On the Dynamic Characteristics
of the Open Harris-Todaro Model[*]

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The dynamic characteristics of the open Harris-Todaro (H-T) economy are considered employing the dynasty model with incomplete international financial markets and one traded good and one non-traded consumption good. Under foreign capital inflow with its full repatriation, the economy is globally stable if the two consumption goods are complements and the non-traded consumption good is more capital intensive. For all the other cases, global stability does not follow.

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1. INTRODUCTION

This paper investigates the Harris-Todaro (Harris and Todaro, 1970) model in the context of a small open country growth economy from the viewpoint of welfare economics. The H-T (1970) model itself tries to explain the persistence of rural to urban migration despite urban unemployment in LDCs. However this model has often been utilized to analyze the possibility of immiserizing growth within a small open country economy. This possibility was first advocated by Bhagwati (1968) and Johnson (1967) — i.e., foreign capital inflows lower the real income (wealth) of the import tariff-ridden host country.

This proposition was furthered by Brecher and Alejandro (1977), Khan (1982), and Beladi and Naqvi (1988). Grinols (1991) also questioned the above immiserizing growth proposition by stressing the role of the urban informal sector within the standard H-T model. Grinols' points are further extended by Chandra and Khan (1993) employing the three sector and two factor model. Many authors have generalized the H-T model in various ways; Gupta (1994, 1995, and 1997), Kar and Marjit (2001), Marjit and Beladi (2002, 2003), Marjit (2003), and Choi and Yu (2006) among others.

However, these contributions are all static, i.e., at most the effects of once and for all foreign capital inflows, technological expansion, immigration or some parameter's change are at issue. Very few attempts have been made to investigate the H-T model from the viewpoint of economic growth, which our paper attempts. Here, recalling that the H-T model treats the distorted labor market, attention must be paid to whether it retains dynamic stability when it is reconsidered in the framework of an open growth economy. Otherwise, it does not make much sense to analyze the welfare aspects of various policies in a static open economy. The following are attempts to investigate the H-T model from the viewpoint of economic growth.

Rauch (1993) investigated the evolution of income inequality during economic growth employing a three sector (urban formal, urban informal and rural sector) discrete time growth model and obtained that inequality tends to

follow an inverted U shape.

Beneivenga and Smith (1997) also employed the three sector overlapping generation model and showed that periodic equilibria with undamped oscillation can arise from adverse selection among groups of two different skill ability types.

These authors are concerned with different aspects of the dynamic H-T model of a closed economy, but both employed the three sector model. In contrast with these two predecessors, we examine an open small country two sector (urban and rural) continuous time H-T growth model, focusing on its dynamic stability.

In order to investigate the global dynamic stability of the H-T model, its framework is minimized. That is, we assume there exist two goods — one is traded and the other is non-traded, and the repatriation of the foreign capital inflow is made in term of the traded good. In other words, the good is exported for the repayment of foreign debt, and since there exists only one kind of traded good, no trade in the good exists. The traded good is a composite good used both for investment and consumption. The other good, the non-traded good is a consumption good. Both traded good and non-traded good are produced under neoclassical technology employing labor and capital.

Next, complying with the H-T model, the traded good is produced in the urban sector, and the non-traded good is produced in the rural sector. In the urban sector, the wage rate is fixed and higher than the rural wage rate, satisfying the condition that the formal urban wage rate times the urban employment rate equals the rural wage rate. That is, in the standard usage of the H-T model, the rural wage rate equals the expected urban wage rate under free mobility of labor between the two sectors.

Third, as is often true for developing countries, the international financial market is incomplete, i.e., the borrowing interest rate is not constant for these countries. Rather if they borrow more they have to pay a higher interest rate. For simplicity we assume foreign debt for the host country equals foreign capital, and the interest rate increases as the debt/total capital stock

ratio increases, in a sense reflecting the risk of default.

Lastly the traded good is a numeraire.   Under these setting it is shown that if the non-traded good of the rural sector is more capital intensive than the traded good of the urban sector, and both consumption goods are complements, then under certain regularity conditions, the economy is globally stable in the sense that given initial per capita capital, it converges toward its unique stationary value (Proposition 1).   However under the same capital intensity assumption if both consumption goods are substitutes, then the economy is unstable, in the sense that it moves away from the stationary state unless the initial state is not equal to the stationary state ((1) of Proposition 2).

Furthermore, under the opposite capital intensity assumption (i.e., the traded good of the urban sector is more capital intensive than the non-traded good of the rural sector), then irrespective of the properties of both consumption goods (i.e., complements or substitutes), the economy is always globally unstable ((2) of Proposition 2).   In short, the global stability follows only when both goods are compliments and good 1 is capital intensive and for all other cases global instability follows.   This is due to the distortion within the labor market.

In the next section, we introduce the framework of our model.   In section 3, employing the dynasty model with representative consumer utility maximization, we derive the global stability in Proposition 1 and the opposite results in Proportion 2.

In section 4 concluding remarks are derived.

2. MODEL

There exist two sectors, an urban sector (sector 1) and a rural sector (sector 2).   In the former sector the composite good used both for consumption and investment is produced with capital and labor, while in the latter sector only the consumption good is produced similarly with capital and labor.

