Spending on Private Education:
Semiparametric Estimation Approach[*]

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In this paper we estimate Engel curve for private education expenditure using the Korea Labor and Income Panel Study 9th waves, assuming different functional forms according to householder's education levels. A semiparametric model, proposed by Powell (1986), is applied in order to solve several problems caused by the likelihood-based models. From the Engel curve, we also calculate income elasticity of the expenditure on private education.

The basic finding is that the Engel curve has the inverted-U shape, showing different patterns according to householder's education levels. The income elasticity tells us that private education service is a 'normal goods' for Korean households. The empirical results imply that the possibility to reproduce the social class through private education in the Korean society is quite high. The paper also points out such phenomenon that the Korean education largely focuses on university entrance exam.

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1. INTRODUCTION

Noticing the marvelous economic growth of East Asian countries, in particular Japan, Hong Kong, Taiwan, and Korea, an increasing attention has been given to what the resources to expedite the great growth are.   Among them, education has been pointed out as a main factor of the miracle.   As such, the past growth of the Korean economy in some decades can be somewhat contributed to the enthusiasm of Korean households for education; however, it is also true that nowadays too extreme concerns on education have brought out new social problem, 'private education'.   In fact, recent statistics report that the total expenditure on private education has been continuously increasing and the amounts are tremendous in recent periods.[1] Excessive expenditure on private education becomes a big burden to households.

Why has private education been spotlighted in the Korean society?   The main reason lies in the intrinsic characteristics of private education.   While public education gives all the students the same opportunity to receive educational service, private education is quite selective according to household's preference, so the market for private education would be ruled by the competition principle.   Therefore, as income levels of households become high, the probability to invest on private education may increase, which indicates that the social class can be reproduced through private education.   This is obviously opposed to Hanusheck (1998)'s assertion that education play a role of demolishing the distinction of classes.   Rather, education may adhere to the social reproduction of classes, especially in Korea where general education depends on private education with high proportion.   Furthermore, the bi-polarization phenomenon accelerated after the Asian financial crisis would make the inequality of the class even worse. As a matter of fact, the difference of household's expenditure on private

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[1] National Income Statistics, provided by the Bank of Korea (BOK), reported that household expenditure on private tutoring institutes was 10.6, 10.9, 11.8, and 12.3 trillion KRW for the 1st quarter in 2004, 2005, 2006, and 2007, respectively. (Yonhap News: the article on June 4, 2008)

education may give a social sense of alienation to low income class because of high burden on the expenditure.   In this sense, we analyze Engel curve for private education expenditure in the Korea economy, which allows us to directly assess the patterns of household expenditure on private education by the classes or various income levels.

The studies of the relationship between commodity expenditure and income, i.e., Engel curve, have been at the center of applied microeconomic welfare analysis.   Earlier studies in this area have mainly examined to empirically prove Engel's law, the expenditure share of food decreases as income level increases. Starting with the early attempts of Working (1943) and Lesser (1963), which show a linear relationship between income (or total expenditure) and the expenditure share of food, i.e., the Working-Lesser specification, sequential studies had proven the effectiveness of the specification by empirical research for various countries (Pollak, 1971; Brown and Heien, 1972; Deaton, 1974).   But a complete description of consumer behavior sufficient for welfare analysis requires a specification of both Engel curve and relative price effects consistent with utility maximization.   Much effort of Muellbauer (1976), Deaton and Muellbauer (1980), Jorgenson *et al*. (1982) makes it possible to place the Working-Lesser Engel curve specification within integrable consumer theory.

More recent studies on Engel curve, however, have questioned whether or not a linear relationship can explain any other Engel curves for various commodities.   Consequently, a series of empirical Engel curve studies have shown that nonlinear relationships do provide more accurate picture of individual behavior for some commodities, but not all, rather than the linear specification.   Examples of those studies include Atkinson *et al*. (1990), Bierens and Pott-Buter (1987), Blundell *et al*. (1993), Hausman *et al*. (1995), and Lewbel (1991).   Furthermore, Kedir and Girma (2003) found that the quadratic Engel curve can also be observed in food insecure countries, using the 1994 Ethiopian Urban Household Budget Survey data set.   For nonlinear specification, Bank *et al*. (1997) proposed QUAIDS (Quadratic Almost Ideal Demand System), which integrates quadratic Engel curve

specification with consumer theory.

While many studies on Engel curve have widely been discussed concerning various commodities such as food, alcohol, clothing, and fuel, Engel curve for private education expenditure has received little attention. However, recently, issues regarding private education expenditure have been emerged as a hot issue especially in the Korean society.   On the basis of the Working-Lesser specification, Kim (2002) analyzed the relationship between total expenditure and the share of expenditure on private education in order to investigate the effect of introducing government-sponsored compulsory middle school education.   His main results report that the increment of private education expenditure, caused by government-sponsored compulsory middle school education, is small for high income classes as a necessary good and large for low classes as a luxury good.   Nam (2007) presented that Engel curve for private education expenditure in Korea has a quadratic form and the shapes.

Model specification can be an important issue in order to more accurately estimate Engel curve for private education expenditure.   In the case of private education expenditure, in particular, we would expect that household's behavior of the consumption on private education services varies with household's income level or preference.   Accordingly, we allow a nonlinear relationship for the curve by adding nonlinear terms in total expenditure.   Nonlinear Engel curve includes the linear relationships of expenditures and income as a special case.   This would provide accurate information concerning household's consumption behavior on private education service.

This paper has several features for Engel curve estimation regarding private education.   First, we assume different Engel curve functional forms according to householder's education levels.   Other studies have widely used dummy variables as a proxy of education levels; however, assuming different functional forms according to education levels has recently been regarded as one of the general methods to analyze the education effects (Kapteyn *et al*., 2005), so we also apply this specification to our model.

Second, the number of children is classified into two groups by their education levels, high school students or lower and college student or higher, to investigate which education level is a heavier consumer of private education. Finally, we apply a semiparametric approach to estimate Engel curve for private education expenditure. Considering the existence of zero expenditure on private education, Tobit model, one of the likelihood-based models, is widely used to solve censored data problem. However, these models have well-known weak points: sensitiveness to the prespecified error distribution and inconsistency caused by heteroscedasticity. The paper solves such problems of likelihood-based models, by using a semiparametric method, proposed by Powell (1986).

In the paper, we mainly estimate Engel curve for private education expenditure according to householder's education levels by employing a semiparametric method, Symmetrically Trimmed Least Squares (hereafter STLS) estimation. Since the method is based on symmetric error distribution, a rigorous test for the conditional symmetry, proposed by Zheng (1998), is applied. From the estimated Engel curves, we also plot the curves to compare each other, and derive income elasticity of private education expenditure according to various total expenditure (or income) levels.

