 periods is larger than that of regime 0 periods.

Figure 1. Dow Jones Industrial Index $Y_t$ and House Price $H_t$ in the U.S.

Figure 2. Dow Jones Industrial Index $\Delta Y_t$ and House Price $\Delta H_t$.

Figure 3. Smoothed probabilities of regime 1 $\Pr(S_t=1|\Delta Y_t)$.
(Figure 1) & (Figure 2) depicts the relationship between house price and stock price in the U.S.. (Figure 3) depicts two-state markov-switching probabilities during the regime 1 periods. From (Figure 3), we can find that the inferred probabilities \( \text{Pr}(S_t = 1|\Delta Y_t) \) accord with U.S. recessionary dates during 1970s’ and early 1990s’ oil shocks periods. This result is consistent with Hamilton (1989), who suggests that there was great uncertainty after two major OPEC oil shocks in 1973-1974 and 1979-1980. And, house price is also procyclical in movement with stock price during 1980s, late 1990s, 2000s IT bubble collapse and financial shock. Thus, we can interpret that there is the co-movement of housing bubble burst and stock market crash with economic big shocks.

Let’s consider the Housing Price Index\(^4\) and Total Share Prices for All Shares Index\(^5\) for the U.K..

(Table 2) reports estimation results in the Hamilton’s Markov-Switching Model.

**TABLE 2: MAXIMUM LIKELIHOOD ESTIMATION OF THE HAMILTON’S MODEL in the U.K.

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>2.7333 (0.5760)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>-2.7156 (3.1398)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>-0.0562 (0.1621)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.3089 (0.7929)</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>5.0258 (0.3680)</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>14.7113 (2.3068)</td>
</tr>
<tr>
<td>( q )</td>
<td>0.9584 (0.0210)</td>
</tr>
<tr>
<td>( p )</td>
<td>0.8300 (0.0944)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>- 599.88</td>
</tr>
</tbody>
</table>

Standard errors of the parameters estimates are reported in the parentheses

The coefficient \( \beta_0 \), \( \beta_1 \) are not significant. However, the variance \( \sigma_0 \), \( \sigma_1 \) are significant. \( (\sigma_1 = 14.7113) > (\sigma_0 = 5.0258) \) showed the uncertainty of regime 1 periods is larger than that of regime 0 periods.

(Figure 4) & (Figure 5) depicts the relationship between house price and stock price in the U.K.. (Figure 6) depicts two-state markov-switching probabilities during the regime 1 periods.

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\(^5\) Source: Federal Reserve Bank of St. Louis (https://research.stlouisfed.org/fred2/)
Figure 4. Total Share Prices for All Shares Index $Y_t$ and House Price $H_t$ in the U.K.

Figure 5. Total Share Prices for All Shares Index $\Delta Y_t$ and House Price $\Delta H_t$

Figure 6. Smoothed probabilities of regime 1 $\Pr(S_t = 1 | \Delta Y_t)$

From (Figure 6), we can find that the inferred probabilities $\Pr(S_t = 1 | \Delta Y_t)$ accord with U.K recessionary dates during 1970s’ oil shocks periods. And, the uncertainty of
house price and stock price has risen during stock collapse in 1980s, IT bubble collapse in early 2000s and financial shock in 2008.

From (Figure 3), (Figure 6), we can find the common business cycle between U.S. and U.K.

To find out common business cycle between housing price and stock price for U.S. and U.K., this paper adopted FIML Markov-Switching Model by Yoon (2006), which assumes common two-state probabilities in the simultaneous equations.

\[
\Delta Y_{us} = \alpha_{S_{t,us}} + \beta_{H_{t,us}} \Delta H_{us} + e_{S_{t,us}} \tag{5}
\]

\[
\Delta Y_{uk} = \alpha_{S_{t,uk}} + \beta_{H_{t,uk}} \Delta H_{uk} + e_{S_{t,uk}} \tag{6}
\]

where \( \alpha_{St} = \alpha_0 S_t + \alpha_1 (1 - S_t) \), \( \beta_{St} = \beta_0 S_t + \beta_1 (1 - S_t) \) for U.S and U.K.

