Outsourcing and Import Restriction Policies

Kai-Hsi Chu and Kar-yiu Wong
University of Washington

April 21, 2008
Abstract

This paper examines the issues of outsourcing and corresponding policy interventions by the government. We consider a model with three countries, the North, the South, and the Rest of the World. There is a single firm in the North producing a final product using heterogeneous intermediate inputs from the local market and heterogeneous intermediate inputs from the South. We begin with a situation in which no government interventions are allowed, and examine how the output and profit of the final-product firm may be dependent on some of the exogenous variables. The government of the North, when it is allowed to intervene, can choose one of the three options: (a) to limit the quantity of each type of variety of the foreign intermediate inputs to be imported; (b) to limit the number of varieties of the foreign intermediate inputs to be imported; and (c) to impose a tariff on the imported intermediate inputs. For each policy, the optimal intervention is derived. The analysis can be used to examine the argument for restricting outsourcing.
1 Introduction

Outsourcing has become a very important feature of the ongoing process of globalization, and a centerpiece of the many policy debates and protectionist arguments. On the one hand, many people recognize the rising significance of outsourcing and the growing reliance of multinational corporations on it as a way to gain international competitiveness, and on the other hand, many people are viewing it as a threat to their jobs and earnings and are using it as a reason to argue for protectionist measures.

Outsourcing is the transfer of parts of a production process, which are initially done within the local plant or factory, to other firms in the same country or in other countries.\(^1\) Recently, it is getting more and more common to have final products consisting of components made in different countries. For example, computer companies such as Dell outsource many of their components, and footwear firm Nike only keeps R&D and coordination at home while outsources most of its production to southeast Asia. Grossman and Helpman (2005) cite from recent annual report of the World Trade Organization (1998) describing the production of a particular “American” car:

30 percent of the car’s value goes to Korea for assembly, 17.5 percent to Japan for components and advanced technology, 7.5 percent to Germany for design, 4 percent to Taiwan and Singapore for minor parts, 2.5 percent to the United Kingdom for advertising and marketing services, and 1.5 percent to Ireland and Barbados for data processing. This means that only 37 percent of the production value ... is generated in the United States. (p.36)

Why would firms choose to outsource? One argument offered is based on the existence of incomplete contracts; See, for example, Antras and Helpman (2003) and Grossman and Helpman (2005). Feenstra and Hanson (1996) argue that the difference in endowments is the fundamental reason for comparative advantage and international outsourcing: Companies in industrialized countries shift labor-intensive stages of their production process to labor abundant countries with lower wage rates. On the other hand, Ethier (1979) and Hummels et al. (2001) use a Ricardian model to explain the pattern of

\(^{1}\)A variety of terms have been used in the literature or the media to refer to this or related phenomena: for example, the “slicing of the value chain”, “international outsourcing”, “fragmentation of the production process”, “vertical specialization”, “global production sharing”, “disintegration of production”, “multi-stage production”, “intra-product specialization,” “offshoring” (for the transfer of the production activity to a firm in another country), and so on.
vertical specialization in intermediate inputs.

The purpose of this paper is to analyze several issues related to outsourcing. It begins with the choice of a local firm in terms of outsourcing. It then examines the impacts of several policies available to a local government, and derives the optimal use of each of these policies. In particular, it will look at the wisdom of restricting outsourcing, which was raised in recent discussion about the appropriate response of a local government to outsourcing. In analyzing outsourcing, this paper takes the convenient approach of considering the use of intermediate inputs by a local firm, some from a local market and some from other countries.\footnote{There is a large literature on trade in intermediate inputs; See, for example, Ethier (1982), Jones (2000), and Feenstra and Hanson (2001). Many papers have focused on the volume of trade in intermediate inputs; for example, Yeats (2001), Hummels et. al.(2001), and Kleinert (2003).} Outsourcing in the present paper refers to the increase in the use of foreign intermediate inputs.\footnote{Feenstra and Hanson (1999) focus on vertical specialization with goods being made in multiple stages located in different countries.} The framework constructed here allows us to examine the impacts of three different government policies: (a) to restrict the quantity of import of each type of intermediate inputs to be used by a local final-product firm; (b) to restrict the number of types of intermediate inputs to be used by a local final-product firm; and (c) to impose a tariff on the imported intermediate inputs.

The rest of the paper is organized as follows. Section 2 describes the model to be used in the present paper. It consists of three countries (the North, the South, and the Rest of the World), a final product, and intermediate inputs. The section explains the technology to produce the final product and the market structure in the North. Section 3 focuses on the production and technologies of the intermediate inputs in the North and the South. The equilibrium under free trade is derived. Section 4 explains how the welfare of the North is measured. It then analyzes the impacts of each of the three policies described above. Whether it makes sense to restrict outsourcing using these policies is examined. Section 5 presents some concluding remarks.