This paper is organized as follows. Descriptive statistics of the data are reported in section 2. Section 3 provides preliminary analysis of the data, and section 4 discusses the estimation method applied in this paper and proposes the test method for conditional symmetry of the error distribution. Section 5 shows the empirical results for the Engel curve estimation and their implication. In section 6, we calculate income elasticity of private education expenditure by taking derivative of the Engel curve. Finally, brief summary and concluding remarks are presented in section 7.

2. THE DATA

In order to estimate Engel curve for private education expenditure, we use

the KLIPS (Korea Labor and Income Panel Study) 9th Waves, recently provided by Korean Labor Institute (KLI). The KLIPS is a longitudinal survey of Korean households and individuals residing in urban areas, well representative of the total population. Since the first survey was conducted with 5,000 households in 1998, a sequential survey has been made by present. Information was collected on socio-economic characteristics: demographics, income and consumption, education, job market participation, etc.

From the raw data, we first found 5,002 observations excluding households with no response on 9th Waves. We have mainly focused on the cases which have children from primary education to college graduate in their households. Since we are focusing on private education expenditure of households, this criterion would be quite valid. As a result, we get 2,516 observations out of 5,002, and from those, 903 households with zero private education expenditure are observed.

Descriptive statistics for the private education expenditure and the expenditure share according to householder's characteristics are provided in table 1. As shown in Panel A, households in full sample spent on average 3,266 thousand won (KRW) per year as private education expenditure, which indicates that they spent approximately 270 thousand KRW per month; and the expenditure share was on average 11.0% of their total expenditure. Furthermore, when we consider the households with only nonzero expenditure on private education, the amount of the expenditure was 5,094 thousand KRW per year, and the share was 17.2%, implying that the realized burden, imposed on households truly spending on private education, was higher than we expected.

Panel B reports households' behaviors on private education expenditure according to householder's education levels. Here, "Middle School," "High School," and "University" indicate householder's final education levels of middle school graduates or lower, high school graduates, and college graduates or higher, respectively. The mean of private education expenditure as well as the expenditure share increase with householder's education levels, showing that investment on private education is quite sensitive to education

Table 1   Descriptive Statistics according to Householder's Characteristics

(Unit: ten thousand KRW, %)

	# of Obs.	Private Education Expenditure			Expenditure Share of Private Education		
		Mean	Median	Std. Dev.	Mean	Median	Std. Dev.
Panel A. Full Sample							
Full Sample	2516 (1613)	326.633 (509.49)	216 (396)	413.666 (416.838)	0.110 (0.172)	0.094 (0.162)	0.113 (0.096)
Panel B. Householder's Education Level							
Middle School	472	142.246	0	252.905	0.059	0	0.09
High School	1,106	305.284	240	353.049	0.114	0.100	0.111
College	938	444.589	324	498.564	0.132	0.127	0.117

Note: The values in parentheses indicate sample statistics excluding household observations with zero private education expenditure.

levels of householders with much difference.   Those results imply that the class reproduction may be possible through private education as a medium; accordingly, the various analyses of Engel curve for private education expenditure based on householder's education levels are required.

There are two noticeable features.   First is the difference between mean and median.   The median expenditure is roughly two-thirds of mean expenditure.   This suggests that the distribution of the expenditure on private education is highly skewed to the right.   Secondly, in some classes with head's final education being middle school graduates, the median expenditure is equal to zero, which reveals that half of the households do not expend on private education.   These features confirm that the class reproduction may be possible through private education as a medium.

3. PRELIMINARY ANALYSIS OF THE DATA

3.1. Nonparametric Density Function Estimation

The observed spending on private education contains some extreme value observations (possible outliers) and a fraction of zero expenditure. The problem caused by outliers is the estimation bias: while the consumption model based on a representative agent holds for the majority of the population there are individuals who exhibit consumption patterns inconsistent with the model. For the observed zero expenditure, there are

Figure 1   Nonparametric Density Function[2] of the Share of Private Education Expenditure

(a) Full Sample

(b) Education Level 1
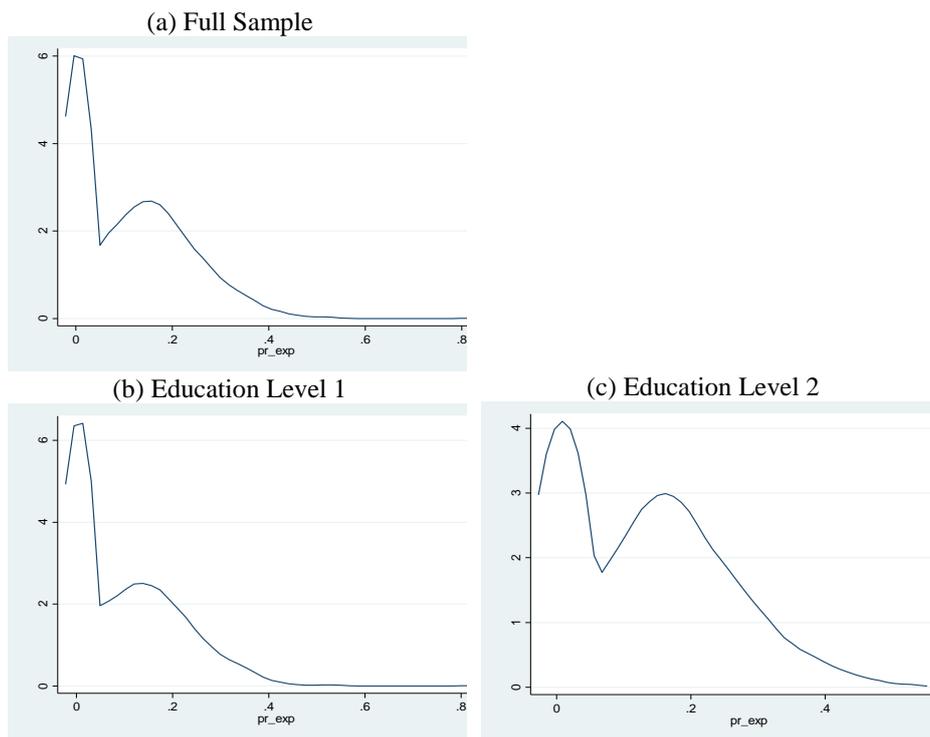
(c) Education Level 2

[2] In order to plot nonparametric density function, Epanechnikov kernel function is used.

Table 2   Skewness and Kurtosis of the Expenditure Share
of Private Expenditure

	Full Sample	Education Level 1	Education Level 2
Variance	0.013	0.012	0.014
Skewness	0.845	1.040	0.563
Kurtosis	3.387	4.178	2.590

two possible explanations: first, it can be resulted from false reporting by either the respondent or the enumerator, and second, the household indeed does not expend on the private education during the period.