Let X denote the amounts of the composite good produced in the urban sector and $X = F_1(K_1, L_1)$ is the neoclassical production function where K_1 and L_1 are respectively the amounts of capital and labor employed, and F_1 is concave, homogenous of degree 1 in K_1 and L_1, and twice continuously differentiable both with respect to capital and labor. Its wage rate w_1 is fixed. In sector 1, there exist population N_1 and $\ell = L_1 / N_1 \leq 1$ is the employment rate in this sector. The capital is freely mobile between the two sectors. In the rural sector, let Y be the amounts of the consumption good and $Y = F_2(K_2, L_2)$ be its neoclassical production function where K_2 and L_2 are respectively the amounts of capital and labor employed there, and again, F_2 is concave, homogeneous of degree 1 in K_2 and L_2, and twice continuously differentiable both with respect to capital and labor. Its wage rate w_2 is flexible so that full employment of labor is realized in sector 2, i.e., $L_2 = N_2$ where N_2 is the population in sector 2. Due to the free mobility of capital, its rental price r is uniquely determined so that full employment of capital is realized.

Let $K = K_1 + K_2$ be the total stock of capital. The Harris-Todaro model is summarized by two characteristics — (1) the urban wage rate w_1 is fixed, and (2) the expected wage rate in the urban sector is equal to the rural wage rate, i.e.,

$$w_2 = \ell w_1 + (1 - \ell) \times 0 = \ell \cdot w_1, \text{ in short}$$
$$w_2 = \ell w_1. \tag{1}$$

Foreign capital is imported from abroad and so its rental rate is also r, since both domestic capital and foreign capital are identical in quality. However the rent for the foreign capital r changes due to the incomplete international financial market, i.e., $r = r(K_f / K)$ with $r' > 0$ where K_f is the amounts of foreign capital imported, K_d is those of domestic capital, and $K = K_f + K_d$. That is, as the ratio of foreign capital to total capital stock increases, the paid interest rate for foreign capital increases.[1]

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[1] This is a conventional assumption to reflect the incompleteness of international financial markets. See, e.g., Chatterjee and Turnovsky (2004, 2005).

The composite good of sector 1 is tradable, while the consumption good of sector 2 is nontraded. Good 1 is the numeraire and its price is 1 and the price of good 2 is p. Consequently, the interest rate payment $r \cdot K_f$ is made in terms of good 1. Let C_i, i=1, 2 be the amounts of domestic consumption in sector i, and I be the amount of domestic investment. Then

$$X = C_1 + I + rK_f, \tag{2}$$

holds for sector 1 as the market equilibrium condition. Since there exists only one traded good, no trade in the good exists, and only good 1 is used for interest payments. Similarly

$$Y = C_2, \tag{3}$$

holds for sector 2, reflecting good 2 as a nontraded consumption good.

Let $u(c_1, c_2)$ be the instantaneous felicity function of the representative consumer where $c_i = C_i / N$ is the per capita consumption of good i, i=1, 2.

To derive the definite conclusions, the felicity function is specified as

$$u(c_1, c_2) = \frac{1}{\gamma}(c_1^\theta c_2^{1-\theta})^\gamma, \text{[2]}$$

where $0 < \theta < 1$ and $-\infty < \gamma < 1$. Both goods are substitutes (respectively complements) if and only if $u_{12} < 0$ (respectively >0) and if and only if $\gamma < 0$ (respectively $0 < \gamma < 1$) where $u_{12} = \partial^2 u / \partial c_1 \partial c_2$.

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[2] We are indebted to Osang and Turnovsky (2000) for this pecification.

3. UTILITY MAXIMIZATION OF THE REPRESENTATIVE CONSUMER

The law of motion of domestic capital K_d is expressed as

$$\dot{K}_d = I = X - C_1 - rK_f,$$

where $\dot{K}_d$ is the time rate of change in K_d (in general $\dot{x}$ is the time rate of change in variable x).

The population growth rate $n = \dot{N}/N > 0$ is constant. Then the above equation can be expressed as

$$\dot{k}_d = F_1(K_1,\,L_1)/N - c_1 - rk_f - nk_d, \tag{4}$$

where k_d and k_f are respectively $k_d = K_D/N$ and $k_f = K_f/N$, i.e., per capita domestic capital and per capita foreign capital (small letters denote per capita amounts).

The equilibrium conditions of the labor market and capital market respectively are then expressed as

$$L_1/\ell + L_2 = L, \tag{5}$$

and

$$K_1 + K_2 = K_d + K_f = K. \tag{6}$$

The utility maximization of the representative consumer is expressed as

$$\max \int_0^\infty u(c_1,\,c_2)e^{-(\rho-n)t}\,dt,$$

subject to (4), (5) and (6).

Here $t=0$ is the present time. $\rho(1 > \rho > n)$ is the time discount rate.

The corresponding current value Hamiltonian is expressed as

$$H = u(c_1, c_2) + \lambda(F_1(K_1, L_1)/N - c_1 - rk_f - nk_d) + P(F_2(K_2, L_2)/N - c_2)$$
$$+ R(k_d + k_f - K_1/N - K_2/N) + W(N - L_1/\ell - L_2). \tag{7}$$

Here it is assumed that although the borrowing interest rate r depends on $K_f/K = k_f/k$, the representative consumer regards it as given, since as a microeconomic agent, he/she can not control it, for it is determined in the international financial market (i.e., the macroeconomic environment).

Then, by letting $P = \lambda p$, the following first order conditions

$$u_1 = \partial u / \partial c_1 = \lambda, \tag{8}$$

$$u_2 = \partial u / \partial c_2 = \lambda p, \tag{9}$$

$$\lambda F_{1K} = \lambda p F_{2K} = R, \tag{10}$$

$$\lambda F_{1L} = WN/\ell = w_1, \tag{11}$$

$$\lambda p F_{2L} = WN = w_2, \tag{12}$$

$$\lambda r = R, \tag{13}$$

$$\dot{\lambda} = \rho\lambda - R, \tag{14}$$

and the transversality condition

$$\lim_{t \to \infty} \lambda k_d e^{-(\rho-n)t} = 0, \tag{15}$$

hold.

