THE JOURNAL OF THE KOREAN ECONOMY, Vol. 10, No. 2 (August 2009), 151-176

Explaining the Equity Premium in Korea*

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The annual equity premium in Korea of 3% or less is smaller than the 6% of the United States. The equity premium puzzle is considered mild. With a Generalized Method of Moments (GMM), Hansen-Jagannathan bounds, and long-run risk approach, the analysis shows that the equity premium puzzle exists in Korea. We find that low consumption growth volatility, together with a low correlation between consumption growth and asset returns, contributes to the equity premium puzzle.

JEL Classification: C01, E21, G19
Keywords: equity premium, GMM, Hansen-Jagannathan bounds, recursive preference, long-run risks

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* Received May 6, 2009. Accepted July 4, 2009. We thank Jinook Jeong, Yongsung Chang, Taeyoon Sung, Seounghoon Cho and seminar participants at Yonsei University for helpful comments.
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1. INTRODUCTION

Mehra and Prescott (1985) show that the consumption-based asset pricing model fails to generate the historically observed equity premium of around 6%. There have been various kinds of approaches to explain it. Epstein and Zin (1989) use a recursive preference to separate risk aversion and intertemporal substitution. Constantinides (1990) and Campbell and Cochrane (1999) modify preference to include habit formation. Constantinides and Duffie (1996) study a model with heterogeneous agents to examine how heterogeneity affects the volatility of the pricing kernel. Heaton and Lucas (2000) study the effect of idiosyncratic risks and how it can affect the equity premium.

Attempts to estimate and explain the equity premium in Korea have been made. However, asset markets in Korea are different from the ones in the U.S. in at least three respects: a short history of asset market data, no representative risk-free rate, and small equity premium. Due to limited availability of the asset market data, it is hard to perform an empirical analysis with yearly observations, as Mehra and Prescott (1985) did. It would be better to estimate with more observations as long as it is assumed that the decision interval of consumers is quarterly and benign.[1)]

The second issue is about the existence of risk-free assets in Korea. While the 3-month Treasury Bill rate is used as a proxy for a risk-free rate in the U.S., there is no such an asset in Korea. Among previous studies, Dokko *et al*. (2001) use the rate of saving deposits as a proxy for risk-free rates. Kim and Hong (2008) attempt to select the right measure among saving deposit rate, certificate of deposits (CD) rate, and a mix of these two rates. In this study (rather than choosing any rates *a priori*) we estimate the beta of candidate assets first and choose the rates with the lowest beta.

As to the equity premium itself, Dokko *et al*. (2001), Kim and Hong (2008) and this study report that the estimated equity premium in Korea is

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[1)] In the following sections, we examine if there is any significant discrepancy between summary statistics based on annualized quarterly data and yearly data.

3% or less annually.[2)] This premium is relatively small, compared to the U.S. case of 6%, which might imply that the equity premium puzzle in Korea would be mild and it could be possible to explain the equity premium with a reasonably low risk aversion. Kim and Hong (2008) report that the Generalized Method of Moments (GMM) estimate of relative risk aversion coefficient is around 0.1, which is very small. The results suggests that the equity premium puzzle does not exist in Korea. Using Epstein-Zin utility function, Dokko *et al*. (2001) show that the level of risk-free rate and the equity premium can be explained if we accept the relative risk aversion coefficient of 6 to 7 and the elasticity of intertemporal substitution (EIS) of 0.5, implying that a reasonably low risk aversion coefficient can explain the equity premium. Considering these findings, it is possible to think that the equity premium is small and the puzzle is mild in Korea.

This study suggests a different finding: The equity premium puzzle does exist in Korea. We start from the basic asset pricing equation to illustrate the equity premium puzzle. The GMM estimation results show that we could explain the risk-free rate with a reasonably low risk aversion coefficient. However, we cannot explain the equity premium with low risk aversion. In doing so, particular attention is paid to interpreting the moment conditions to carefully track down the pricing errors, depending on different values of the risk aversion coefficient. This analysis suggests that a low risk aversion coefficient could not satisfy two moment conditions related to bond returns and the equity premium at the same time. To confirm this finding, we take a backward approach using Hansen-Jagannathan bounds (HJ bounds). Rather than coming up with a new model to test, we ask what properties the stochastic discount factor (SDF) must have in order to explain the GMM asset return data. This experiment shows that the volatility of the SDF is too low to explain the equity premium in Korea. In terms of risk aversion coefficient, a higher coefficient is needed to raise the volatility of the SDF and qualify HJ bounds. This analysis with HJ bounds provides additional

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[2)] This return is calculated using only capital gains. If dividend yields are included the equity premium would be larger.

insight on what causes the puzzle to exist: low consumption growth volatility contributes not only to the existence of the puzzle itself but also to different behavior of SDF compared to the U.S. case.

After showing that a very high risk aversion coefficient is needed to explain both stock and bond returns in Korea, we also present evidence against using any model based on i.i.d. consumption growth to explain the equity premium in Korea. As mentioned, Dokko *et al*. (2001) suggest that relative risk aversion coefficient of 6-7 and the EIS of 0.5 can explain the equity premium puzzle. In doing so, they assume the consumption growth follows i.i.d. process. However, this assumption is not desirable theoretically and empirically for this study. Theoretically, one reason for using Epstein-Zin utility function is to raise the volatility of the SDF by adding additional stochastic term to the SDF. The term is the return on wealth, which gives a consumption stream. But this term disappears when we assume i.i.d. consumption growth, implying one of the main reasons to use a recursive preference is erased. To show that consumption growth process is not i.i.d., we estimate the long-run risk model of consumption growth suggested by Bansal and Yaron (2004) using the maximum likelihood estimation and Bayesian technique. The results say that the consumption process in Korea is far from the random walk and its growth has a slow-moving part inside, which is called “long-run risk”. Moreover, we provide the empirical evidence against taking the EIS at less than one. In this regard, it is questionable to say that the equity premium in Korea can be easily resolved by using Epstein-Zin utility function. Different from previous studies, this study delves more into consumption growth process and the relation to asset returns. The findings suggest this approach is rewarding. This paper consists of the following order. In the next section, we describe the data for empirical analysis and illustrate the equity premium puzzle using the simple basic pricing equation. The results of the GMM estimation are then reported with more attention given to how to interpret the moment conditions. Section 3 adopts a nonparametric way to explain the puzzle and Hansen-Jagannathan bounds. Following this, we discuss related issues using an Epstein-Zin utility function and long-run risk models in section 4. Section 5 concludes the paper.