To figure out the overall properties of the share of private education expenditure, the kernel estimates of the share of the private education is plotted in figure 1.   As we see in figure 1, two modes are observed: one apparently originates from zero private education expenditure and the other is expected as a true sample mode.   The estimated nonparametric density functions reveal long tail to the positive direction.   This finding is consistent with the higher moments of the share of the private education expenditure as described in table 2.

The values of skewness of the share are 0.845, 1.040, and 0.563 in full sample, education level 1 and education level 2, respectively.   This indicates that the density of the expenditure share is highly right-skewed.   Moreover, the values of skewness and kurtosis imply that the density of the variable is neither normally distributed nor symmetric.

In this case, we would consider STLS estimation method as a semiparametric approach.   Since STLS estimation trims both ends under the assumption of symmetrically distributed errors, some parts of right-skewed density caused by outliers are eliminated.   More importantly, some parts of zero expenditure expected censored observations are also removed in order to make the distribution symmetric.   Therefore, if the symmetrically trimmed distribution satisfies the key assumption of symmetry, STLS estimation would obtain more robust estimates than general MLE models, for example Tobit model.

3.2. Nonparametric Regression Function Estimation

As a preliminary analysis, nonparametric regression estimation is applied to investigate how Engel curve for private education expenditure looks like. The shape of Engel curve demonstrates the consumption behaviors on private education varying with household's income levels; as a result, for more accurate description on Engel curve, a nonlinear relationship in total expenditure should be taken into account.   Therefore, we present the shape of Engel functions by applying nonparametric method, prior to specifying the form of Engel function.

In figure 2, we plot Engel curves for private education expenditure according to householder's education levels by using nonparametric regression method.   We put the bandwidth $h = 0.15$ and employ the Gaussian kernel function for estimating nonparametric regression. Horizontal axis in the figure represents log of total expenditure in panel (a) and total expenditure in panel (b), ranging from 5 percentile through 95 percentile of the variables.

**Figure 2   Engel Curves for Private Education Expenditure
by Nonparametric Regressions**

(a)   (b)
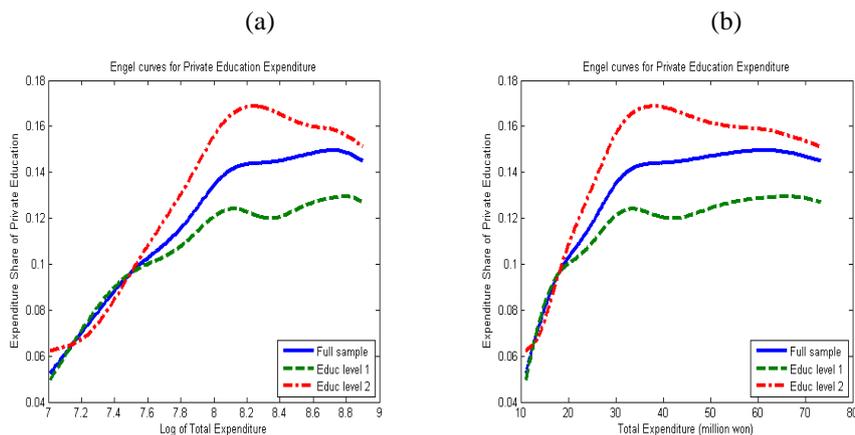
Even though the simple nonparametric regressions in figure 2 do not reflect the effect of household's demographic information on the estimation, we can derive meaningful implications regarding household's behaviors from the regression results. The first thing we found from the figure is the inverted-U shape of the Engel curves. Therefore, a quadratic model specification in log of total expenditure would be applied to estimations. In addition, the figure shows that Engel curves according to householder's education levels have different shapes, which justifies the different functional forms of the Engel curve according to householder's education levels.

4. ECONOMETRIC FRAMEWORK

4.1. Symmetrically Trimmed Least Squares Estimation

The variable to be explained is taken throughout to be the share of total household expenditure on private education, denoted by y^*. The basic formulation that we have used is the following:

$$y_i^* = x_i'\beta_0 + u_i, \tag{1}$$

where y_i^* is the share of expenditure on private education, x_i includes log of total household expenditure and variables of household characteristics. We have a simple quadratic approximation in the space of log of total household expenditure.

Private education expenditure of households is very selective, which gives dependent variable of Engel curve, the expenditure share of the commodity, zero values. In this case, the first thing we should consider is the censored data problem, caused by nonnegativity of the dependent variable.

The best-known method of treating the possibility that expenditures may be zero is to assume that y_i^* is censored at zero. The observed y_i is related to y_i^* by:

$$y_i = \max(0, \ y_i^*).$$ \hfill (2)

In effect, the density is 'piled up' at zero. With the assumption that u_i is normally distributed, this gives the Tobit model. We have then a probability $\phi(y_i / \sigma)$ of observing a positive expenditure share, where $\phi(\bullet)$ denotes the cumulative distribution function of standard normal, and a probability $1 - \phi(y_i / \sigma)$ of observing zero expenditure.

The Tobit model is restrictive. In particular, when the error density function does not follow the normal distribution, those estimators are quite sensitive to the specification of the (assumed) error distribution. Therefore, semiparametric approach which does not prespecify the error distribution can be applied to this model.

The presence of outliers and zero expenditures problem leads us to consider robust estimators with various desired properties. In this paper, we consider Symmetrically Trimmed Least Squares estimation, proposed by Powell (1986). Under the relatively weak assumption on the error distribution, i.e., symmetrically distributed error terms, STLS estimator is consistent and asymptotically normally distributed; moreover, the estimator is robust to heteroscedasticiy.

Figure 3   Distribution of y_i in Symmetrically Censored Sample
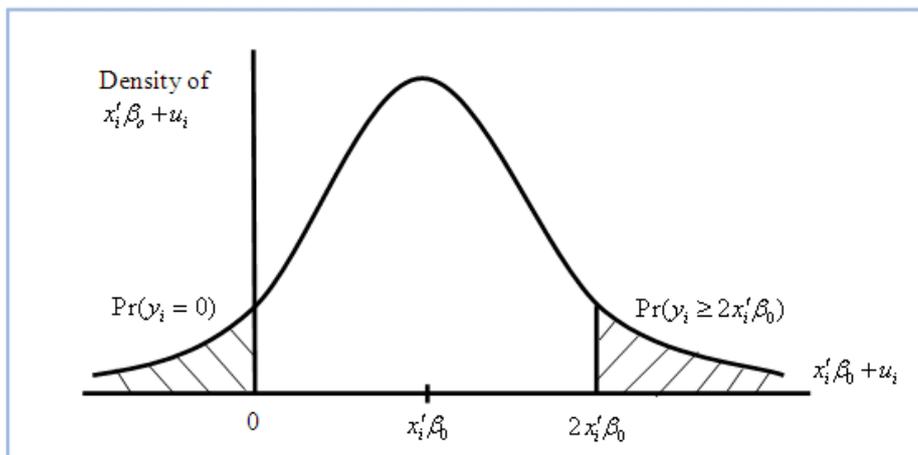
To make STLS explicit, consider figure 3, presented by Powell (1986). Suppose that the error density function is symmetric.   As shown in figure 3, we only observe zero value for which $y_i \leq 0$.   As we exclude observations of $y_i \geq 2x_i'\beta_0$ or $u_i \geq x_i'\beta_0$, then remaining observations would have error terms lying within $(-x_i'\beta_0, x_i'\beta_0)$.   Due to the symmetry of the distribution of the original error term, the residuals for the symmetrically truncated sample will also be symmetrically distributed about zero; the corresponding dependent variable would take values between zero and $2x_i'\beta_0$.   The STLS can be obtained from the least squares regression from symmetrically truncated sample.