From (13) and (14), we obtain

$$\dot{\lambda} = (\rho - r)\lambda. \tag{16}$$

Here we have to assume

$$\rho = r, \tag{17}$$

holds always. In fact from (10) and (13) the rental price of domestic capital R/λ must be equal to the borrowing interest rate r. Here recalling the equivalence between the social planner's optimum and the competitive equilibrium, from cost minimization or from duality theory

$$1 = G_1(r, w_1) \text{ and } p = G_2(r, w_2),$$

hold where G_i is the unit cost function of good i, $i=1$, 2. (Good 1 is numeraire.) Since w_1 is fixed, so is r from the first equation. Then from (15) $\rho = r$ must follow, from which $\lambda = \bar{\lambda}$ (const) follows in view of (16). That is, the marginal utility of wealth λ must be constant always. Then from (8) and (9), we obtain

$$c_1 = a_1 p^{\frac{-(1-\theta)\gamma}{1-\gamma}}, \tag{18}$$

where $a_1 = \theta^{\frac{1}{1-\gamma}} \left(\frac{1-\theta}{\theta} \right)^{\frac{(1-\theta)\gamma}{1-\gamma}} \bar{\lambda}^{-/(1+\gamma)}$, and

$$c_2 = a_2 p^{\frac{\theta\gamma-1}{1-\gamma}}, \tag{19}$$

where $a_2 = \theta^{\frac{1}{1-\gamma}} \left(\frac{1-\theta}{\theta} \right)^{\frac{1-\theta r}{1-r}} \bar{\lambda}^{-/(1+\gamma)}$.

By construction $c_1 = \frac{\theta}{1-\theta} p c_2$ and $c_2'(p) < 0$ hold always, and

$c_1'(p) < 0$ (resp. >0) $\Leftrightarrow 0 < \gamma < 1$ (resp. $\gamma < 0$) $\Leftrightarrow$ both goods are complements (resp. substitutes).

Henceforth we assume the production functions of both sectors are Cobb-Douglass, i.e.,

$$f_i(k_i) = k_i^{\alpha_i}, \quad i = 1, 2,$$
$$0 < \alpha_i < 1, \tag{20}$$

where $f_i = F_i / L_i$ being the labor productivity function of good i, and $k_i = K_i / L_i$ is the capital intensity of good i. First we note that L_i, i=1, 2 is obtained from

$$\begin{pmatrix} k_i & k_2 \\ 1/\ell & 1 \end{pmatrix}\begin{pmatrix} L_1 \\ L_2 \end{pmatrix} = \begin{pmatrix} K \\ N \end{pmatrix},$$

which is obtained from (5) and (6), as

$$\begin{pmatrix} L_1 \\ L_2 \end{pmatrix} = \frac{1}{k_1 - k_2/\ell}\begin{pmatrix} K - k_2 N \\ k_1 N - K/\ell \end{pmatrix}. \tag{21}$$

Then, regarding the rankings of the factor intensities, there exist two cases;

I. $k_1 < k < k/\ell < k_2$, i.e., good 2 is capital intensive where $k = K/L$, and

II. $k_1 > k/\ell > k > k_2$, k_2, i.e., good 1 is capital intensive. The first case (resp. second case) corresponds to $\alpha_1 < \alpha_2$ (resp. $\alpha_1 > \alpha_2$). Then we obtain the following propositions. First

Proposition 1: *If good 2 is capital intensive and both goods are complements ($0 < \gamma < 1$), then under the following regularity condition the economy is globally stable, i.e., for any given initial per capita domestic*

capital k_d, *it converges toward the unique stationary point* k_{dE}. *Here the regularity condition is*

$$\frac{1-\gamma}{1-\theta} < \left(\frac{\alpha_2}{1-\alpha_2} - \frac{\alpha_1}{1-\alpha_1}\right)\frac{(1-\alpha_2)}{\rho}$$

$$\left\{\frac{(1-\alpha_2)(1-\theta)\gamma(1-\alpha_1)\rho}{\alpha_1} + (\rho-\gamma)\varepsilon(1-\gamma+(1-\alpha_2)(1-\theta)\gamma)\right\}.$$

[3] (22)

Here we note $k \geq k_m > 0$ *holds always by construction where* k_m *is the fixed minimum amount.* *Here we note* $k_d = \varepsilon k$ *with* ε $(0 < \varepsilon < 1)$ *const. holds from* $\rho = r(k_f / k)$ *with* $r'(k_f / k) > 0$ *and* $k = k_f + k_d$.

Proof (see Appendix A1)

Intuitively Proposition 1 can be understood using the following reasoning; suppose initially that $k_0 > k^*$ (resp. $k_0 < k^*$). Then $p_0 < p^*$ (resp. $p_0 > p^*$). Since *X* and *Y* are complements, the consumptions of both *Y* and *X* increase (resp. decrease). An increased (resp. a decreased) consumption of *X* in turn will make less (resp. more) capital investment available in the economy, which implies $k \downarrow$ (resp. $k \uparrow$). This ensures convergence.[4]

Here the crucial point for stability is that *k* and *p* move in the opposite direction.

For I. $k_1 < k < k / \ell < k_2$, $0 < \alpha_1 < \alpha_2$ holds. Hence the above regularity condition likely holds unless θ is sufficiently close to 1. (see footnote 3)

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[3] As $\gamma \nearrow 1$, LHS $\searrow 0$ and RHS $\to \left(\frac{\alpha_2}{1-\alpha_2} - \frac{\alpha_1}{1-\alpha_1}\right)(1-\alpha_2)(1-\alpha_1)^2(1-\theta) / \alpha_1 > 0.$

Similarly RHS $\nearrow +\infty$ as $\alpha_1 \to 0$ or $\alpha_2 \to 1$. Hence (22) holds if γ is close to 1 or α_1 is close to 0, or α_2 is close to 1.