2. EMPIRICAL STUDY

2.1. Data

The estimation is based on quarterly variables due to the small sample size while Mehra and Prescott (1985) used the yearly observations from 1890-1979. Market returns from the stock market, risk-free rates, and consumption data are needed for the empirical analysis. Data was collected from all from the Economic Statistics System (ECOS) of the Bank of Korea and the website of the Korea Exchange. The sample period is determined by data availability. For example, when we use the returns of the Monetary Stabilization Bond (MSB), the sample period is 1987:I-2007:IV. The sample period is 1991:I-2007:IV when we use the CD rates available from 1991:I.

For stock market returns, we calculate them using the Korea Composite Stock Price Index (KOSPI), which is an index of all common stocks traded on the stock market of Korea Exchange. For risk-free rates, there is a consensus that the three month Treasury bill return is used as risk-free rate in the U.S. However, one difficulty in the estimation is that there does not exist an ideal proxy for risk-free rates in Korea. Previous studies use returns of MSB, CD rates, and rates of saving deposit. Since finding a good measure of risk-free rates is not the focus of this paper, we simply take the rates used in previous studies. Instead, we calculate beta of those rates because asset returns (whose beta is very close to zero) can be regarded as risk-free rates as they are found to be very low. For MSB, its beta is 0.001. For the CD, it is –0.007.[3] In this regard, it is admissible to use them as risk-free rates. Table 1 reports the summary statistics of asset rates together with that of inflation.

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[3] The *t*-values are 0.19 and –0.82 respectively.

Table 1 Summary Statistics of Interest Rates and Inflation

	Mean	Mean without Year of 1998	Mean after Winsorizing	Standard Deviation	Min.	Max.
Real Market Return	2.76	2.05	1.68	18.6	–42.71	80.35
Real MSB Rate	1.40	1.39	1.35	1.08	–1.51	3.58
Real CD Rate	1.33	1.27	1.30	1.16	–0.62	4.31
CPI	1.15	1.14	1.18	0.92	–0.3	5.3

Notes: This table shows the summary statistics of interest rates used in GMM estimation. The real market return is the real return from KOSPI index. MSB and CD stand for ‘Monetary Stabilization Bond’ and ‘Certificate of Deposit’ respectively. These statistics are based on quarterly observations from 1987:I-2007:IV. For real CD rates, it began from 1991:I due to availability. Winsorizing is made at the upper 10% and lower 10%.

For consumption data, we use aggregate consumption growth while Mehra and Prescott (1985) used per capita consumption growth. The reason for not adjusting the population is the low frequency of population data. Population data is available only on a yearly basis and we need to use interpolation to make quarterly per capita consumption growth, which might make the data imprecise. Another reason is the high correlation between per capita consumption growth and aggregate consumption growth. The correlation during the sample period is 0.99 and we use aggregate data in addition to increasing the sample size.

Observations of the year 1998 are outliers. Market returns are –42% and 79% for the second and fourth quarter. The consumption growth dropped by 11% in the first quarter, which is the minimum during the sample period. In this regard, we estimate the model after dropping four observations of 1998.

Table 2 shows the summary statistics of key variables used in the empirical analysis. There are two things noticeable. One is relatively low equity

Table 2 Comparison of Summary Statistics

Country		Consumption Growth		Real Market Return		Real Risk-free Rate		Equity Premium	
		Mean	STD	Mean	STD	Mean	STD	Mean	STD
Korea	(1)	4.79	3.06	11.03	37.2	5.60	2.16	5.44	37.4
	(2)	5.35	1.72	8.20	32.4	5.56	2.04	2.64	32.4
U.S.		1.83	3.57	6.98	16.54	1.04	5.56	5.94	16.84

Notes: This table shows the key statistics of Korean and U.S data. The U.S. data is based on yearly observations from 1890-1979, which Mehra and Prescott (1985) used. The Korean data is quarterly for 1987:I-2007:IV and reported after being annualized. In case of Korea, statistics in row (1) are based on the whole sample period and statistics in row (2) are obtained after dropping 4 quarters of 1998. For a proxy of riskfree rate, a MBS rate is used. STD refers to standard deviation.

premium in Korea, 2.64%, compared to the equity premium in the U.S. of 5.94%. The other is the relatively low standard deviation of consumption growth. While the standard deviation of consumption growth is 3.57% in the U.S., it is 3.06% in Korea and lowers to 1.72% when the observations of 1998 are dropped.[4] These two things make the difference in the coefficient of variation and Sharpe ratio. Table 3 displays this difference clearly. While the coefficient of variation, $\sigma(\Delta c) \,/\, E(\Delta c)$ where $\Delta c = c_{t+1}/c_t - 1$, in U.S. is almost 2, the same coefficient in Korea is not higher than 1.[5] For the Sharpe ratio, $E(R^e)/\sigma(R^e)$ where R^e is the excess returns of stocks over bonds, low equity premium and high standard deviation of excess returns generates a very low Sharpe ratio in Korea. It is not higher than 0.1 while it is 0.35 in U.S. data. These characteristics will affect the result of the GMM estimation and the shape of Hansen-Jagannathan bounds. These are discussed in detail later.