The resulting "normal equation" is[3]:

$$0 = \sum_{i=1}^{n} 1\left(x_i'\beta_0 > 0\right) \cdot \left(\min\left\{y_i, 2x_i'\beta_0\right\} - x_i'\beta_0\right) \cdot x_i, \tag{3}$$

where $1(A)$ is the indicator function, i.e., $1(A) = 1$ if event A occurs and $1(A) = 0$ otherwise.   From the "normal equation," the objective function can be derived as follows:

$$S_n(\beta) = \sum_{i=1}^{n} \left(y_i - \max\left\{\frac{1}{2}y_i, x_i'\beta\right\} \right)^2 $$
$$+ \sum_{i=1}^{n} 1\left(y_i > 2x_i'\beta\right) \cdot \left[\left(\frac{1}{2}y_i\right)^2 - \left(\max\left\{0, x_i'\beta\right\}\right)^2 \right]. \tag{4}$$

Now, the STLS estimator can be obtained by minimizing the objective function, $S_n(\beta)$, and Powell (1986) proposed the STLS estimator just as

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[3] Here, the normal equation of equation (3) depends on the properties of the symmetry of error density function.   However, for the STLS to work actually, even if the original error density function is not symmetric, symmetry of the trimmed error density function is enough.

$$\hat{\beta}_n = \left[\sum_{i=1}^{n} 1\left(x_i' \hat{\beta}_n > 0 \right) \cdot x_i x_i' \right]^{-1} \sum_{i=1}^{n} 1\left(x_i' \hat{\beta}_n > 0 \right) \cdot \min \left\{ y_i, 2 x_i' \hat{\beta}_n \right\} \cdot x_i . \qquad (5)$$

By the iterative computation method, we can easily get $\hat{\beta}_n$ which is strongly consistent and asymptotically normal.

4.2. Tests for Conditional Symmetry

The key assumption on STLS estimation is the symmetric distribution of the underlying error terms; so, in order to make this estimation effective, it is essential to test conditional symmetry of the distribution. Fortunately, Zheng (1998) provided a useful consistent specification test method for conditional symmetry by using a kernel estimation method.

The specification test, proposed by Zheng (1998), is based on the null hypothesis of symmetric conditional distribution such as:

$$H_0 : \Pr\left[F\left(u_i \middle| x_i \right) = 1 - F\left(-u_i \middle| x_i \right) \right] = 1, \qquad (6)$$

where $F(u_i | x_i)$ indicates the conditional distribution function of u_i given x_i. Therefore, the alternative to be tested is that the conditional distribution is not symmetric.

To construct the test statistics, we denote that for any $t \in \mathbb{R}$, $\xi_i(t) \equiv 1(u_i \geq t)$ $-1(u_i \leq t)$, which indicated that $E\left[\xi_i(t) \middle| x_i \right] = 1 - F(t | x_i) - F(-t | x_i) = 0$ for all t with probability one. Note that since u_i has been already normalized at mean zero by STLS estimation, t would be zero. Additionally, denote $F_1(t)$ as the marginal distribution function of u_i. Then the following equation (7) holds if and only if H_0 is true.

$$W \equiv \int E\left\{ \xi_i(t) E\left[\xi_i(t) \middle| x_i \right] p(x_i) \right\} dF_1(t)$$

$$= \int E\left\{ \left[1 - F\left(t \middle| x_i \right) - F\left(-t \middle| x_i \right) \right]^2 p(x_i) \right\} dF_1(t) \geq 0. \qquad (7)$$

Denote the estimated $u_i(\hat{\beta}) \equiv g(y_i, x_i; \hat{\beta})$ where $\hat{\beta}$ is an estimator of β. Then, the marginal distribution of u_i can be estimated by $F_{1n}(t|\hat{\beta}) = \frac{1}{n}\sum_{i=1}^{n} 1(u_i(\hat{\beta}) \leq t)$, and $\xi_i(t)$ by $\xi_i(t, \hat{\beta}) = 1(u_i(\hat{\beta}) \geq t) - 1(u_i(\hat{\beta}) \leq -t)$.

As shown by Zheng (1998), the sample analog of W can be estimated by:

$$W_n \equiv \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{h^m} K\left(\frac{x_i - x_j}{h}\right) \int \xi_i(t, \hat{\beta}) \xi_j(t, \hat{\beta}) dF_{1n}(t|\hat{\beta}). \qquad (8)$$

For statistical inference to test the conditional symmetry of the residuals, the test statistics for symmetric conditional distribution under the null hypothesis can be derived as follows:

$$T_n \equiv nh^{m/2}W_n / \hat{\sigma} \xrightarrow{d} N(0, 1). \qquad (9)$$

From now on, we refer this test statistics as T_n statistics.

5. EMPIRICAL RESULTS

5.1. Conditional Symmetry Tests

If we get consistently estimated parameters for the Engel curve function by STLS estimation, testing conditional symmetry of error distribution would be necessary in order to make the estimators more reliable.   In this subsection, we apply a rigorous test method, proposed by Zheng (1998), to test the conditional symmetry.

As known, the distribution of the original dependent variable exhibits a right-skewed distribution, so does the error distribution, because of the censored data problem.   However, the semiparametric estimation method only requires the conditional symmetry of the 'symmetrically trimmed' error

term.   Therefore, we use the symmetrically trimmed residuals in this subsection.