[4] We thank an anonymous referee for this suggestion.

Figure 1 $\dot{k}_d(k_d)$ Curve
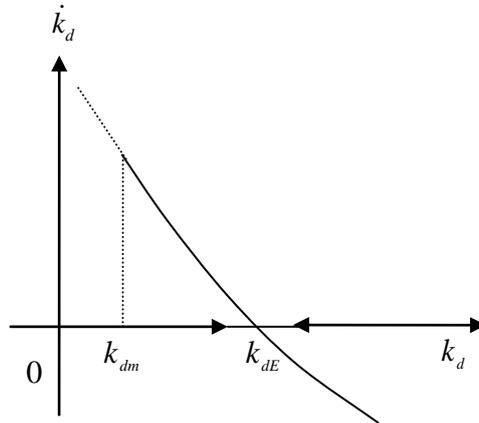
Figure 2 $k(p)$ Curve
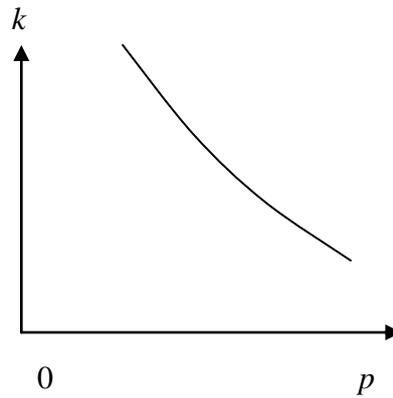
Then as shown in figure 1, where the per capita domestic capital k_d converges toward the unique stationary state k_{dE} globally with $c_i = c_i(k)$ and $c_1'(k) > 0$ and $c_2'(k) > 0,$ $i = 1, 2,$ i.e., per capita consumption of both goods 1 and 2 increases as per capita capital increases.   Furthermore since $\ell = \ell(p)$ and $\ell'(p) > 0$ holds, the employment rate increases (resp. decreases) if and only if per capita capital decreases (resp. increases). However except I, when $k_1 < k < k/\ell < k_2$ and $0 < \gamma < 1,$ the economy is globally unstable for all other cases, that is,
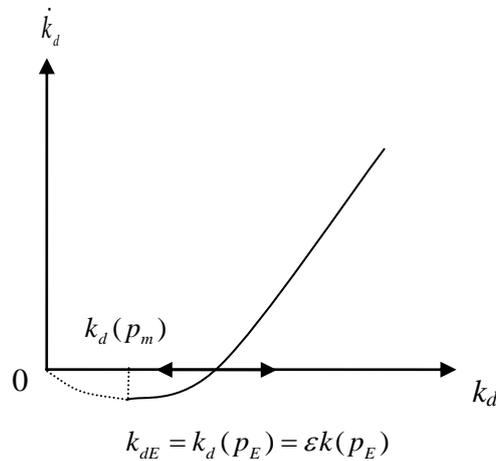
Proposition 2

(i) For the following cases

I-2. $k_1 < k < k/\ell < k_2$, (i.e., good 2 is capital intensive) and $\gamma < 0$ (i.e., both goods are substitutes), and

(ii) II. $k_1 > k/\ell > k > k_2$ (i.e., good 1 is capital intensive), the economy is globally unstable in the sense that the initial per capita domestic capital k_d diverges from k_{dE} unless the initial k_d equals k_{dE}.

Figure 3 $\dot{k}_d(k_d)$ Curve (I. $k_1 < k < k/\ell < k_2$ and $\gamma < 0$)

$$k_{dE} = k_d(p_E) = \varepsilon k(p_E)$$

Proof (see Appendix A2)

Intuitively Proposition 2 can be understood by the following; here for all cases both k and p move in the same direction.   Suppose initially $k_0 > k_E$ (resp. $k_0 < k_E$). Then $p_0 > p_E$ (resp. $p_0 < p_E$). Then demand for *Y* decreases (resp. increases) hence its production must decrease (resp. increases), implying higher (resp. lower) migration of labor from the rural sector to the urban sector and higher (resp. lower) employment in the urban sector and hence higher (resp. lower) production of *X*.   In the case where both goods are complements (subacase of (2) of Proposition 2), consumption of good 1 decreases (resp. increases), implying more (resp. less) investment

is available, and hence $k \uparrow$ (resp. $k \downarrow$).   This shows the divergence of the economy.   In the case where both goods are substitutes ((1) and subcase of (2) of Proposition 2), although the consumption of good 1 increases (resp. decreases), the increase (resp. decrease) in the production of X surpasses the increase (resp. decrease) in its consumption, implying more (resp. less) investment is available, and hence $k \uparrow$ (resp. $k \downarrow$).   This again shows the divergence of the economy.[5]

4. CONCLUDING REMARKS

As seen from Proposition 1 and 2, for most cases, the global stability of the economy does not follow.   Intuitively this is because the economy is distorted due to the fixed wage rate (and fixed rental price of capital) in comparison with the flexible rural wage rate.   The distortion is reflected by the urban unemployment, which never is resolved throughout the transitional path.   In other words, this distortion makes the economic system unstable, which has not been noted with the static arguments.   Perhaps one possible way to wash out this instability would be to adopt autarky.

However, even under autarky as long as the above distortion exists, exactly the same instability follows.[6]   As is widely known from the conventional arguments, the most ready policy to recover full employment is to give equal employment subsidies both to urban employers and rural employees so that the employment of both sectors increases and the wage rates received by employers of both sectors are equalized (see Bhagwati and Srinivasan, 1974; Basu, 1980).

These subsidies are financed by nondistortionary lump-sum taxation. However as Kasselman (1979) shows, sometimes the subsidy bill becomes so

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[5] Again we thank an anonymous referee for this suggestion.