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[4] Calculating the statistics based on yearly observations, the average stock market return and its standard deviation are 4.98% and 3.68% respectively. They are 5.57% and 2.53% without the year of 1998.

[5] When calculating the number using the annual data, not by annualizing as shown in table 3, it is 0.45 (=2.53/5.57).

Table 3 Comparison of Coefficient of Variation and Sharpe Ratio

	This Study	Kim and Hong (2008)	Dokko *et al*. (2001)	U.S. Data
Coefficient of Variation	0.32 (=1.72/5.35)	N/A	0.81 (=5.18/6.39)	1.95 (=3.57/1.83)
Sharpe Ratio	0.08 (=2.64/32.40)	0.06 (=1.56/27.59)	0.06 (=2.21/33.33)	0.35 (=5.94/16.67)

Notes: This table compares coefficient of variation and Sharpe ratio from the existing literature. The sample period of Kim and Hong (2008) and Dokko *et al.* (2001) is 1980-2004 (after dropping 1987, 1998, 2000) and 1975-1999 (after dropping 1998) respectively. Both studies used saving deposit rates for riskfree rates. The U.S. data is the same that Mehra and Prescott (1985) used.

2.2. Diagnostics and Illustration of the Puzzle

In this section, we illustrate the equity premium puzzle using the basic pricing equation before going into deeper analysis using GMM estimation and Hansen-Jagannathan bounds. Known is that the first-order condition for an optimal consumption and portfolio choice can be used to price any assets. It is given by

$$E_t\left[m_{t+1}x_{t+1}\right] = p_t,$$

where m_{t+1} is the SDF, x_{t+1} is the payoff, and p_t is the price. The power utility case, m_{t+1} is $\beta(c_{t+1}/c_t)^{-\gamma}$. Since the price of return is one, it can be written as $E(mR) = 1$. For excess returns, it can be expressed as $E(mR^e) = 0$. We will illustrate the equity premium puzzle using this simple equation. Manipulating $E(mR^e) = 0$, we have

$$\frac{E(R^e)}{\sigma(R^e)} \le \frac{\sigma(m)}{E(m)},$$

since the correlation coefficient between the SDF and excess returns cannot be larger than one. An approximation in discrete time with a time-separable utility implies

$$\frac{E(R^e)}{\sigma(R^e)} \leq \frac{\sigma(m)}{E(m)} \approx \gamma\sigma(\Delta c), \tag{1}$$

where $\gamma \equiv -u''(c)c / u'(c)$ is the local curvature of the utility function and risk aversion coefficient for the power utility case.

For the U.S. annual data, we have $E(m) = 1/R^f = 1/1.01 = 0.99$, the standard deviation of per capita consumption growth is 3.57%, and Sharpe ratio is $E(R^e)/\sigma(R^e) \approx 5.94\% / 16.84\% = 0.35$. Following Cochrane (2005), the correlation between the SDF and excess returns is about 0.2 or less.[6] Then,

$$\frac{\sigma(m)}{E(m)} = \frac{1}{|\rho|}\frac{E(R^e)}{\sigma(R^e)} = \frac{1}{0.2}\frac{5.94}{16.84} = 1.75.$$

It implies

$$\gamma\sigma(\Delta c) = \gamma \times 3.57\% = 1.75.$$

To make this equation hold, we need a risk aversion coefficient of 49.02 (= 1.75/3.57%), which is too big to be justified.

For the case of Korea, we found

$$\gamma\sigma(\Delta c) \approx \frac{\sigma(m)}{E(m)} = \frac{1}{|\rho|}\frac{E(R^e)}{\sigma(R^e)} = \frac{1}{|0.03|}\frac{2.64}{32.4}.$$

It implies $\gamma\sigma(\Delta c) = \gamma \times 1.71\% = 2.71$, and gives too high of a risk aversion coefficient. This highlights the puzzle: we cannot have a reasonably low risk aversion coefficient which can explain the given asset market data.

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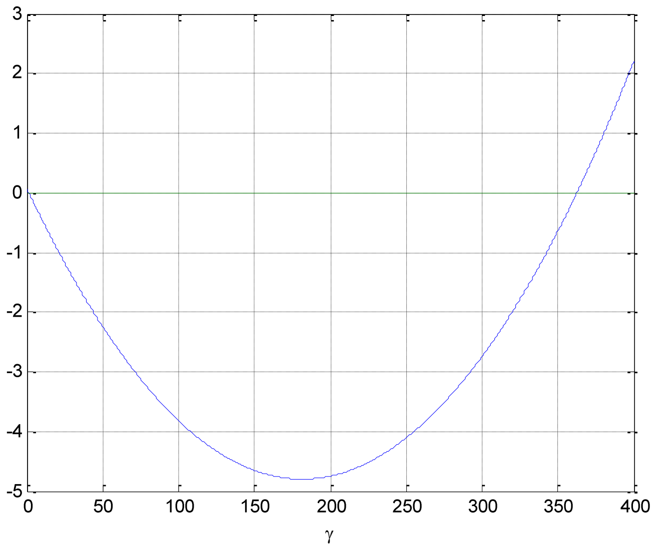
6) The correlation coefficient is calculated with log utility function, in which relative risk aversion coefficient is one. This is calculated it in the same way for the case of Korea.

Figure 1 Finding Risk Aversion Coefficient from $R^f = 1/E(m)$



Note: This figure shows a preliminary search of relative risk aversion coefficient using the equation of (2).

It is possible to make another hypothesis about the value of a risk aversion coefficient under the normality assumption of consumption growth and returns. From the condition of $E(mR^f) = 1,$ we can derive the following:

$$r^f = -\ln\beta + \gamma E(\Delta c) - 0.5\gamma^2\sigma^2(\Delta c). \tag{2}$$

Substituting $\beta = 0.99,$ annualized $E(\Delta c) = 5.35\%,$ annualized $\sigma^2(\Delta c)$ $= (1.72\%)^2,$ and $r^f = 5.56\%$ into the equation, we could get the estimate for γ. They are 1.23 and 362.4. It is shown in figure 1. Which one would we choose? We do not know *a priori*. That is one of reasons why we turn to GMM estimation in the following section.