\[
Pr(S_t = 0 | S_{t-1} = 0) = q, \ Pr(S_t = 1 | S_{t-1} = 1) = p,
\]

\[
p = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix},
\]

To solve the equation (5) and (6) together, we can rewrite it as follows:

\[
\begin{bmatrix} \Delta Y_{us} & \Delta Y_{uk} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \Delta H_{us} & \Delta H_{uk} \end{bmatrix} \begin{bmatrix} \beta_{H_{t,us}} & 0 \\ 0 & \beta_{H_{t,uk}} \end{bmatrix} - \begin{bmatrix} \alpha_{S_{t,us}} & \alpha_{S_{t,uk}} \end{bmatrix} = \begin{bmatrix} e_{S_{t,us}} & e_{S_{t,uk}} \end{bmatrix}
\]

where \( e_{S_{t,us}} \sim i.i.d. N(0, \Sigma_{St} \otimes I_T) \).

\[
\Sigma_{St} = \begin{bmatrix} \sigma_{S_{t,us}} & 0 \\ 0 & \sigma_{S_{t,uk}} \end{bmatrix}, \quad \alpha_{St} = \alpha_1 S_t + \alpha_0 (1 - S_t), \quad \beta_{St} = \beta_1 S_t + \beta_0 (1 - S_t),
\]

\[
Pr(S_t = 0 | S_{t-1} = 0) = q, \ Pr(S_t = 1 | S_{t-1} = 1) = p, \quad p = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}
\]

(Table 3) reports estimation results in the FIML Markov Switching Model using quarterly data for 1970:II-2015:I. The variance \( \sigma_0 \) is significant. The variance \( \sigma_1 \) is also significant and showed large volatility during the regime 1 period in the model because \( \sigma_1 \) is larger than \( \sigma_0 \).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>US</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>4.9341 (0.7901)</td>
<td>3.4337 (0.5782)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-5.7988 (2.2781)</td>
<td>-3.6262 (2.5833)</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-0.9416 (0.3681)</td>
<td>-0.0974 (0.1654)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>1.1624 (0.7439)</td>
<td>0.1204 (0.7124)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>29.4105 (4.8361)</td>
<td>22.7819 (3.2466)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>126.9756 (31.3374)</td>
<td>176.6978 (45.5228)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.8994 (0.0367)</td>
<td>0.6594 (0.1094)</td>
</tr>
</tbody>
</table>

Log Likelihood: -1205.40

Standard errors of the parameters estimates are reported in the parentheses.

Figure 7. Common smoothed probabilities of regime 1 $Pr(S_t = 1 | Y_t)$ for U.S and U.K

(Figure 7) show that common smoothed probabilities $Pr(S_t = 1 | Y_t)$ accord well with U.S. smoothed probabilities $Pr(S_t = 1 | Y_t)$ in Figure 3. The common periods of regime 0 is $1/(1-0.8994) = 9.9$ quarters. However, the common periods of regime 1 is $1/(1-0.6594) = 2.9$ quarters which is shorter than regime 0 periods.

(Figure 7) show that common smoothed probabilities accord well with oil price shock periods during 1970s’, and 1990’s. However there was another common business cycle during S&L crisis (1987:I-1988:I) and housing bubble bursts (2008:II-2009:II). From
these results in (Table 3) and (Figure 7), there was the evidence of the international common business cycle between housing price and stock price with large shocks.

4 Conclusion

As we applied to the Markov-Switching Model for 1970-2015 periods, the findings of this paper are as follows: House price is common business cycle with stock price during 1970s’ and 1990s’ oil shocks periods, and house price is also common business cycle with stock price during 1980s, IT bubble collapse and financial shock in 2008. These results suggest another explanation that when there are big shocks such as oil shocks, house price is common business cycle with stock price and the common business cycle also occurs in conjunction with big shocks including housing bubble bursts.

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References