[6] Under autarky, for the existence of the optimal path, $\rho = r$ (17) must hold.   Then (A1) through (A8) in Appendix I hold where k_d is replaced by k (i.e., $k_f = 0$).

enormous it exceeds income.[7]   Furthermore, in reality governments must spend money for various alternatives.

Then whether the above subsidy policy is implemented becomes doubtful. Here one alternative way to ease the distortion by raising the rural sector's wage rate w_2 and hence the employment rate ℓ.   In fact let its labor productivity function be $f_2(k_2) = T_2 k_2^{\alpha_2}$ where T_2 is the parameter representing the technology level.   Then noting $\rho = r$ holds always, it follows that $(1 - \alpha_2)\hat{k}_2 = \hat{p} + \hat{T}_2$ where e.g., $\hat{k}_2 = dk_2 / k_2$, i.e., the relative rate of change in k_2 from $\rho = r = pT_2\alpha_2 k_2^{\alpha_2 - 1} = pf_2'(k_2)$.   Similarly $\hat{w}_2 = \hat{p} + \hat{T}_2 + \alpha_2\hat{k}_2 = \hat{k}_2$ follows   from $w_2 = pT(1 - \alpha_2)k_2^{\alpha_2} = p(f_2 - k_2 f_2')$. Next recalling $T_2 y(p, k) = c_2(p)$ where $y(p, k)$ is the per capita supply function of good 2 when $T_2 = 1$, we obtain $py_p\hat{p} / y + \hat{T}_2 = (\theta\gamma - 1)(1 - \gamma)^{-1}\hat{p}$ or $\hat{T}_2 = ((\theta\gamma - 1) / (1 - \gamma) - py_p / y)\hat{p}$.   Clearly, if T_2 rises and $0 < \lambda < 1$, then p falls from $T_2 y(p, k) = c_2(p)$.   Hence it follows that $(1 - \alpha_2)\hat{k}_2 = \hat{p} + \hat{T}_2$ $= ((\theta - 1)\gamma / (1 - \gamma) - py_p / y)\hat{p} > 0$ from $(\theta - 1)\gamma / (1 - \gamma) < 0$ if $0 < \gamma < 1$, $y_p > 0$ and $\hat{p} > 0$, implying $\hat{w}_2 > 0$ if T_2 rises and both consumption goods are complements.

On the other hand, technological progress in the urban sector merely raises the wage rate w_1, and decreases its employment rate, thereby worsening the distortion.

Our model assumes that social welfare is represented by the over time summation of the felicity function of the representative consumer.   This further implies that the urban unemployed are subsidized by say, an unemployment insurance program.   However if we take into consideration the lack of such social security programs in the developing countries, the existence of the urban unemployed becomes less likely and may even be replaced by the informal urban employee.   Then the model would be more realistic but more complicated.

Lastly we admit a limitation of our model — only one good is traded so that no trade in goods occurs.   Goods are exported as a means to pay the interest payment $r \cdot K_f$.   If trade in goods is allowed, i.e., there are not

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[7] We owe Basu (1997), Chapter 8 for pointing out Kesslman (1979)'s contribution.

composite goods but both consumption good and investment good produced under different technologies, then it is impossible to perform analysis within a small open economy framework because the prices of traded goods must be endogenously determined in a general equilibrium model, or a two country, two traded good growth model must be considered.   This is challenging and beyond the scope of our present model.

APPENDIX

A1. Proof of Proposition 1 (Henceforth only the sketch of proof is given. The detailed proof is sent upon request.)

First we obtain the expression of k from $y = f_2(k_2)L_2 / N = c_2$ and (21), as

$$k = \ell(k_1 + f_2^{-1}(k_2 / \ell - k_1)a_2 p^{\frac{\theta\gamma-1}{1-\gamma}}),\qquad\qquad\text{(A1)}$$

where $k_2 / \ell = \dfrac{\alpha_2}{1-\alpha_2}\dfrac{w_1}{r} = \dfrac{\alpha_2}{1-\alpha_2}\dfrac{1-\alpha_1}{\alpha_1}k_1$ from $\ell = w_2 / w_1$, $w_2 = (1-\alpha_2)f_2$

and $k_2 / w_2 = \dfrac{1}{p(1-\alpha_2)}k_2^{1-\alpha_2} = \alpha_2 / p(1-\alpha_2)\alpha_2 k_2^{\alpha_2-1} = \alpha_2 / (1-\alpha_2)r.$ Here from $r = \alpha_1 k_1^{\alpha_1-1} = \rho,$ k_1 and r and hence w_1 are always constant.   This further implies k_2 / l is constant.

Here from $pf_2' = r,$

$$f_2 = k_2^{\alpha_2} = (\alpha_2 / r)^{\frac{\alpha_2}{1-\alpha_2}} p^{\frac{\alpha_2}{1-\alpha_2}},\qquad\qquad\text{(A2)}$$

and from (20), (A1) is rewritten as

$$k = \frac{1-\alpha_2}{w_1}\left(\frac{\alpha_2}{r}\right)^{\frac{\alpha_2}{1-\alpha_2}}\left\{k_1 p^{\frac{1}{1-\alpha_2}} - p\left(\frac{\alpha_2}{r}\right)^{\frac{-\alpha_2}{1-\alpha_2}}(k_1 - k_2/\ell)a_2 p^{\frac{\theta\gamma-1}{1-\gamma}}\right\}$$

$$= \frac{1-\alpha_2}{w_1}\left(\frac{\alpha_2}{r}\right)^{\frac{\alpha_2}{1-\alpha_2}}\left\{k_1 p^{\frac{1}{1-\alpha_2}} - \left(\frac{\alpha_2}{r}\right)^{\frac{-\alpha_2}{1-\alpha_2}}(k_1 - k_2/\ell)a_2 p^{\frac{(\theta-1)\gamma}{1-\gamma}}\right\} = k(p). \tag{A3}$$

That is, k is always expressed as function of p only.