2.3. GMM Estimation

Though GMM estimation is a well-known procedure, it would be good to have a quick refresher on it because we are going to look at the moment conditions and weights in them in detail.[7] We define GMM estimation as follows: we have parameters $b_{K\times 1}$ and n moment conditions

$$E\left[f(x_t, b)\right] = 0,$$

for $n > K$. Let $g_T(b) \equiv \sum_{t=1}^{T} f(x_t, b)/T$ be the sample analogue of the moment conditions so that $g_T(b)$ is a $(n \times 1)$ vector. Hansen (1982) proposes choosing a_T and b such that

$$\underset{K\times n}{a_T}\ \underset{n\times 1}{g_T}(b) = 0.$$

If we estimate b by $\text{Min}\ g_T(b)' W g_T(b)$, the first-order conditions are

$$\frac{\partial g_T(\hat{b})}{\partial b} W g_T(\hat{b}) = 0,$$

with $a_T \equiv D' \equiv (\partial g' / \partial b)W$. The weight matrix W is usually an identity matrix in the first stage estimation. In the second stage, we use the optimal weighting matrix S instead of W. The general GMM procedure allows the choice of arbitrary linear combinations of the moments so that they can be set to zero.

In the context of asset pricing, $f(x_t, b)$ would be pricing error $u_{t+1} = m_{t+1}x_{t+1} - p_t$. In the power utility case, we try to find the risk aversion coefficient γ, which minimize the objective function with $g_T(\gamma) \equiv \Sigma[\beta(c_{t+1}/c_t)^{-\gamma} x_{t+1} - p_t]/T$, given asset market and consumption data. We set $\beta = 0.99$ and focus on the value of γ. We perform a GMM estimation for three cases: excess returns alone, risk-free rate alone, both excess returns and risk-free rate. For excess returns, we have $x_{t+1} = R^e_{t+1}$ and $p_t = 0$. For vector returns, we have

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[7] See Cochrane (2005) on the application of GMM to asset pricing.

$$x_{t+1} = \begin{bmatrix} R^e_{t+1} \\ R^f_{t+1} \end{bmatrix} \text{ and } p_t = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Table 4 displays the estimation results. For the first case of excess returns, it shows that γ estimate is too high at 110.9, with a large standard error of 114.8. Figure 2 shows how GMM procedure finds the value of 110.9. The top figure in figure 2 clearly shows that the pricing error $E(mR^e - 0)$ reaches zero around 110. However, it also shows that the standard deviation at the value is somewhat large, supporting high standard errors of 114.8 obtained in GMM estimation. This result suggests that, with reasonably low values of γ less than 5-10, we cannot explain the equity premium.

The second estimation with risk-free rates gives more promising result in terms of the magnitude of γ. Table 4 shows that the estimate is 0.28 with standard error of 0.078, making the estimate statistically significant. The bottom figure in figure 2 confirms it. The pricing error passes the zero line around 0.28 and the standard deviation at the value is very small. This value is very similar to the ones reported in Kim and Hong (2008). This result is good news to those who believe that the relative risk aversion coefficient is relatively low and not higher than 5 or 10. However, it seems inappropriate to argue that the equity premium observed in the asset market can be explained with a low value of γ, because the low value does not come from the moment condition of excess returns. The low estimate of 0.28 is the value that satisfies only the moment condition of risk-free rates.

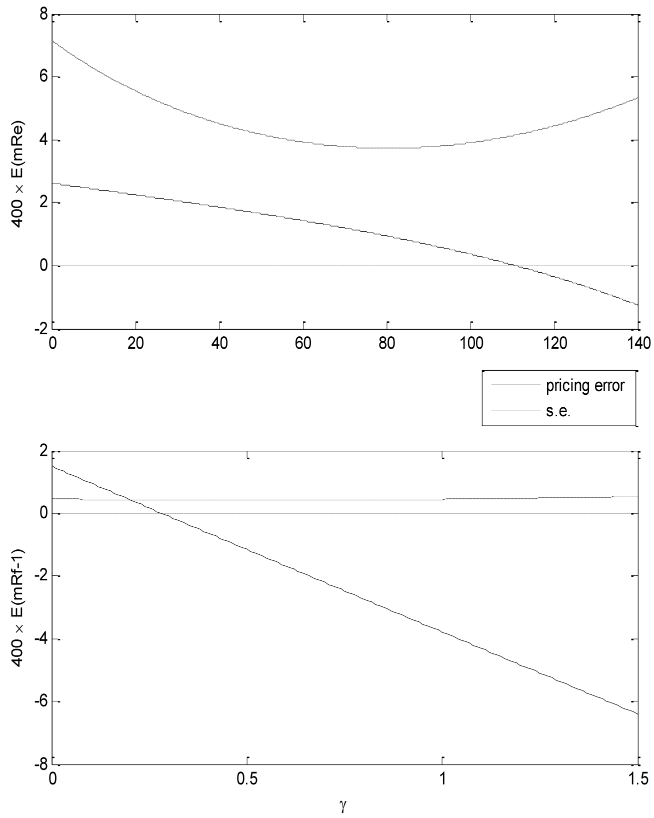
For the third case, we estimate γ using the two moment conditions together. Table 4 reports the statistically significant estimate of γ. Note that GMM finds a value of γ which makes both moment conditions close to zero. The estimate is reasonably low at 0.28 with low standard error. This value is quite close to the one from risk-free rate moment condition. The $a = D'$ and $a = D'S^{-1}$ diagnostics in table 4 show why. Since the risk-free rate moment condition is far more sensitive to the value of γ than is the excess return moment condition, GMM pays more attention to the former.