Since the test method is based on nonparametric approach, we require some mechanism to choose the amount of "smoothing" imposed, so how to select the bandwidth h is an important issue.   In the paper, the smoothing parameter h is chosen to minimize the "generalized cross-validation (GCV) function," developed for nonparametric regression by Craven and Wahba (1979).   The GCV function can be expressed by:

$$GCV(h) = \sum_{i=1}^{n} \left[\hat{u}_i - \hat{g}_h(x_i) \right]^2 \Big/ \left[n(1 - tr(H)/n)^2 \right], \qquad (10)$$

where $\hat{g}_h(\bullet)$ is a kernel estimator of the regression function $E\left[u_i \middle| x_i\right]$ and calculated by:

$$\hat{g}_h(x) = \sum_{j=1}^{n} K\left(\frac{x - x_j}{h}\right) \hat{u}_i \Big/ \sum_{j=1}^{n} K\left(\frac{x - x_j}{h}\right), \qquad (11)$$

and $tr(H)$ is the trace of the matrix $H = (h_{i,j})_{n \times n}$ with

$$h_{i,j} = K\left(\frac{x_i - x_j}{h}\right) \Big/ \sum_{l=1}^{n} K\left(\frac{x_i - x_l}{h}\right). \qquad (12)$$

It is known that minimizing GCV function will yield an asymptotically optimal bandwidth which is proportional to $n^{-1/5}$ (see Härdle, 1990). Therefore, the optimal bandwidth can be expressed by $h^* = cn^{-1/5}$ where c is constant.   To find the optimal c, we use the grid search from 0.01 to 40, i.e., $0.01 \leq c \leq 40.00$, with increment 0.01.   Also, the kernel function $K(\bullet)$ is chosen to be the Gaussian kernel such as equation (13).

$$K(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right). \qquad (13)$$

**Table 3   Conditional Symmetry Test Results by
Householder's Education Levels**

	Full Sample	Education Level 1	Education Level 2
Skewness of Error Term	–0.098	–0.067	0.055
Optimal c^*	38.14	40.00	21.28
Optimal Bandwidth ($h^* = c^* \times n^{-1/5}$)	9.267	10.996	6.037
T_n Statistics	1.809	0.108	2.689
P-values	**0.070**	**0.914**	0.007

Table 3 reports the test results of conditional symmetry for the Engel curve estimations according to householder's education levels.   Skewness of error term in first row is close to zero, which supports the symmetric distribution of the residual.   As a visual test, kernel density estimations of the symmetrically trimmed residuals are also presented in figure A2 of appendix A.   The figure tells us that distributions of the residuals look like normal distribution, implying symmetric distributions of the residuals by STLS estimation.   However, when we see the T_n statistics in table 3, the null hypothesis of conditional symmetry of the error term is not rejected only in full sample and education level 1 at 5% significant level.

5.2. Engel Curve Estimation

In this section, we assess empirical evidence on Engel curve for private education expenditure according to householder's education levels.   Before presenting the main empirical results, we need to discuss about the assumption that the Engel curves have different functional forms according to householder's education levels.   What are the main factors to affect household's expenditure on private education?   Two factors are mainly considered in this paper: One is related to consumption ability of households

such as income (or total expenditure) levels and the other is related to student's intellectual ability.   While the former determines the budget capacity to invest on private education, the latter provides the incentive to increase or decrease the consumption on private education service.   A well-known variable which represents student's intellectual ability would be mother's education levels.   Though the paper, in fact, analyzed both cases according to householder's and mother's education levels, any significant difference has not been found.[4]   This implies that the assumption of different functional forms according to householder's education levels is quite valid because of the high correlation between householder's education levels and mother's.

For model specification, we assume that the Engel curve functions are quadratic to log of total expenditure[5] and age of householder, denoted by $\ln(EXP)$ and $HAGE$, respectively.   In the model, we also include demographic information such as residential distinct ($SEOUL$ for residents of Seoul, and $GWANG$ for residents of metropolitan area), sex of householder ($HSEX = 1$ if male), status of employment ($UNEMP = 1$ if unemployed), and ownership of house ($HOUSE = 1$ if householders own their house).   More importantly, the number of children as an independent variable is classified into two groups, high school students or lower (denoted by $NUMSON1$) and college student or higher (denoted by $NUMSON2$), according to their education levels.   Here, education level 1 refers to householder's education of high school or lower, and education level 2 to

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[4] Indeed, the estimation of Engel curves according to mother's education levels caused a large loss of observation; nonetheless, the significance and the values of the estimated coefficients did not show us much variation.   Further, R^2 of the estimation based on mother's education, which represents the explanatory power of the model, was quite below that of the estimation based on householder education.

[5] In the paper, we use total expenditure as a proxy of total income.   In fact, it is more meaningful to use total income to analyze income elasticity or the expenditure pattern on private education according to household's income levels.   Nonetheless, total income may cause bias, which is not negligible, due to measurement error (see Liviatan, 1961).   Since the measurement error caused by total expenditure is smaller than that of total income, and total expenditure of households is close to the concept of "Permanent Income," we include total expenditure to our model.   Examples of those studies which employ total expenditure in their models are Hausman *et al*. (1995), Kedir and Girma (2003), and You (2003).

college or higher.

In order to make the STLS estimation effective, two prerequisite conditions should be satisfied: sample selection bias and symmetric distribution of the true error term. We employ the two-step Heckman model[6] to test whether or not sample selection bias exists. Since sample selection bias in two-step Heckman model appears in inverse mills ratio, we can test the existence of sample selection bias by the significance of the coefficients of inverse mills ratio in the estimation. Test results show that the coefficients of inverse mills ratio are 0.051, 0.050 and 0.032 with t-statistics of 2.35, 1.66 and 1.03 in full sample, education level 1 and education level 2, respectively.[7] Therefore, the null hypothesis that sample selection bias does not exist is rejected at 10% significance level in full sample and education level 1.[8]

The next prerequisite condition, conditional symmetry of the true error distribution, is a controversial issue. Even if Powell (1986) assumes the symmetrically distributed error as a weaker assumption than that of likelihood-based models, there is no way to check the conditional symmetry of the 'true' error distribution. Therefore, this paper provides the conditional symmetry test of the error term, proposed by Zheng (1998), by using the Tobit residual prior to the STLS estimation. Note that the residual, used in the prior test for conditional symmetry, are also symmetrically trimmed on the basis of the same criterion as STLS model. The T_n statistics[9] report that 1.76, 0.44, and 2.48 in full sample, education level 1,

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[6] For the application of two-step Heckman model, we apply the Probit model as the first step with full sample. For the second step, all the explanatory variables and inverse mills ratio as independent variables are used, using OLS with uncensored sample.

[7] This indicates that p-values are 0.019, 0.096, and 0.303 in full sample, education level 1 and education level 2, respectively.

[8] In the paper, sample selection bias largely depends on zero private education expenditure. Education level 1 has 636 zero values (approximately 40%) out of 1578 observations and education level 2 has 267 zero values (approximately 40%) out of 938. From the test results for sample selection bias, we also found some decreasing patterns as the number of zero private education expenditure decreases. We would expect that the main reason why sample selection bias does not exists in education level 2 lies in the insufficiency of zero value in the data set.