\section{The Model}

Consider three countries and a final product. The countries are labelled N (North), S (South), and R (rest of the world). The final product is produced
only in country N by a single firm using two types of heterogeneous intermediate inputs, one of which comes from the local economy and the other type is imported from country S. All transport costs are negligible. For the time being, free trade is assumed with no government intervention.

The demand for the final product exists in country R only, and thus the output of the firm in country N will be exported to country R. The demand can be represented by \( p = p(y) \), where \( p \) is the market price and \( y \) is the output, which is equal to the level of export. Using a prime to denote a derivative, we assume that \( p' < 0 \), and \( p'' \) is less than a sufficiently small positive number. There are \( N^n \) varieties of intermediate inputs in country N and \( N^s \) varieties of intermediate inputs in country S, where \( N^n \) and \( N^s \) are to be determined endogenously but are taken as given by the final-product firm. The production function of the final product is given by

\[
y = \sum_{i=1}^{N^n} \gamma^n(x^{in}) + \sum_{i=1}^{N^s} \gamma^s(x^{is}),
\]

where \( x^{ij} \) is the quantity of intermediate input of variety \( i \) from country \( j \), \( j = n, s \), and function \( \gamma^j(x^{ij}) \) satisfies the following conditions: \( \gamma'^j > 0 \), \( \gamma''^j < 0 \), and \( \gamma'''^j \) is less than a sufficiently small positive number. Intermediate inputs from each country enter the production function in a symmetric way.

The profit of the final-product firm can be defined as

\[
\Pi = py - \sum_{i=1}^{N^n} r^{in}x^{in} - \sum_{i=1}^{N^s} r^{is}x^{is},
\]

where \( r^{ij} \) is the price of variety \( i \) from country \( j \). Taking the numbers of varieties and their prices as given, the firm chooses the intermediate inputs to maximize its profit, subject to the production function defined in (1). Define \( \phi \equiv \phi(y) \equiv p + p'y \) as the marginal revenue of the final product, which falls with \( y \), \( \phi' < 0 \). The first-order conditions are (assuming an interior solution):

\[
\phi(y)\gamma'^j(x^{ij}) = r^{ij}.
\]

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4 The production function is simplified by assuming no primary inputs.

5 For simplicity, we assume that the final-product firm exploits the monopoly power on the output side but not the monopsony power on the input side because of a large number of intermediate inputs.

6 This is due to the assumption that \( p'' \) is less than a sufficiently small positive number.
Equation (3) represents an inverse derived demand for intermediate input \( x^{ij} \), and can be written as \( r^{ij} = r^{ij}(x^{ij}, y) \). Differentiate (3) to give

\[
dr^{ij} = \phi \gamma^j \phi' \gamma^j d\phi + \phi' \gamma^j dy,
\]

which shows that \( r^{ij} \) depends negatively on \( x^{ij} \) and \( y \). We assume that the number of varieties of intermediate inputs in each country is sufficiently large so that a small change in one of the input prices will not affect much the total output. As a result, each intermediate-input firm takes the value of \( y \) in equation (3) as given, implying that equation (3) can be regarded as the derived demand for an intermediate input, with a slope equal to (evaluated at a given level of \( y \)):

\[
\frac{dx^{ij}}{dr^{ij}} = \frac{1}{\phi \gamma^j (x^{ij})} < 0.
\]

3 Intermediate Sectors

The intermediate good sector in each country is characterized by monopolistic competition, heterogeneous products, and increasing returns. Each variety is produced by a firm using a technology that exhibits increasing returns, with the labor input required to produce \( x^{ij} \) units of variety \( i \) in country \( j \), \( j = n, s \), given by

\[
\ell^{ij} = \alpha^j + \beta^j x^{ij},
\]

where \( \alpha^j, \beta^j > 0 \). The profit of the firm producing variety \( i \) in country \( j \) is

\[
\pi^{ij} = r^{ij} x^{ij} - w^j(\alpha^j + \beta^j x^{ij}),
\]

where \( w^j \) is the wage rate in country \( j \). The assumption of a large number of varieties of intermediate inputs in each country implies that the two intermediate input sectors can be solved separately, and in the same way.