Table 4 GMM Estimation

	1st Stage	2nd Stage
(1) One Excess Return: R^e		
γ_{GMM}	110.9233	110.9233
Standard Error	114.7812	114.7812
J-statistics (*p*-value)	N/A	N/A
$a = D'$; $a = D'S^{-1}$	–8.9751e-005	–0.0106
Minimized $g_T \times 400$	7.2283e-009	7.2283e-009
$D'g_T$; $D'S^{-1}g_T$	1.8071e-011	–1.9103e-013
(2) One Risk-free Rate: R^f		
γ_{GMM}	0.2810	0.2810
Standard Error	0.0775	0.0775
J-statistics (*p*-value)	N/A	N/A
$a = D'$; $a = D'S^{-1}$	–0.0133	–156.9622
Minimized $g_T \times 400$	1.4162e-007	1.4162e-007
$D'g_T$; $D'S^{-1}g_T$	–1.3402e-004	–5.5574e-008
(3) Two Returns: R^e and R^f		
γ_{GMM}	0.2826	0.2820
Standard Error	0.0774	0.0774
J-statistics (*p*-value)	N/A	0.1357 (0.287)
$a = D'$; $a = D'S^{-1}$	–0.0000, –0.0133	–0.3347, –157.2807
Normalized to $\Sigma D_i = 1$	0.0033, 0.9967	0.0021 0.9979
Minimized $g_T \times 400$	2.6132, –0.0087	2.6132, –0.0056
$D'g_T$; $D'S^{-1}g_T$	–1.3402e-004	1.4990e-008

Notes: This table reports the results from GMM estimation. The first order condition in the first stage is given by $a_T g_T = 0$ where $a_T \equiv D' \equiv (\partial g' / \partial b)W$ and $g_T(\gamma) \equiv \Sigma[\beta(c_{t+1}/c_t)^{-\gamma} x_{t+1} - p_t]/T$. a is $D'S^{-1}$ in the second stage, where S is the optimal weighting matrix. In estimation, we set $\beta = 0.99$.

Figure 2 Pricing Errors from GMM Estimation



Notes: This figure shows pricing errors from GMM estimation. The top figure shows pricing errors of $E(mR^e - 0)$ and the standard deviation given the value of γ. The bottom figure is for the case of the risk-free rate, $E(mR^f - 1)$.

The first-stage GMM from a minimization problem does not set $[1 \ \ 1]g_T = 0$, but it sets $D'g_T = 0$. Thus moments which are not sensitive to γ get lower weight. Table 4 reports the values of elements in a_T (‘$a = D'$; $a = D'S^{-1}$’) and the weighted values (‘Normalized to $\Sigma D_i = 1$’). The two values, 0.0033 and 0.9967, can be interpreted as how much attention GMM pays to each moment conditions. The relative size of two values implies that GMM pays almost no attention to the first moment, which is from excess returns.[8] Also note that *J*-statistics suggests the two moment conditions are not jointly close to zero in statistical sense. From this experiment, we can see that a low estimate of γ in vector return case is almost entirely derived from the risk-free rate moment condition, not from the moment condition related to the equity premium, which we are trying to explain. Thus it is not reasonable to believe that the low value of γ can explain the equity premium observed in Korean asset markets. The following section adds another evidence for this argument using Hansen-Jagannathan bounds.

3. ANOTHER LOOK AT THE EQUITY PREMIUM: HJ BOUNDS

Hansen and Jagannathan (1991) describe a nonparametric way of summarizing the equity premium puzzle by asking the conditions for admissible stochastic discount factors given the data of payoffs and prices.[9] Specifically, given data on *p* and the distribution of returns *R*, they want to infer the properties of SDF while imposing no more structure than linearity or the pricing functional.[10] Imposing only this, they construct bounds on the

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[8] We can perform an interesting experiment to examine this reasoning. What would be the result with a different weighting matrix like

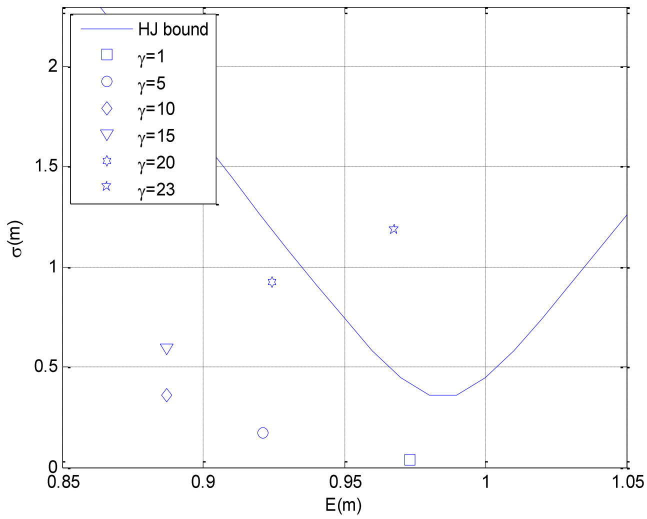
$$W = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}.$$

It implies that we put 100 times more weight on the excess return moment condition and estimate. In the first stage, we obtain $\widehat{\gamma} = 0.446$ with a standard error of 0.076. The estimate gets a little bit higher because we put more weight on the excess return moment condition, which produces higher γ estimate. In the second stage, the GMM considers the fact that the excess return moment condition is more volatile, so its entry into S is larger, and the entry into S^{-1} is smaller. The second stage estimate is 0.282 with standard error of 0.076. Thus we come back to the initial situation.

[9] Also see Hansen and Jagannathan (1997).

[10] This linearity is guaranteed by the law of one price.

Figure 3 Hansen-Jagannathan Bounds: Case of U.S.