[9] The method for the choice of optimal bandwidth and the kernel function has been shown in

and education level 2, respectively.[10]   Hence, the null hypothesis of conditional symmetry is not rejected at 5% significance level in full sample and education level 1.   Furthermore, all the cases cannot reject the null at 1% significance level.   This result indicates that conditional symmetry of the error term holds as a prior test for STLS estimation.   Additionally, kernel density estimations of the residuals are also provided in figure A1 of appendix.   As seen, the figure shows, by and large, symmetric distributions of the estimated residuals.

Empirical results estimated by OLS, Tobit and STLS estimation methods are presented in table 4.   To compare the explanatory power of model specification based on householder's education levels, we also report the empirical results using education dummy variable (UNIV=1 if householder's education level is college or higher level) in the first, fourth, and seventh columns.

We can see that the coefficients on both log of total expenditure and the squared are highly significant in all the estimation models excluding education level 1 in OLS estimation, so it is predictable that the Engel curves for private education expenditure have an inverted-U shape from the positive coefficients of log total expenditure and the negative of the squared.   For residential distinct, most of the coefficients of *SEOUL* are significant at 10% significant level while those of *GWANG* are insignificant.   This result supports the hypothesis that Seoul inhabitants invest the highest amount on private education among any other cities in Korea.   In addition, the coefficients of *HSEX* and *HUNEMP* are not significant in any estimation methods, and those of *HOUSE* are significant only in education level 1 of OLS and STLS estimations, which implies that householders holding education level 1 have a higher priority on house than private education.

From the empirical evidence above, we found two interesting points.   First, the coefficients of UNIV are insignificant, which means that

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the previous section.
[10] This indicates that *p*-values are 0.078, 0.657, and 0.013 in full sample, education level 1 and education level 2, respectively.

educational difference of householders have no effect on the Engel curves by their intercepts.   However, the Chow test statistics are 7.423 in OLS, and 5.825 in STLS.[11]   This implies that there is a structural difference between education level 1 and education level 2 of householders.   The result supports that assuming different functional forms according to householder's education levels are more reliable to capture the effects of householder's education.

Second, the table shows the significant coefficients with positive sign in *NUMSON1* and negative sign in *NUMSON2*.   This means that an increase in the number of children of high school students or lower causes an increase in the share of expenditure on private education, whereas an increase in the number of children of college or higher induces a decrease in the share of the expenditure.[12]   Thus, private education expenditure for college students or higher is quite sensitive; however, the expenditure for high school students or lower is not.   We conclude that the result reflects such social phenomenon that Korea education has focused on university entrance exam.

Following the study of Newey *et al*. (1990), in which they compare the difference of various semiparametric estimations for selection models, we also employ Hausman test to check whether or not there is difference between OLS and STLS estimation.   We formulate the test statistics as follows:

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[11] The critical value at 1% significance level is 2.18 in both cases, i.e., $\Pr(F > 2.18) = 0.01$. This means that the null hypothesis that there is no structural difference between two model specifications is rejected at 1% significance level.

[12] The reasons for negative sign in *NUMSON2* result mainly from the following two effects. First, this is because the consumption behavior of households on private education is more flexible with respect to an increase of the number of children of college or higher than an increase of the number of children of high school or lower.   The second reason is attributed to the characteristics of the variable for private education expenditure used in the paper.   Since the KLIPS data set we used does not include college tuition fee in the private education expenditure, the substitution effect between the private education expenditure and college tuition fee would occur.   In other word, the coefficients of *NUMSON2* are negative since the large amount of college tuition fee give more burden to each household than the expenditure on private education as the number of children of college or higher increases.

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Table 4   Engel Curve Estimation according to Householder's Education Levels

Variables	OLS			Tobit			STLS		
	Full Sample	Education Level 1	Education Level 2	Full Sample	Education Level 1	Education Level 2	Full Sample	Education Level 1	Education Level 2
$\ln(EXP)$	0.272*** (0.081)	0.143 (0.097)	0.570*** (0.180)	0.837*** (0.140)	0.624*** (0.203)	1.039*** (0.234)	1.218*** (0.242)	1.348*** (0.297)	1.114*** (0.402)
$[\ln(EXP)]^2$	−0.013** (0.005)	−0.005 (0.007)	−0.032*** (0.011)	−0.046*** (0.009)	−0.032** (0.013)	−0.060*** (0.015)	−0.071*** (0.015)	−0.079*** (0.019)	−0.065*** (0.025)
$HAGE$	0.005*** (0.001)	−0.001 (0.001)	0.013*** (0.002)	0.006*** (0.002)	−0.002 (0.003)	0.025*** (0.005)	0.014*** (0.003)	0.002 (0.003)	0.029* (0.016)
$HAGE^2$	-4.8×10^{-5}*** (1.2×10^{-5})	-1.8×10^{-6} (1.4×10^{-5})	-1.2×10^{-4}*** (2.3×10^{-5})	-7.1×10^{-5}*** (2.1×10^{-5})	-4.8×10^{-6} (2.5×10^{-5})	-2.5×10^{-4}*** (6.3×10^{-5})	-1.5×10^{-4}*** (2.9×10^{-5})	-3.2×10^{-5} (3.3×10^{-5})	-3.2×10^{-4}* (1.9×10^{-4})
$SEOUL$	0.015*** (0.005)	0.011* (0.006)	0.027*** (0.008)	0.016** (0.007)	0.011 (0.010)	0.028*** (0.010)	0.024*** (0.008)	0.026** (0.010)	0.024** (0.012)
$GWANG$	0.001 (0.004)	0.003 (0.005)	−0.003 (0.007)	0.002 (0.006)	0.006 (0.008)	−0.005 (0.010)	0.001 (0.007)	0.010 (0.009)	−0.010 (0.010)
$HSEX$	0.000 (0.007)	0.006 (0.008)	−0.016 (0.015)	0.003 (0.010)	0.012 (0.013)	−0.025 (0.017)	0.018 (0.015)	0.027 (0.019)	−0.018 (0.022)