The profit of each intermediate-input firm is maximized by choosing the optimal output, taking the inverse demand function, final-product output level \( y \), and the wage rate as given. The first-order condition is

\[
\frac{d\pi^{ij}}{dx^{ij}} = r^{ij} + r^{ij}_x x^{ij} - w^j \beta^j = 0.
\]

Let us denote the marginal revenue by \( \theta^{ij} \equiv r^{ij} + r^{ij}_x x^{ij} = \phi (\gamma^j + x^{ij} \gamma^j) \), where a subscript for \( r^{ij} \) represents a partial derivative. The derivative of
the marginal revenue is \( \theta_{ji}^{ij} = 2r_{xi}^{ij} + r_{xix}^{ij} = \phi(2\gamma_{xj}^{ij} + x^j \gamma_{xx}^{ij}) \), which is assumed to be negative.\(^7\) Note that all input firms in the same country are symmetric both on the demand side and on the cost side. Thus in equilibrium they must make the same production decision. This allows us to drop the subindex “i” for each firm unless confusion may arise. Thus, using (3), the profit-maximization condition can be written as

\[
\phi(\gamma^j + x^j \gamma_{xx}^{ij}) = w^j \beta^j. \tag{9}
\]

Condition (9) can be illustrated in Figure 1. The demand \( D^j \) is obtained from (3), which yields the marginal revenue function \( \theta^j \), or \( MR^j \), which is assumed to be declining with \( x^j \). The marginal cost is equal to \( w^j \beta^j \). Point E, the intersection point between the \( MR^j \) curve and the horizontal line at \( w^j \beta^j \), gives the equilibrium output that satisfies condition (9). Since it is assumed that \( \theta^j < 0 \), the second-order condition holds.

Condition (9) gives a relation between \( w^j \) and \( x^j \), which can be illustrated by curve PP in Figure 2. The slope of PP is

\[
\left. \frac{dw^j}{dx^j} \right|_{PP} = \frac{\phi(2\gamma_{xj}^{ij} + x^j \gamma_{xx}^{ij})}{\beta} < 0. \tag{10}
\]

Note that in the region left (right) of curve PP, \( \theta^j > (\leq) w^j \beta^j \).

Free entry and exit exists drives the profit of each existing firm to zero. From (7), zero profit implies

\[
\phi \gamma^j x^j = w^j (\alpha^j + \beta^j x^j), \tag{11}
\]

which gives another relation between \( w^j \) and \( x^j \), and can be illustrated by curve ZZ in Figure 2. The slope of ZZ can be obtained by differentiating condition (11) and rearranging terms:

\[
\left. \frac{dw^j}{dx^j} \right|_{ZZ} = \frac{\theta^j - w^j \beta^j}{\alpha^j + \beta^j x^j}. \tag{12}
\]

As explained above, \( \theta^j - w^j \beta^j \) is the derivative of the profit function with respect to output, and is positive (negative) in the area left (right) of curve

\(^7\)The derivative \( \theta_{ji}^{ij} \) involves the third derivative of \( \gamma^j \), and thus its sign is ambiguous, but we assume that it is less than a sufficiently small positive number. This guarantees that the second-order condition for the input firm’s profit maximization is satisfied.
PP and condition (12) implies that curve ZZ is positively (negatively) sloped, as shown in Figure 2. Curve ZZ reaches a maximum at the intersection point with PP. The region below (above) curve ZZ represents positive (negative) profit.

The intersection between curves PP and ZZ in Figure 2, point F, represents the free-trade equilibrium $(x^j^f, w^j^f)$, where the superscript “$f$” denotes the value of a variable under free trade. This equilibrium is also shown in Figure 1. The $AC^j$ curve represents the average cost, $w^j(\beta^j + \alpha^j/x^j)$. Zero profit means that the demand for the firm’s output $D^j$ will be such that it touches $AC^j$ at the equilibrium input price $r^j$. Equations (9) and (11) are solved for the input production level and wage rate in country $j$, $x^j$ and $w^j = w^j(y)$. Note that the value of $x^j$ is independent of the value of $y$, but that of $w^j$ is.

From the profit-maximization or zero-profit condition, it is easy to get

$$w^j = \frac{\partial \gamma^j x^j}{\partial \alpha^j + \beta^j x^j} < 0. \quad (13)$$

From (3), we can write the corresponding input price as $r^j = \tilde{r}^j(x^j, y) = \phi(y)\gamma^j(x^j)$.

To explain how the number of varieties is determined, suppose that labor for the intermediate input production is withdrawn from other parts of the economy according to the following function,

$$L^j = L^j(w^j), \quad (14)$$

where $L''(w) > 0$. Thus $w^j$ can be interpreted as the opportunity cost of using one extra unit of labor in this market. Function (14) can be inverted to give

$$w^j = \omega^j(L^j), \quad (15)$$

which can be interpreted as the minimum wage rate required to attract an amount of $L^j$ labor from the rest of the economy. The derivatives of functions $L^j(\cdot)$ and $\omega^j(\cdot)$ depend on the cost of attracting labor from the rest of the economy. If the market considered is only a small part of the economy so that the wage rate is not too much affected by the employment in the market,

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8. Note that when equation (9) is divided by equation (11) both $\phi(y)$ and $w^j$ are cancelled out, leaving $x^j$ to be determined. Thus with no government intervention, $x^j$ is independent of $y$. 