First we consider the case of $0 < \gamma < 1$, i.e., of complements. Recalling $k_1 < k_2/\ell$ for capital intensive good 2, we obtain the function of $k = k(p)$ as shown in figure A1 where $k'(p) = 0$ at $p = p^*$. p^* is defined implicitly by

$$\frac{k_1}{1-\alpha_2}p^{\frac{1}{1-\alpha_2}} = (k_2/\ell - k_1)\left(\frac{\alpha_2}{r}\right)^{\frac{-\alpha_2}{1-\alpha_2}}\cdot a_2\frac{(1-\theta)\gamma}{1-\gamma}p^{\frac{-(1-\theta)\gamma}{1-\gamma}}. \tag{A4}$$

Let $k_m = k(p^*)$, i.e., the minimum value of k for $p = p^*$ be k_m.

Figure A1 $k(p)$, $\dfrac{k_1(p)}{1-\alpha_2}p^{\frac{1}{1-\alpha_2}}$ **and**

$(k_2/\ell - k_1)\left(\dfrac{\alpha_2}{r}\right)^{\frac{-\alpha_2}{1-\alpha_2}} a_2\dfrac{(1-\theta)\gamma}{1-\gamma} p^{\frac{-(1-\theta)\gamma}{1-\gamma}}$ **Curves**
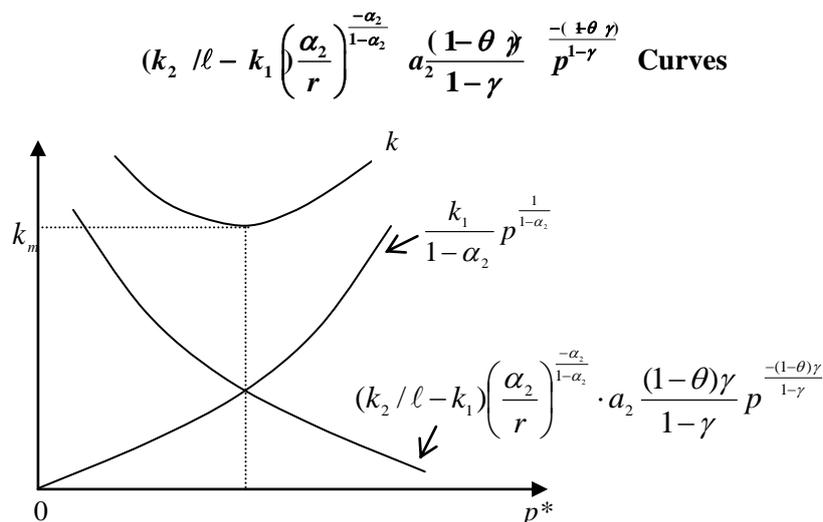
Next let

$$\dot{k}_d = x - c_1 - rk_f - nk_d = w_2 + rk - py - c_1 - rk_f - nk_d = h(p).$$

From (A2), and $c_2 = a_2 p^{\frac{\theta\gamma-1}{1-\gamma}}$, $h(p)$ is expressed as

$$\dot{k}_d = h(p) = (1-\alpha_2)(\alpha_2/r)^{\frac{\alpha_2}{1-\alpha_2}} p^{\frac{1}{1-\alpha_2}} + (\rho-n)\varepsilon k(p) - \frac{1}{1-\theta} a_2 p^{\frac{(\theta-1)\gamma}{1-\gamma}}. \quad (A5)$$

Here noting that

$$w_2 - \frac{1}{1-\theta} pc_2 = (1-\alpha_2)(\alpha_2/r)^{\frac{\alpha_2}{1-\alpha_2}} p^{\frac{1}{1-\alpha_2}} - \frac{1}{1-\theta} a_2 p^{\frac{(\theta-1)\gamma}{1-\gamma}}. \quad (A6)$$

is an increasing function of p, we observe $h(p)=0$ at $p=p_E$ if

$$(1-\alpha_2)(\alpha_2/r)^{\frac{\alpha_2}{1-\alpha_2}} \left(1+(\rho-n)\varepsilon \frac{k_1}{w_1}\right) p^{\frac{1}{1-\alpha_2}}$$

$$= a_2 \left(\frac{1}{1-\theta} - (\rho-n)\varepsilon \frac{1-\alpha_2}{w_1}\left(k_2/\ell - k_1\right)\right) p^{\frac{(\theta-1)\gamma}{1-\gamma}} \quad (A7)$$

holds.

However (A7) holds always since

$$\frac{1}{1-\theta} > 1 > \frac{(\rho-n)\varepsilon}{\rho}(1-\alpha_2)\left(\frac{\alpha_2}{1-\alpha_2}\right) = \frac{(\rho-n)\varepsilon}{\rho}\alpha_2,$$

holds by construction.