NUMSON1	0.048*** (0.003)	0.038*** (0.004)	0.068*** (0.005)	0.071*** (0.004)	0.060*** (0.006)	0.086*** (0.007)	0.052*** (0.006)	0.041*** (0.008)	0.069*** (0.008)
NUMSON2	–0.032*** (0.005)	–0.029*** (0.006)	–0.038*** (0.008)	–0.059*** (0.008)	–0.057*** (0.010)	–0.061*** (0.013)	–0.080*** (0.012)	–0.092*** (0.015)	–0.055*** (0.018)
HUNEMP	0.006 (0.007)	0.011 (0.008)	–0.01 (0.013)	0.007 (0.010)	0.016 (0.012)	–0.021 (0.017)	0.006 (0.016)	0.007 (0.020)	–0.018 (0.028)
HOUSE	0.006 (0.004)	0.013** (0.005)	–0.004 (0.006)	0.002 (0.006)	0.011 (0.008)	–0.008 (0.009)	0.008 (0.006)	0.023*** (0.008)	–0.003 (0.009)
UNIV	0.003 (0.004)	- -	- -	–0.002 (0.006)	- -	- -	0.007 (0.006)	- -	- -
CONS	–1.384*** (0.313)	–0.739* (0.365)	–2.751*** (0.709)	–3.873*** (0.551)	–2.841*** (0.782)	–5.017*** (0.925)	–5.478*** (0.971)	–5.677*** (1.184)	–5.348*** (1.631)
Obs.	2,516 [903]	1,578 [636]	938 [267]	2,516 [903]	1,578 [636]	938 [267]	1,181	637	544
R^2 or χ^2	0.338	0.302	0.410	1216.7	689.5	550.0	0.334	0.316	0.370

Notes: 1) Values in parentheses present standard deviation of the estimated coefficients.
2) *, **, and *** indicate significance at 10%, 5%, and 1% level, respectively.
3) Values in square brackets indicate the number of observation with zero private education expenditure.
4) The observations and R^2, presented in STLS estimation, are calculated with the symmetrically trimmed data.

$$H_n = (\beta_{STLS} - \beta_{OLS})'\big[Cov(\beta_{STLS}) - Cov(\beta_{OLS})\big](\beta_{STLS} - \beta_{OLS}) \sim \chi^2(k), \quad (14)$$

where k is the number of independent variable of each model specification.

Formal test statistics for the null hypothesis of no difference in the probability limits of the estimators are tabulated in table 5. As seen, the null hypothesis is rejected in full sample and education level 1 at any significance level; the test statistics in education level2, however, does not reject the null.[13]

Figure 4 plots the Engel curves according to householder's education levels by STLS estimation. The left side of figure 4 plots the Engel curve with respect to log of total expenditure on the horizon axis, and the right side

Table 5　Hausman Test Results

	Full Sample	Education Level 1	Education Level 2
Test Statistics	89.467	64.995	17.610
p-value	0.000	0.000	0.128

Figure 4　Engel Curves according to Householder's Education Levels

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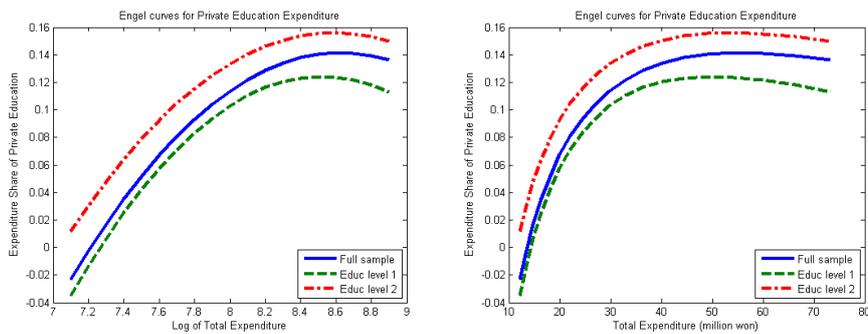
[13] As a matter of fact, STLS estimation is also based on Least Squares estimation. Therefore, the reason why Hausman test statistics is not rejected in education level 2 may arise from the deficiency of zero private education expenditure observations.

with respect to total expenditure.   Here, (log of) total expenditure, ranging from 5 percent value to 95 percent value, is also used.

Those figures tell us that Engel curves for private education expenditure have the inverted-U shape even though they show the increasing pattern by relatively high threshold value of total expenditure.   On the basis of the right side figure, the turning points of the Engel curves appear at some point ranging from 40 million won to 50 million won of household's expenditure.   One more thing we can specify from those figures is that the share of expenditure on private education has highest values in education level 2 and lowest in education level 1 at any total expenditure levels.   This result indicated that householder with education level 2 put relatively a higher priority on private education of their expenditure than those with education level 1.

6. INCOME ELASTICITY ESTIMATION

While Engel curve allows us to analyze the relationship between the share of expenditure on some commodities and total expenditure, it cannot directly tell us how much the expenditure is changed as total expenditure varies.   In this case, estimation of income elasticity can be a useful tool to explore the expenditure pattern on private education more dynamically.   Because we use total expenditure of households as a proxy of total income, the estimated elasticity would be 'expenditure' elasticity instead of 'income' elasticity for sure.   Nonetheless, if we consider the fact that total expenditure is more close to the concept of "Permanent Income" than total income, it may be excused to call the elasticity as 'income' elasticity.   In this paper, we calculate income elasticity for private education expenditure by taking derivative of the estimated Engel curves. Now, consider the following equation (15).

$$\frac{y_i}{x_i} = \beta_1 \ln x_i + \beta_2 \left[\ln x_i \right]^2 + C, \tag{15}$$

where y_i is private education expenditure, x_i is total expenditure, and C is a constant, representative of the effects of all other independent variables excluding log of total expenditure and the squared.   To make other variables constant, we calculate the values C by multiplying the means of other independent variables by the corresponding estimated coefficients.   Then we get equation (16) by multiplying x_i in both sides of equation (15).

$$y_i = \beta_1 x_i \cdot \ln x_i + \beta_2 x_i \cdot \left[\ln x_i\right]^2 + C \cdot x_i. \tag{16}$$

By taking derivative of equation (16), we can easily derive the following equation (17) to estimate income elasticity for private education expenditure.[14]

$$\varepsilon = \frac{\partial y_i}{\partial x_i}\frac{x_i}{y_i} = \left(\frac{y_i}{x_i} + \beta_1 + 2\beta_2 \ln x_i\right) \cdot \frac{x_i}{y_i} = 1 + (\beta_1 + 2\beta_2 \ln x_i) \cdot \frac{x_i}{y_i}$$

$$= 1 + (\beta_1 + 2\beta_2 \ln x)\left(\frac{1}{\beta_1 \ln x + \beta_2 \left[\ln x\right]^2 + C}\right). \tag{17}$$

From the STLS estimation results, the income elasticity according to householder's education levels is presented in table 6 and 7.   Since the equation (17) is for calculating the point elasticity, we can obtain income elasticity for private education expenditure according to various expenditure levels.   Here, $\bar{X}_{0\sim p20}$, for example, indicates the mean value of total expenditure ranging zero percent value to 20 percent value.   The difference between table 6 and 7 depends on which sample is used to get the various total expenditure levels.   Moreover, confidence intervals are presented in the parentheses, using the bootstrapping method.

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[14] Note that the estimated income elasticity can be applicable only if total expenditure is used to estimate Engel curve.