6
is very large (or \( \omega_j \) very small). Using the production function in (6), the equilibrium of the labor market is

\[ L^j = N^j(\ell^j) = N^j(\alpha^j + \beta^j x^j). \]

Using the free-trade equilibrium described above, the number of varieties in country \( j \) under free trade is

\[ N^j = N^j(w^j, x^j) = \tilde{N}^j(y) = \frac{L^j(\tilde{w}^j(y))}{(\alpha^j + \beta^j x^j)}. \]

Differentiation gives

\[ \tilde{N}^j_y = \frac{L^j d\tilde{w}^j / dy}{(\alpha^j + \beta^j x^j)} < 0. \]

The number of varieties can also be shown in Figure 2, which shows the contour NN representing \( \tilde{N}^j(y) \) passing through point F. Contours below (above) NN represent smaller (larger) number of varieties. We now close the model by explaining how the final-product output is determined. Making use of the production function:

\[ y = N^n(y)\gamma^n(x^n) + N^s(y)\gamma^s(x^s), \]

which can be solved for the free-trade output of the final product, \( y^f \). The corresponding values of the intermediate-input markets are \( w^j = \tilde{w}^j(y^f) \), and \( N^j = N^j(y^f) \). It should be noted that the derivative of the RHS of (18) is assumed to be less than unity. This assumption is needed for normal comparative-static results, as will be shown later.

## 4 Policies to Control Outsourcing

We now examine the following policies for the government of country N to control outsourcing. We will derive the optimal policy parameter under each of the policy options under the assumption that the governments of countries S and R remain passive in policy choice. The policy options analyzed in the present paper are:

1. to restrict the quantity of each of the imported intermediate inputs;
2. to restrict the number of varieties of the imported intermediate inputs;
3. to impose a tariff on all the imported intermediate inputs.
4.1 Welfare of Country N

The social welfare of country N can be defined as follows:

\[ W^n = \Pi + N^n \pi^n + \left( w^n L^n - \int_0^{L^n} \omega^n(v) dv \right). \]  \hspace{1cm} (19)

In (19), the first term is the profit of the final-product firm, and the second term is the total profit of the intermediate-input firms. The terms inside the parentheses is the surplus generated by hiring \( L^n \) from the rest of the economy: \( w^n L^n \) is the income generated and \( \int \omega^n(v) dv \) is the total opportunity cost of hiring \( L^n \).

Substitute the firms’ profit functions in (2) and (7) into (19) to get

\[ W^n = py - N s r s x s - \int_0^{L^n} \omega^n(v) dv. \]  \hspace{1cm} (20)

Condition (20) can be used to evaluate the social welfare level at different equilibria. For example, under free trade, the social welfare level is

\[ W^{nf} = p^f y^f - N s r s x^s - \int_0^{L^n f} \omega^n(v) dv. \]  \hspace{1cm} (21)

The welfare level in (21) will be used as the benchmark, with which the welfare levels under different policies are compared. The three policies mentioned above are now analyzed one by one.

4.2 Quantitative Restriction

The first policy for country N we examine is quantitative restriction. Suppose that government N imposes a ceiling \( \bar{x}^s < x^s \) on the quantity of each of the imported intermediate inputs, i.e., \( x^{is} \leq \bar{x}^s \) for all \( i \). This policy will likely create a gap between the price of each input in country S and that in country N. We assume that it is the producers, not the local government, who pick up this quota premium.

Let us first examine the impacts of this policy. Because the output of each intermediate-input firm in country S goes to country N, the restriction on import means a direct restriction on the output of the intermediate-input firms in country S. As a result, the firms may not be able to maximize their profits. This implies that condition (9) and curve PP for country S are no
longer applicable. On the other hand, because of free entry and exit, the
profit of each of the firms in country S will be zero at the new equilibrium.
This means that equation (11) and curve ZZ still hold. Treating \( x^s \) as a
parameter, write the wage rate in country S as \( w^s = \tilde{w}^{sq}(y; x^s) \). As Figure 3 shows, a drop in \( x^s \), when given \( y \), will lower \( w^s \), measured along curve ZZ:

\[
\frac{\partial \tilde{w}^{sq}}{\partial x^s} = \frac{\theta^s - w^s \beta^s}{\alpha^s + \beta^s x^s} > 0,
\]

(22)

where for \( x^s < x^{sf} \), \( \theta^s > w^s \beta^s \). The derived demand implies that a quantitative restriction will raise the wage rate, when \( y \) is given. The effect of a change in the final-product output on the wage rate is

\[
\frac{\partial \tilde{w}^{sq}}{\partial y} = \frac{\delta^s \gamma^s x^s}{\alpha^s + \beta^s x^s} < 0,
\]