Figure A2 $(\rho - n)\varepsilon k(p)$ and $w_2(p) - \dfrac{1}{1-\theta}pc_2(p)$ **Curves**
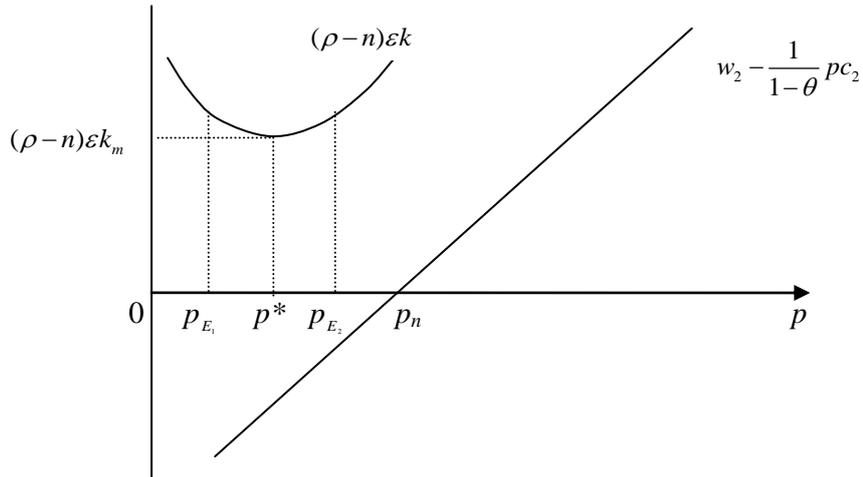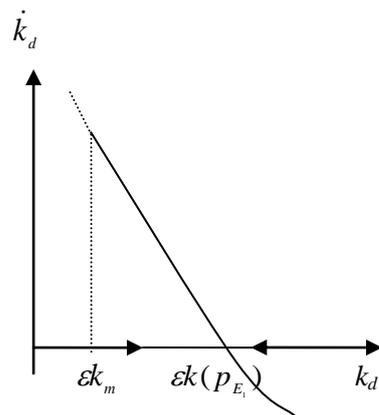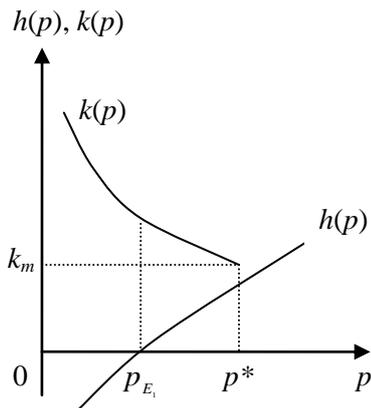

Figure A2 shows two possible cases of P_E's, P_{E_1} and P_{E_2}. By comparing the values of coefficients of $p^{1/(1-\alpha_2)}$ and $p^{-(1-\theta)\gamma/(1-\gamma)}$ of (A4) and (A7), and recalling (22), we observe $p_E(=p_{E_1}) < p^*$ holds always.

Figure A3 $h(p)$ and $k(p)$ **Curves**   **Figure A4** $k_d(k_d)$ **Curve**

As seen from figures A2 and A3, for $p < p^*$ (resp. $p > p^*$), $k'(p) < 0$ (resp. $k'(p) > 0$) holds, and since $h(p) = 0$ holds only at $p = p_E$, $\dot{k}_d = w_2 - pc_2 + (\rho - n)\varepsilon k = 0$ holds also only at $k = k(p_E)$.

In short as seen from figure A4, the economy is globally stable if I. $k_1 < k < k/\ell < k_2$ and (22) hold, in the sense that given an arbitrary initial per capita domestic capital k_d it moves toward the unique stationary value $k_{dE} = \varepsilon k(p_E)$.

A2.

First we consider

I-2. $k_1 < k < k/\ell < k_2$ and $\gamma < 0$.

Here first we note there exists a positive minimum value of p, $p_m\,(> 0)$ from

$$f_2 = k_2^{\alpha_2} = c_2 \frac{N}{L_2} \geq c_2, \text{(A2)}, \quad c_2 = a_2 p^{\frac{\theta\gamma-1}{1-\gamma}} \text{ (19) and } (\alpha_2 / r)^{\frac{\alpha_2}{1-\alpha_2}} p^{\frac{\alpha_2}{1-\alpha_2}} \geq a_2 p^{\frac{\theta\gamma-1}{1-\gamma}}.$$
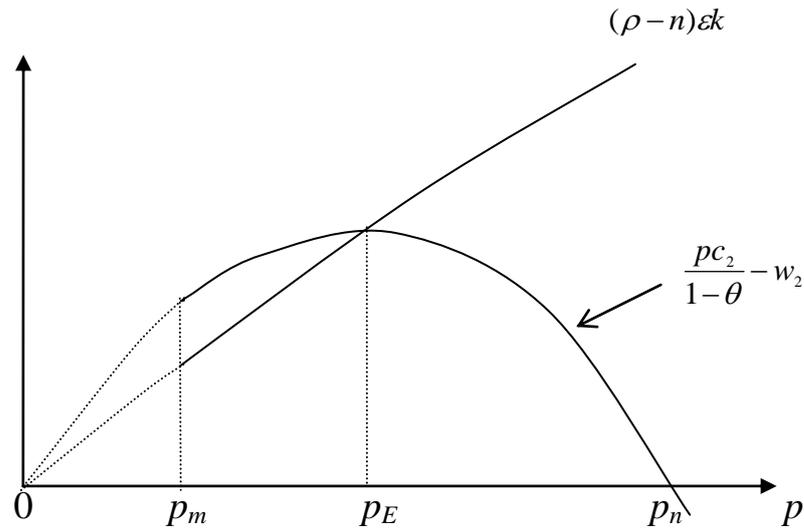
Next we note

$$\frac{1-\alpha_2}{w_1}(k_2 / \ell - k_1)(\rho - n)\varepsilon < \frac{1}{1-\theta}, \tag{A8}$$

holds always from the following inequality

$$\frac{\rho}{(1-\theta)(\rho - n)\varepsilon} > \alpha_2 - \frac{(1-\alpha_1)}{\alpha_1}(1-\alpha_2).$$

By letting $\dfrac{pc_2}{1-\theta} - w_2 = 0$ at $p = p_n$ and $h(p) = 0$ at $p = p_E$ (see figure A5) we obtain by comparing the value of coefficients of $p^{1/(1-\alpha_2)}$ and $p^{(\theta-1)\gamma/(1-\gamma)}$ of (A5) and (A7), observing $1/(1-\alpha_2) > 1 > (\theta-1)\gamma/(1-\gamma) > 0$ with $\gamma < 0$, $P_E < P_n$ holds always.