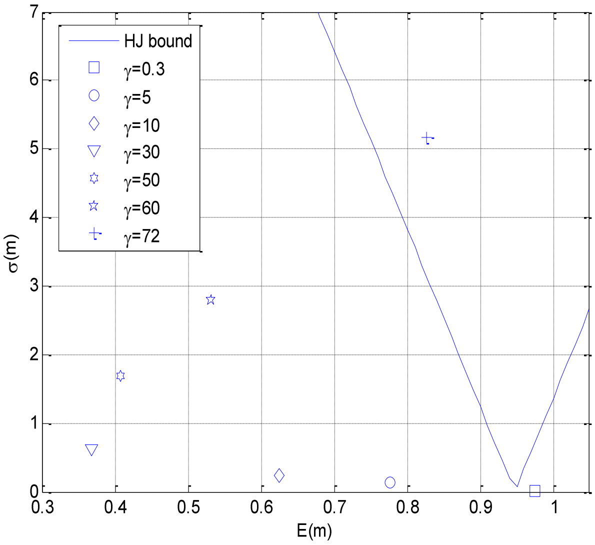


Note: This figure shows Hansen-Jagannathan bound based on U.S. stock market data of 1890-1979 and the combinations of $\{E(m),\ \sigma(m)\}$ depending on various values of relative risk aversion coefficient.

first and second moments of SDF that are consistent with a given distribution of payoffs on a set of primitive securities. This is like asking "what is the set of $\{E(m),\ \sigma(m)\}$ consistent with a given set of asset prices and payoffs?" They derive the lower bound of standard deviation of SDF $\sigma(m)$ given the mean of the SDF $E(m)$, which is constructed from data. It is called the Hansen-Jagannathan bound.

We will begin with the U.S. case as an illustrative purpose. Figure 3 shows the HJ bounds constructed from annual data of the U.S. from the years 1890-1979, which Mehra and Prescott (1985) initially used. The U-shape line is the HJ bound. If any SDF can explain the given asset market and consumption data of U.S., it should be on or above the U-shape line. We also plot $\{E(m),\ \sigma(m)\}$ with the power utility case. At $\gamma = 1$, we have $E(m) = 0.97$ and $\sigma(m) = 0.02$. It implies that, given $E(m) = 0.97$, the volatility of SDF $\sigma(m)$ is too small. If we can raise $\sigma(m)$ up to 0.5, the equity premium does not exist for the power utility case.

Figure 4 Hansen-Jagannathan Bounds: Case of Korea



Note: Figure 4 shows Hansen-Jagannathan bound based on the Korean stock market data of 1987:I-2007:IV (after dropping 4 observations of 1998) and the combinations of $\{E(m),\ \sigma(m)\}$, depending on various values of relative risk aversion coefficient.

Then, what about accepting a higher risk aversion and raising it? Another puzzle, called "risk-free rate puzzle," arises. Note that the inverse of SDF $1/E(m)$ is the risk-free rate, following $E(mR^f)=1$. As we raise γ higher and higher, $E(m)$ decreases, implying higher risk-free rate. At $\gamma=10\sim15$, $E(m)\approx0.89$, meaning that $R^f=1/0.89\approx1.124$. 12.4% of risk-free rate is too high for the U.S. data.

In figure 3, $\{E(m),\ \sigma(m)\}$ moves northwest as γ increases. When γ is over 15, it changes the direction, heading northeast.[11)] This finally goes into the region of the HJ bound when γ is larger than 22-23. This result implies that we need the risk aversion coefficient at least higher than 22-23,

11) The change of its direction is due to precautionary saving motive. It's easier to explain with normality assumption and (2). As γ increases, the third tem $(-0.5\gamma^2\sigma^2(\Delta c))$ starts to dominate the second term $(\gamma E(\Delta c))$, lowering the risk-free rate (raising $E(m)=1/R^f$).

in order to explain the U.S. asset market characteristics with a power utility. Following Kocherlakota (1996), it remains a puzzle.

Figure 4 shows the HJ bound for Korea. The HJ bound for Korea looks similar to the U.S. case in terms of the overall U-shape. However, the lowest bound is closer to zero compared to the U.S. case, reflecting the lower Sharpe ratio of Korea.[12] And, the behavior of $\{E(m),\ \sigma(m)\}$ is somewhat different. As we increase γ up to 30, $\{E(m),\ \sigma(m)\}$ is still having a long detour to northwest, making both the equity premium puzzle and the risk-free rate puzzle more severe.[13] When γ is raised larger than 70, it finally lands in the region of the HJ bound. Hence, the equity premium puzzle is still alive.

It is useful to see how the first and second moment of the SDF respond to the risk aversion coefficient. Table 5 shows how $E(m)$ and $\sigma(m)$ vary with different values of γ for the case of Korea and U.S. Both $E(m)$ and $\sigma(m)$ of Korea seem to be less responsive to γ's. What makes this difference? We can see what's happening more easily with normality assumption of returns and consumption growth. Firstly, using (2), we can find the value of risk aversion coefficient at which $E(m)$ changes direction. By differentiating r^f, which is the inverse of $E(m)$, with respect to γ, we have $\partial r^f / \partial\gamma = E(\Delta c) - \gamma\sigma^2(\Delta c)$. As long as $\gamma > E(\Delta c)/\sigma^2(\Delta c)$, r^f increases and the inverse $E(m)$ decreases. The value of γ at which it changes direction will be larger as we have higher $E(\Delta c)$ and lower $\sigma^2(\Delta c)$. Note that, from table 3 the coefficient of variation $(\sigma(\Delta c)/E(\Delta c))$ is very small in Korea, suggesting that the risk-free rate will change direction at a higher value of γ compared to the U.S. case.