(23)

where \( x^s \) is evaluated at \( \bar{x}^s \). The number of varieties in country S is given by (16), and we can write \( N^s = \tilde{N}^{sq}(y; x^s) \). When taking \( y \) as given, the effect of the policy is

\[
\frac{\partial \tilde{N}^{sq}}{\partial x^s} = \frac{(\alpha^s + \beta^s x^s) L^s \partial \tilde{w}^{sq} / \partial x^s - L^s \beta^s}{(\alpha^s + \beta^s x^s)^2}.
\]

(24)

The sign of the expression in (24) is ambiguous, but if the current market is a small part of the economy so that \( L^s \) is sufficiently large and if \( \partial \tilde{w}^{sq} / \partial x^s \) is finite, then \( \partial \tilde{N}^{sq} / \partial x^s > 0 \). The effect of a change in \( y \) on the number of varieties is

\[
\frac{\partial \tilde{N}^{sq}}{\partial y} = \frac{L^s (\partial \tilde{w}^{sq} / \partial y)}{\alpha^s + \beta^s x^s} < 0.
\]

(25)

We now examine the effects of the policy on country N. Refer back to equation (18), except that \( x^s \) is now treated as a parameter. Differentiate it and rearrange the terms to give

\[
\frac{dy}{dx^s} = \frac{N^s \gamma^s + \gamma^s \partial \tilde{N}^{sq} / \partial x^s}{1 - \gamma^s dN^s / dy - \gamma^s \partial \tilde{N}^{sq} / \partial y} > 0,
\]

(26)

where the denominator is positive.\(^9\) Condition (26) means that a restriction on \( x^s \) for each of the intermediate inputs imported from country S will lower

\(^9\)See the explanation given above.
the output of the final product. Using equations (24) to (26), the total effects of a change in $x^s$ on the number of varieties in country $S$ is
\[
\frac{dN^s}{dx^s} = \frac{\partial \tilde{N}^{sq}}{\partial x^s} + \frac{\partial \tilde{N}^{sq}}{\partial y} \frac{dy}{dx^s}.
\]
Equation (26) can also be used to examine the effects of a change in $x^s$ on the wage rate in country $N$,
\[
\frac{dw^n}{dx^s} = \frac{dw^{nf}}{dy} \frac{dy}{dx^s} < 0.
\]
We now examine how the policy may affect the welfare of the market. Again, treating $x^s$ as a parameter, differentiate the welfare function in (20) with respect to $x^s$:
\[
\frac{dW^n}{dx^s} = -r^s x^s \frac{dN^s}{dx^s} - N^s x^s \frac{dr^s}{dx^s} - w^n L^n w^{nf} \frac{dy}{dx^s},
\]
where $L^n = dL^n/dw^n$ and $w^{nf} = dw^{nf}/dy$. Conditions (23), (24), and (27) can be used to simplify (28). In general, the sign of the derivative in (28) is ambiguous. However, we can evaluate it in two special cases. First, consider the small region near the free-trade equilibrium. As Figure 2 shows, the zero-profit curve, ZZ, has a horizontal tangent, which means that a small change in $x^s$ will marginally not affect the wage rate. Condition (24) reduces to
\[
\frac{dN^s}{dx^s} = -\frac{L^s \beta^s}{(\alpha^s + \beta^s x^s)^2} < 0.
\]
Similarly, equation (28) reduces to
\[
\frac{dW^n}{dx^s} = -r^s x^s \frac{dN^s}{dx^s} - N^s x^s \frac{dr^s}{dx^s} > 0.
\]
Equation (30) implies that a small restriction on the volume of import is detrimental. This result is not surprising, as the policy severely restricts the choice of the local final-product firm and the use of labor.

Second, assume that $x^s$ is very small so that only small amounts of intermediate inputs are allowed from country $S$. Equation (28) reduces to
\[
\frac{dW^n}{dx^s} = -w^n L^n w^{nf} \frac{dy}{dx^s} > 0.
\]
Equation (31) implies that in the small region with small trade volume, allowing more import of intermediate inputs benefits country N. Another way to put it is that (at least a small volume of) trade is good.\(^{10}\)

The above results are summarized by the following proposition:

**Proposition 1** In the present framework, the gain from (at least a small) trade is positive for country N.

**Proposition 2** With the government N collecting none of the quota premia, the market is hurt by the following quantitative restriction: (i) a small restriction; (ii) a large restriction so that the imported quantity of each type of intermediate input is small.