Figure A5 $(\rho - n)\varepsilon k(p)$ and $\dfrac{1}{1-\theta}pc_2(p) - w_2(p)$ **Curves**

Then under (A8), we obtain the $h(p)$ curve as shown in figure A3, and hence the $\dot{k}_d$ curve as shown in figure 3, from $k'(p) > 0$ with $k(0)=0$ and $k_d = \varepsilon k$. As seen from figure 3, the economy is unstable if the initial k_d is not equal to k_{dE}.

Next we consider

II-1. $k_1 > k/\ell > k > k_2$ and $0 < \gamma < 1$.

From (A3) we observe $dk/dp > 0$ always holds for this case, and $k(p_k) = 0$ where $p = p_k$ is implicitly defined by $k = 0$ in (A3).

Furthermore from (A5), we obtain that $h'(p) > 0$ holds always, and $p = p_E$ is implicitly defined from (A7).

Then, just as in the previous case, $p_k < p_E$ holds always in this case.
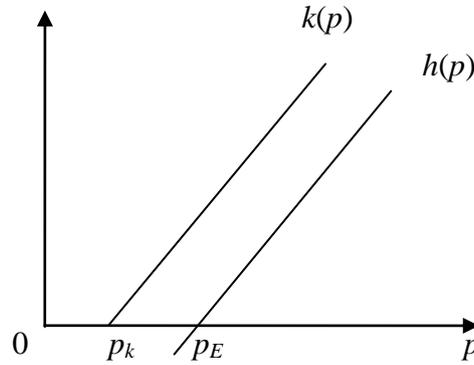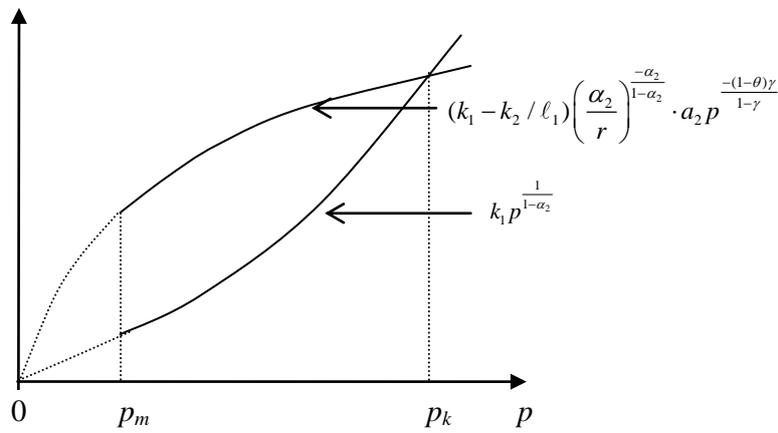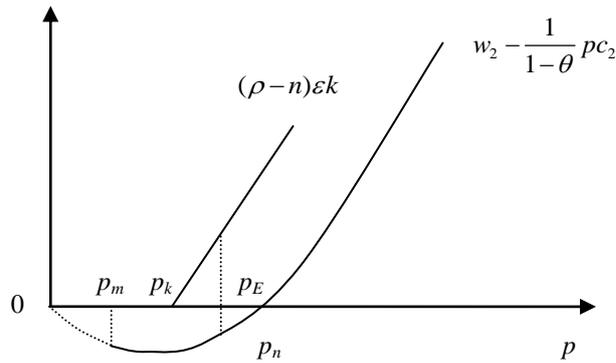
Figure A6 $h(p)$ and $k(p)$ Curves

Figure A7 $k_1 p^{\frac{1}{1-\alpha_2}}$ and $(k_1 - k_2/\ell)\left(\dfrac{\alpha_2}{r}\right)^{\frac{-\alpha_2}{1-\alpha_2}} \alpha_2 \dfrac{(1-\theta)\gamma}{1-\gamma} p^{\frac{-(1-\theta)\gamma}{1-\gamma}}$ Curves

Then we obtain two positively sloped curves $k(p)$ and $h(p)$ in figure A6, and hence the positively sloped $\dot{k}_d$ curve as shown in figure 3, implying the global instability.

Lastly we discuss

Figure A8 $(\rho-n)\varepsilon k(p)$ and $w_2(p)-\dfrac{1}{1-\theta}pc_2(p)$ Curves

II-2. $k_1 > k/\ell > k > k_2$ and $\gamma < 0.$

Here again $p \geq p_m > 0$ follows from $\gamma < 0.$ From (A4), we obtain the following figure in view of $0 < (\theta-1)\gamma/(1-\gamma) < 1$;

Then we obtain the curve of k such that $k'(p) > 0$ and $k(p_k) = 0.$

The existence of a unique $p = p_E$ such that $h(p) = 0$ is assumed from (A7).

Furthermore from (A6), we obtain the curve $w_2 - pc_2/(1-\theta)$ in figure A8 and p_n is again implicitly defined by $w_2 - \dfrac{1}{1-\theta}pc_2 = 0$, and hence by comparing the values of the coefficients of $p^{1/(1-\alpha_2)}$ and $p^{(\theta-1)\gamma/(1-\gamma)}$ of (A3) and (A6),

$$p_k < p_n \Leftrightarrow \quad 0 < \frac{k_1 - k_2/\ell}{k_1} < \frac{1}{(1-\alpha_2)(1-\theta)},$$

which holds always for this case.   Then we obtain the same figures as figure A6 and figure 3, and hence the global instability follows.

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