The response of $\sigma(m)$ is also problematic in Korea. As shown in figure 4 and table 5, $\sigma(m)$ increases slowly at low values of γ. If we use the assumption of normality and power utility again, it shows that the problem lies in the low volatility of consumption growth in Korea. From (1), $\sigma(m)$ can be rewritten as

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12) Higher Sharpe ratio makes the restriction on $\sigma(m)$ tighter, given the value of $E(m)$.

13) $E(m)$ of 0.37 implies that the risk-free rate is higher than 207% (=1/0.37), which is unrealistic.

Table 5 Mean and Standard Deviation of Stochastic Discount Factor

γ	Korea		U.S.	
	$E(m)$	$\sigma(m)$	$E(m)$	$\sigma(m)$
0.5	0.97	0.02	0.98	0.02
1	0.94	0.03	0.97	0.03
3	0.85	0.09	0.95	0.10
5	0.78	0.13	0.92	0.17
10	0.62	0.23	0.89	0.36
15	0.52	0.31	0.89	0.60
20	0.45	0.39	0.92	0.92
30	0.37	0.63	1.15	2.12
45	0.37	1.31	2.23	7.51
60	0.53	2.79	5.84	27.70

Note: This table shows (given consumption data) how the first and second moment of stochastic discount factor behave as the relative risk aversion coefficient (γ) varies.

$$\sigma(m) \approx \gamma\sigma(\Delta c)E(m).$$

Differentiating with respect to γ gives

$$\frac{\partial\sigma(m)}{\partial\gamma} = \sigma(\Delta c)E(m) + \gamma\sigma(\Delta c)\frac{\partial E(m)}{\partial\gamma}.$$

Its sign is positive as shown in table 5 and figure 4. Again, because of small $\sigma(\Delta c)$, we have $\partial E(m)/\partial\gamma < 0$ at low values of γ and the value of $\partial\sigma(m)/\partial\gamma$ is also small. This is why we need a high-risk aversion coefficient enough to put $\{E(m),\ \sigma(m)\}$ into the HJ bound.

The low volatility of consumption growth $\sigma(\Delta c)$ plays an important role in characterizing the behavior of $E(m)$ and $\sigma(m)$ in Korea. Note that, if we can find a way to raise $\sigma(m)$ with minimally affecting $E(m)$, we can go directly into the HJ bound. This insight made economists create ways to raise $\sigma(m)$ by modifying the preference. They include habit formation and recursive utility function. In the following section, we will see how a recursive utility helps explain the equity premium puzzle and what factors contribute to generating low $\sigma(\Delta c)$.

4. RECURSIVE PREFERENCES AND LONG-RUN RISKS

One shortcoming in using power utility function is that the coefficient γ plays two roles: risk aversion and the elasticity of intertemporal substitution (more accurately, inverse of intertemporal substitution). This is one motivation for Epstein and Zin (1989) to present a recursive utility function which disentangles risk aversion with the elasticity of intertemporal substitution. They propose a recursive formulation of utility:

$$U_t = \left[(1-\delta) c_t^{\frac{1-\gamma}{\theta}} + \delta \mu_t^{1-1/\psi} \right]^{\frac{1}{1-1/\psi}},$$

where $\theta \equiv (1-\gamma)/(1-1/\psi)$ and $\mu_t = [E_t(U_{t+1}^{1-\gamma})]^{1/(1-\gamma)}$. The latter is the certainty equivalence of future utility or risk correction for uncertain future utility. Note that it reduces back to power utility function when the inverse of the elasticity of intertemporal substitution is equal to the coefficient of risk aversion, $\gamma = 1/\psi$.

There are many interesting properties related to the Epstein-Zin utility function (henceforth, EZ utility function). However, there is a need to focus on how this setup can help understand the equity premium puzzle. Derivation of the SDF for EZ utility function gives

$$m_{t+1} = \left[\delta \left(\frac{c_{t+1}}{c_t} \right)^{-1/\psi} \right]^{\theta} \left(\frac{1}{R_{t+1}^W} \right)^{1-\theta},$$

where R^W is the return on wealth, which gives a consumption stream.[14)]

$$E_t\left[\delta^{\theta}\left(\frac{c_{t+1}}{c_t}\right)^{-\theta/\psi}\left(\frac{1}{R_{t+1}^{W}}\right)^{1-\theta}R_{t+1}^{i}\right]=1. \tag{3}$$

Since the standard deviation of SDF $\sigma(m)$ has another term of return on wealth (R^W), the SDF now has extra variability and higher covariance with stock returns, thereby "fixing" two crucial elements that cause the low equity premium in the power utility framework: low $\sigma(m)$ and Cov(m, R). In addition, separation between risk aversion and intertemporal substitution implies that a higher risk aversion does not cause a decrease in $E(m)$ and an increase in the risk-free rate. In the HJ bound context, it implies that we can go directly into the bound by moving upward, without unnecessarily making a roundabout trip.

Dokko *et al*. (2001) explore the possibility of explaining the puzzle using EZ preference. They conclude that, if we use EZ utility and the consumption growth follows the random walk, it is possible to explain the equity premium in Korea with $\gamma \approx 10$ and $\psi \approx 0.5$. There are two problems in the conclusion. One is that they use a simplifying assumption, which makes the specification too simple. In reaching the conclusion, they assume that consumption growth process is i.i.d.. This assumption drops the term related to R^W and makes (3) return to the standard Euler equation with power utility case. In the context of the HJ bounds, benefit of using EZ utility function is that we can raise $\sigma(m)$ without affecting $E(m)$. However, if the term R^W is dropped, a route to potential resolution of the puzzle is lost by adding an additional term to the SDF and raising $\sigma(m)$. Moreover, empirical evidence does not support this assumption. We estimate the following equations simultaneously to see if the consumption process in Korea follows the random walk.