### 4.3 Variety Restriction

We now turn to the second policy option for country N, namely, restricting the number of types of intermediate inputs imported from country S, i.e., the government of country N setting the maximum number of the varieties to be less than the free-trade number, \(\bar{N}^s < N^s_f\).

The restriction on the number of varieties of intermediate inputs in country S prevents free entry and exit, and thus zero profit is not guaranteed. In other words, condition (11) and curve ZZ in Figure 2 are no longer applicable. Firms, however, can still maximize their profits by choosing the optimal outputs, and equation (9) holds:

\[
\theta^s(x^s, y) = w^s\beta^s,
\]

recalling that firms take the output level \(y\) as given. Because of the constraint, condition (16) can be written as

\[
\bar{N}^s = \frac{L^s(w^s)}{(\alpha^s + \beta^s x^s)}.
\]

\(^{10}\)Equation (31) assumes that with small import of intermediate inputs the profit of the final-product firm is positive. If the profit of the firm is zero so that it chooses not to produce when no or a small volumes of intermediate inputs are allowed, then \(dW^s/dx^s = 0\) as the welfare of the market is zero.
Conditions (32) and (33) can then be solved for \( x = \tilde{x}^{sv}(y; \tilde{N}^s) \) and \( w = \tilde{w}^{sv}(y; \tilde{N}^s) \). Keeping \( N^s \) as a parameter, differentiate (32) and (33) to give:

\[
\begin{bmatrix}
2r_x^s + x^s_{xx} & -\beta^s \\
N^s \beta^s & -L^{st}
\end{bmatrix}
\begin{bmatrix}
dx^s \\
dw^s
\end{bmatrix}
= -
\begin{bmatrix}
r_y^s \\
0
\end{bmatrix}
dy -
\begin{bmatrix}
0 \\
\alpha^s + \beta^s x^s
\end{bmatrix}dN^s,
\]

which are solved to give:

\[
dx^s = \frac{r_y^s L^{st} dy - \beta^s (\alpha^s + \beta^s x^s) dN^s}{D},
\]

\[
dw^s = \frac{N^s \beta^s y dy - (2r_x^s + x^s_{xx}) (\alpha^s + \beta^s x^s) dN^s}{D},
\]

where \( D = -(2r_x^s + x^s_{xx}) L^{st} + N^s (\beta^s)^2 > 0 \). From (35) and (36), we have the following signs of the partial derivatives: \( \tilde{x}_y^{sv} < 0, \tilde{x}_N^{sv} < 0, \tilde{w}_y^{sv} < 0, \) and \( \tilde{w}_N^{sv} > 0 \).

The equilibrium is illustrated in Figure 4. Point F is the free-trade equilibrium point, through which the number-of-variety contour NN representing \( N^{sf} \) passes. The present constraint can be illustrated by contour NN', which is lower than contour NN. The intersecting point V between curve PP and contour NN' gives the new equilibrium, representing a rise in \( x^s \) but a drop in \( w^s \).

The market in country N can be solved in the same way as before. The determined functions of intermediate outputs and number of varieties are then substituted back to the production function:

\[
y = N^{nf}(y) \gamma^n(x^{nf}) + N^s \gamma^s (\tilde{x}^{sv}(y, N^s)),
\]

which can be solved for the final-product output. With \( N^s \) treated as a parameter, differentiate both sides of (37) totally and rearrange terms to give:

\[
\frac{dy}{dN^s} = \gamma^s + N^s \gamma^s \tilde{x}_N^{sv} \frac{\gamma^n}{B^v},
\]

where \( B^v = (1 - \gamma^n N^{nf} - N^s \gamma^s \tilde{x}_y^{sv}) > 0 \). Consider the following condition:

\[
\gamma^s + N^s \gamma^s \tilde{x}_N^{sv} < 0.
\]

Condition (C) is satisfied if the number of varieties in country N is sufficiently large, and if it is satisfied, the total effect of \( N^s \) on \( y \) is negative. So if and
only if condition (C) is satisfied, \( \frac{dy}{dN^s} < 0 \). Making use of (38), the total effects of a change in \( N^s \) are as follows:

\[
\frac{dx^{sv}}{dN^s} = x^{sv} + \tilde{x}^{sv} \frac{dy}{dN^s} \tag{39}
\]

\[
\frac{dw^{sv}}{dN^s} = w^{sv} + \tilde{w}^{sv} \frac{dy}{dN^s} \tag{40}
\]

If condition (C) is not satisfied, then \( \frac{d\tilde{x}^{sv}}{dN^s} < 0 \), or if condition (C) is satisfied, then \( \frac{d\tilde{w}^{sv}}{dN^s} > 0 \).