Table 6   Income Elasticity Estimation Based on Each Sample

	Full Sample	Education Level 1	Education Level 2
$\bar{X}_{0\sim p20}$	−8.029 (−10.316, −5.741)	−3.236 (−4.022, −2.450)	4.568 (2.652, 6.484)
$\bar{X}_{p20\sim p40}$	3.708 (3.513, 3.903)	5.411 (4.352, 6.471)	2.280 (2.117, 2.443)
$\bar{X}_{p40\sim p60}$	2.371 (2.317, 2.425)	2.963 (2.847, 3.078)	1.795 (1.728, 1.863)
$\bar{X}_{p60\sim p80}$	1.778 (1.742 , 1.814)	2.119 (2.004, 2.235)	1.409 (1.356, 1.461)
$\bar{X}_{p80\sim p100}$	1.218 (1.193, 1.244)	1.331 (1.227, 1.434)	1.011 (0.974, 1.047)
$\bar{X}$	2.343	2.948	1.673
Median	2.306	2.660	1.682

Note: Values in parentheses indicate 95% confidence intervals obtained from bootstrapped standard errors. 1000 different samples are randomly generated to calculate the standard errors.

Table 7   Income Elasticity Estimation Based on Full Sample

	Education Level 1	Education Level 2
$\bar{X}_{0\sim p20}$	−3.241 (−6.296, −4.186)	18.439 (−701.933, 738.811)
$\bar{X}_{p20\sim p40}$	4.305 (4.016, 4.594)	2.644 (2.561, 2.727)
$\bar{X}_{p40\sim p60}$	2.513 (2.446, 2.580)	1.961 (1.928, 1.993)
$\bar{X}_{p60\sim p80}$	1.799 (1.756, 1.842)	1.574 (1.549, 1.600)
$\bar{X}_{p80\sim p100}$	1.136 (1.105, 1.167)	1.149 (1.129, 1.170)
$\bar{X}$	2.479	1.944
Median	2.434	1.922

Note: Values in parentheses indicate 95% confidence intervals obtained from bootstrapped standard errors. 1000 different samples are randomly generated to calculate the standard errors.

The first thing we found in table 6 is that the income elasticity of education level 1 is largest in all the total expenditure levels except those below 20 percent value, the next is full sample, and the lowest is education level 2. On the other hand, we found somewhat different results in table 7. When we compare the elasticity based on the same total expenditure levels in table 7, the elasticity of education level 2 is slightly larger in the highest level of total expenditure than that of education level 1. Nonetheless, we can conclude that the elasticity of education level 1 is larger than other cases, i.e., as householder's education levels are relatively low, their households are generous to increase the expenditure on private education. We define the phenomenon as the "compensation psychology effect,"[15] householders with relatively low education level want to be compensated by giving their children more chance to receive education service. Considering the confidence intervals, we also found that nearly all the confidence intervals of income elasticity are not superposed and distances of the intervals are quite compact. This implies that the values of income elasticity calculated in table 6 and 7 are robust estimates and meaningful.

The tables also show that as household's income (or total expenditure) increase, the elasticity decreases.   But most of the elasticities are larger than one even in the highest income level, indicating that private education is 'normal goods.'   Therefore, we can interpret that the definite amount of household's investment on private education always tends to increase at any income levels.   This implies that the gap between the rich and the poor would become larger due to educational descending from generation to generation.

7. SUMMARY AND CONCLUDING REMARKS

Using the KLIPS 9th Waves, the paper estimates Engel curve for private

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[15] Even though the term, "compensation psychology effect," is used in the paper, note that this is not a formal definition of the term, proposed by the study area of Psychology.

education expenditure, assuming different functional forms according to householder's education levels. In doing so, we employ semiparametric estimation method which does not assume prespecified error distribution, and is robust to heteroscedasticity.   In order to make the estimation effective, we also present test results for conditional symmetry of error distribution.   In addition, income elasticity for private education expenditure is calculated from the estimated Engel curve.

Our basic finding is that Engel curve for private education expenditure has an inverted-U shape.   From the Engel curves, we found that householders with college or higher education level put a higher priority on private education of their expenditure than those with high school or lower education level.   In contrast, income elasticity of householder with high school or lower education level was larger than that of householder with college or higher education level in most cases, as we defined the phenomenon as the "compensation psychology effect" in this paper.   These results indicate that householders with relatively low education level do not increase more of investment on private education than those with relatively high education level until their income level approach to some level, but more investment on private education is started after the level is attained.   Another finding on income elasticity tells us that private education is 'normal goods,' indicating that households invest more and more at any income levels as their income increases.   In addition, the paper point out such social phenomenon that Korean education has focused on university entrance exam by analyzing the effect of the number of children classified into two groups.

APPENDIX

Figure A1 Kernel Density Function of the Residuals by Tobit Estimation

Kernel distribution of the residuals by Tobit estimation
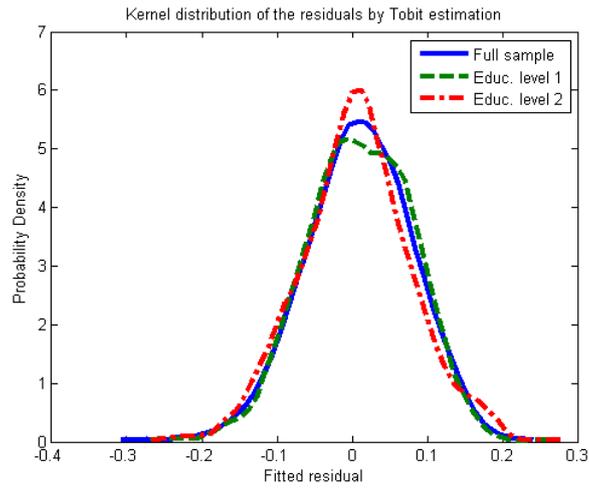
Figure A2 Kernel Density Function of the Residuals by STLS Estimation

Kernel distribution of the residuals by STLS estimation
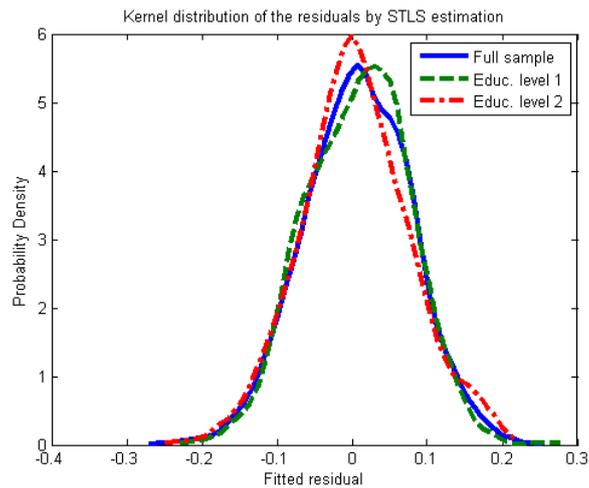
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