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14) We do not include the derivation since it is a known procedure and is not the focus of this study.

$$x_t = \rho x_{t-1} + \sigma_x \varepsilon_t,$$
$$\Delta c_t = \mu + x_{t-1} + \sigma_u u_t. \qquad (4)$$

If $\rho = 0$, consumption process c_t follows the random walk. As ρ gets closer to 1, consumption growth process contains a slow-moving component, which is called “long-run risk” by Bansal and Yaron (2004).[15) This experiment has two purposes. One is to check the plausibility of the assumption of i.i.d. consumption growth. The other is to see what factors might contribute to low consumption volatility. The standard deviation of consumption growth $\sigma(\Delta c)$ is affected by three parameters, σ_u, ρ, and σ_x.[16)

We first estimate the four parameters in the model $(\mu,\ \sigma_u,\ \rho,\ \sigma_x)$ with maximum likelihood estimation (MLE), imposing few restrictions on the parameters. Then we use the MLE estimates as means of prior distributions used in Bayesian estimation.[17) Table 6 shows the estimation results of MLE and Bayesian estimation. Figure 5 shows the mean, prior, and posterior distribution from Bayesian estimation. MLE estimate of ρ in table 6 is reported as 0.91, very close to one and far away from zero, with high *t*-value. Additionally, Bayesian estimate of ρ is 0.87, with 95% confidence interval of (0.7573, 0.9964). All these add validity to the argument that consumption growth is not i.i.d.. Therefore, assuming i.i.d. consumption growth and taking away the returns from consumption stream from the SDF of the EZ utility function is undesirable.

Additionally, taking a low value of EIS less than one is not empirically consistent. Bansal *et al*. (2005) theoretically and empirically show that a rise in consumption volatility lowers asset valuations. Theoretically, a necessary

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15) This specification comes from Bansal and Yaron (2004). They argue that combining EZ utility function and long-run risks can potentially explain most asset market anomalies.

16) By comparing the estimates of σ_u, ρ, and σ_x with counterparts in the U.S., it is possible to see which parameter contributes more to low standard deviation of consumption process in Korea. We leave this for future research.

17) For prior distributions, we use beta distribution for ρ, uniform distribution for μ, and inverse gamma distribution for σ_u and σ_x. Further details on Bayesian estimation are available by request to the authors.

Table 6 ML and Bayesian Estimation of Consumption Growth Process

(1) Maximum Likelihood Estimation

	Estimates	Standard Deviation	*t*-statistics
ρ	0.9104	0.0709	12.84
μ	0.0133	0.0026	5.04
σ_x	0.0022	2.5017	2.50
σ_u	0.0065	9.7950	9.80

(2) Bayesian Estimation

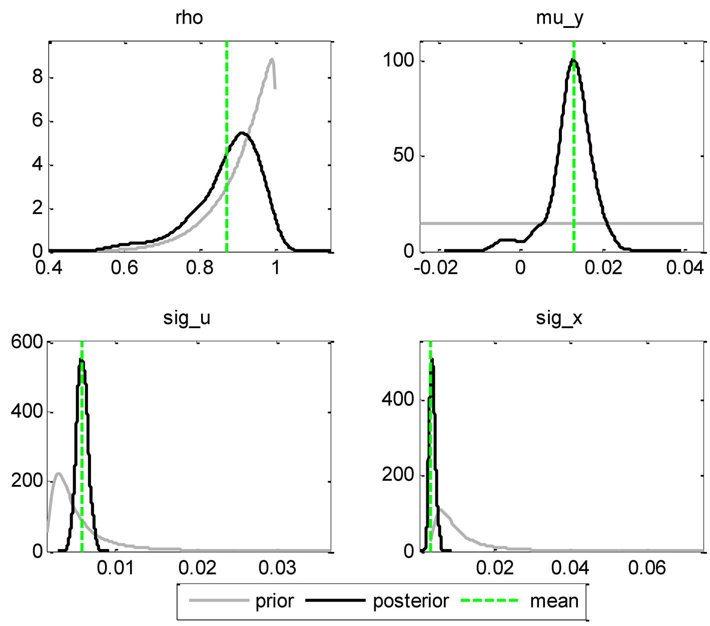
	Prior Mean	Posterior Mean	95% Confidence Interval
ρ	0.910	0.8724	(0.7573, 0.9964)
μ	0.010	0.0124	(0.0033, 0.0195)
σ_x	0.002	0.0040	(.00028, 0.0051)
σ_u	0.006	0.0060	(0.0048, 0.0070)

Note: This table shows estimation results from maximum likelihood estimation and Bayesian technique of (4).

condition for having this prediction is that EIS should not be less than one. They show that increased consumption volatility lowers PE ratio (ratio of price to earnings) for major countries.[18] This finding supports adopting the specification with EIS larger than 1. Thus, with these two things in mind, simply taking the values of $\gamma \approx 10$ and $\psi \approx 0.5$ with i.i.d. consumption growth cannot explain the equity premium observed in Korea.

[18] We investigate the correlation between consumption volatility and lagged PE ratio in Korea, which shows negative signs as predicted by Bansal *et al.* (2005). However, since it is not a main concern of this paper, we do not it report here.

Figure 5 Prior and Posterior Distribution of Parameters



Note: Figure 5 shows the prior/posterior distributions and means of four parameters from the equation system of (4).

5. CONCLUSION

We find that the equity premium puzzle is highly likely to exist in Korea, despite the small equity premium. The findings also suggests that the volatility of consumption growth and the correlation with asset returns matter rather than the absolute magnitude of excess returns.

At this stage there are at least two things to do in the future. Firstly, we need to wait until we add more observations. The history of asset return data is very short in Korea and the time span during which we have reliably collected data is even shorter. In this regard, analyzing with a large sample size may add to the credibility of any study.

Secondly, we need to see what factors contribute to the properties of consumption growth observed in Korea. A better understating of consumption behavior will help resolve the equity premium puzzle and understand other asset market phenomena since asset pricing theory says that returns of any assets are determined through the relation to consumption and how much they contribute to consumption smoothing.

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