We now try to examine the welfare impacts of this policy for country N. The welfare function is still given by equation (20), with \( N^s = \bar{N}^s \). Treating \( N^s \) as a parameter, differentiate the welfare function:

\[
\frac{dW^n}{dN^s} = -r_s x^{sv} - N^s x^{sv} \left( r_x^{sv} \frac{dx^{sv}}{dN^s} + r_y^{sv} \frac{dy}{dN^s} \right) - w^m L^m w^nu \frac{dy}{dN^s}. \tag{41}
\]

The sign of the derivative in (41) is in general ambiguous. If we consider a small drop in the number of varieties from the free-trade level, \( w^nu \) would be very small. Equation (41) reduces to

\[
\frac{dW^n}{dN^s} = -r_s x^{sv} - N^s x^{sv} \left( r_x^{sv} \frac{dx^{sv}}{dN^s} + r_y^{sv} \frac{dy}{dN^s} \right). \tag{42}
\]

Thus either (i) if condition (C) is not satisfied and if \( r_y^{sv} \) is sufficiently small, or (ii) if condition (C) is satisfied and if \( \frac{dx^{sv}}{dN^s} < 0 \), then the derivative in (42) is negative, which means that a small drop in \( N^s \) is beneficial to country N. On the other hand, we showed earlier that the gain from free trade is positive. This means that the welfare of the market when \( N^s = 0 \) is lower than that when the economy is under free trade. Thus there exists an optimal number of imported varieties that will maximize the local welfare, where the optimal number is obtained by solving the following equation:12

\[
-r_s x^{sv} - N^s x^{sv} \left( r_x^{sv} \frac{dx^{sv}}{dN^s} + r_y^{sv} \frac{dy}{dN^s} \right) - w^m L^m w^nu \frac{dy}{dN^s} = 0. \tag{43}
\]

The above results are summarized by the following proposition:

\[\text{11} \text{Recall that the free-trade equilibrium occurs at a point on the zero-profit curve with a zero slope. See Figure 2.} \]

\[\text{12} \text{The second-order condition for a maximum is assumed.} \]
Proposition 3 If either (i) condition (C) is not satisfied and if \( r^s \) is sufficiently small, or (ii) condition (C) is satisfied and if \( dx^s/dN^s < 0 \), then a small cut in the number of imported varieties benefits country N. The optimal number of varieties to be imported from country S is the one that solves equation (43).

4.4 The Tariff Policy

We now examine the use of a tariff on the imported intermediate inputs. Denote the per unit tariff by \( t > 0 \), which can drive a wedge between the price of the Southern intermediate input in country N, \( r^s \), and that in country S, \( \hat{r}^s \). In equilibrium, we have

\[ r^s = \hat{r}^s + t. \]  

(44)

Despite the tariff, for the intermediate-input firms in country S, the profit-maximization and zero-profit conditions are still satisfied. More specifically, for country S,

\[ \theta^s = w^s \beta^s + t \]  

(45)

\[ r^s x^s = w^s(\alpha^s + \beta^s x^s) + tx^s, \]  

(46)

where (45) describes profit maximization and (46) represents zero profit. These two equations are solved for the output and wage rate in country S, \( x^{st} = x^{st}(t, y) \) and \( w^{st} = w^{st}(t, y) \). It should be noted that \( x^s \) is now generally a function of \( t \) and \( y \). Differentiate (45) and (46) totally and rearrange terms to give

\[
\begin{bmatrix}
2r^s + x^{st}r^s_{xx} & -\beta^s \\
0 & -(\alpha^s + \beta^s x^{st})
\end{bmatrix}
\begin{bmatrix}
dx^{st} \\
dw^{st}
\end{bmatrix} = \begin{bmatrix}
1 \\
x^{st}
\end{bmatrix} dt - \begin{bmatrix}
r^s_y \\
x^{st}r^s_y
\end{bmatrix} dy,
\]

which are solved to give

\[ dx^{st} = \frac{-(\alpha^s + \beta^s x^{st})dt + \alpha^s r^s_y dy}{D''} \]  

(47)

\[ dw^{st} = \frac{\theta^{st} x^{st} dt - \theta^{st} x^{st} r^s_y dy}{D''}, \]  

(48)

where \( D'' = -\theta^{st}(\alpha^s + \beta^s x^{st}) > 0 \) and \( \theta^{st} = 2r^s_x + x^{st} r^s_{xx} < 0 \). Note that all intermediate-input firms take \( y \) as given. Equations (47) and (48) imply that
an increase in the tariff rate or the output of the final product will lower both $x^s$ and $w^s$. The dependence of the variables on the tariff rate can be illustrated in Figure 5. A rise in the tariff rate will shift curves PP and ZZ to the left (to $P'P'$ and $Z'Z'$, respectively), resulting in a drop in both $x^s$ and $w^s$. The resulting change in the number of varieties is

$$N^s = N^s(t, y) = \frac{L^s(\tilde{w}^s(t, y))}{\alpha^s + \beta^s \tilde{x}^s(t, y)}.$$  (49)

Differentiate (49) totally to give:

$$dN^s = \frac{(\alpha^s + \beta^s x^s) L^s(\tilde{w}^s dt + \tilde{w}^s dy) - L^s \beta^s (\tilde{x}^s dt + \tilde{x}^s dy)}{(\alpha^s + \beta^s x^s)^2}.$$  (50)

The sign of the term on the right-hand side of (50) for each country is ambiguous. If the labor market is not too wage sensitive so that $L^s$ is sufficiently large, then $N^s$ will react negatively to an increase in $t$ or $y$.

The effect of the tariff on the final-product output can be obtained by making use of (18):

$$y = N^n(y) \gamma^n(x^{nf}) + \tilde{N}^s(t, y) \gamma^s(\tilde{x}^s(t, y)),$$

which can be differentiated to give

$$\frac{dy}{dt} = \frac{N^s \gamma^s \tilde{x}^s + \gamma^s \tilde{N}^s}{B^t} < 0,$$  (51)

where $B^t = 1 - \gamma^n N^n - \gamma^s \tilde{N}^s \tilde{x}^s > 0$, where $(\gamma^s + \gamma^s \tilde{x}^s) > 0$ is the marginal revenue faced by an intermediate-input firm in country S, and where the number of varieties in each country depends negatively on $y$. Condition (51) implies that an increase in $t$ will lower the final product output level.

We now examine the welfare impact of the tariff policy for country N. The welfare function of the country is still given by condition (20). Differentiate the welfare function with respect to the tariff rate to yield:

$$\frac{dW^n}{dt} = -r^s x^s \frac{d\tilde{N}^s}{dt} - N^s x^s \left( r^s \frac{d\tilde{x}^s}{dt} + r^s \frac{dy}{dt} \right) - w^n L^n u^n \frac{dy}{dt}.$$  (52)

In general, the sign of the derivative in (52) is ambiguous. We can examine two special cases. First, consider the case in which $t$ is zero or small. This is
close to the free-trade equilibrium and the change in \( w^n \) is small. Then (52) reduces to
\[
\frac{dW^n}{dt} = -r^n x^{st} \frac{dN^{st}}{dt} - N^n x^{st} \left( r^n \frac{dx^{st}}{dt} + r^n y^{dy} \right). \tag{53}
\]
If, furthermore, the resulting change in the price of the intermediate inputs from country S is small, then the small tariff is beneficial. Second, consider the case when \( t \) is near prohibitive. Then the volume of import, \( x^s \), is small. Equation (52) reduces to
\[
\frac{dW^n}{dt} = -w^n L^n w^{nu} \frac{dy}{dt} < 0. \tag{54}
\]
Equation (54) means that when \( t \) is near prohibitive, a small reduction in the tariff rate is beneficial. This result confirms Proposition 1, i.e., (at least a small) trade is gainful. Equations (53) and (54) imply that if a small tariff imposed by country N does not change much the price of the intermediate inputs from country S, there exists an optimal tariff that maximizes the welfare of country N.

The above results are summarized by the following proposition:

**Proposition 4** If a small tariff imposed by country N does not change much the price of the intermediate inputs from country S, it is beneficial. Then there exists a positive optimal tariff that maximizes country N’s welfare.

5 Concluding Remarks

In this paper, we constructed a model to analyze the phenomenon of outsourcing. We explained the production of a final product by a firm and its use of intermediate inputs from the local market and those from a foreign market. We then used the model to analyze three policy options for the local government to restrict outsourcing. In the present model, the welfare of the local economy depends mainly on the profit of the final-product firm. While the firm is able to maximize its own profit, subject to certain conditions, externality arises because the firm does not take into consideration the effects of its decision on the price and the number of varieties of the imported intermediate inputs. Our analysis suggests that general restrictions on outsourcing are not the right policies to remove the externality. In many cases, restricting outsourcing could hurt the welfare of the economy.
One main reason why in the present model restricting outsourcing could be damaging is that all the policies considered will lead to a rise in the price of the imported intermediate inputs. In the case of a tariff, both the price of the imported inputs that the final-product firm is facing and the price of the inputs in the foreign country rise. This means that outsourcing restriction could jack up the cost of the imported intermediate inputs, and it will not be optimal unless it can substantially raise the profit of the final-product firm.
Figure 1
Zero Profit and Profit Maximization
Figure 2
Equilibrium under Free Trade
Figure 3
Quantitative Restriction
Figure 4
Variety Restriction
Figure 5
Tariff Restriction
References


