Testing the Uncorrelatedness of Aggregate Supply and Aggregate Demand Shocks in VAR Models[*]

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In structural vector autoregression models, the underlying shocks are assumed to be uncorrelated with one another. We examine the empirical relevance of this uncorrelatedness assumption in a well-known model by Blanchard and Quah (1989). To derive a testable form, the Blanchard and Quah model is transformed into a cointegration representation. This alternative setup is extended to allow for testing of the uncorrelatedness between aggregate supply and aggregate demand shocks. Empirical evidence reveals that the two structural shocks are not correlated in any of the six G-7 countries under study. However, when a different identification scheme is adopted, they are correlated in some cases.

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1. INTRODUCTION

In the structural vector autoregression (VAR) literature, it is common to assume that underlying shocks are mutually uncorrelated. The rationale for this is that they are primitive, exogenous forces and hence, lack a shared origin. The uncorrelatedness assumption also serves to reduce the number of identifying restrictions required. Its absence necessitates $n(n-1)/2$ additional restrictions for exact identification in an n-variable model. Examples include the studies that identify the effects of aggregate supply (AS) and aggregate demand (AD) shocks in a bivariate model. In particular, the Blanchard and Quah (BQ, 1989) model of real output and the unemployment rate is most well-known and numerous applications and extensions have followed subsequently.[1] They employ the identifying assumption that AS and AD shocks are mutually uncorrelated.

The uncorrelatedness assumption is not without criticism, however. Cover *et al*. (2006) suggest economic reasons as to why the AS and AD shocks, identified through the BQ procedure, can be correlated with each other. Pagan (2003) argues that when a VAR model is of small dimension, a factor common to the explanation of more than one variable may be omitted. In this case, the factor would be included in the error terms, rendering the uncorrelatedness assumption invalid. Shiller (1986) and Pesaran and Smith (1998) find that assuming uncorrelatedness between shocks may not be reasonable for many econometric applications of the structural VAR methodology. Cooley and Dwyer (1998) and Giordani (2004) provide examples in which this assumption produces specification errors and biased impulse response estimators.

We examine the extent to which the uncorrelatedness assumption of the BQ model is consistent with actual data. This issue is important because the model implications are dependent on the adequacy of the assumptions in use. Where the assumptions are inconsistent with data, their imposition

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[1] To name a few, Bayoumi and Eichengreen (1994), Quah and Vahey (1995), Keating and Nye (1999), Cecchetti and Rich (2001), and Cover *et al*. (2006).

may result in misrepresentation of the true dynamic structure of the model. Nevertheless, the uncorrelatedness assumption of the BQ model has rarely been evaluated for empirical relevance. The main reason is that it is an exact-identifying restriction and hence, cannot be tested. Cover *et al*. (2006) offer one resolution in applying the BQ procedure to a U.S. model of real output and inflation. Upon abandoning the uncorrelatedness assumption, they introduce an alternative identifying restriction, which assumes that the AD curve has a slope of –1.[2] The correlation between AS and AD shocks is estimated at 0.72. While their approach is plausible, the results would also be conditional on the assumed slope of the AD curve.

In this paper, we propose a different and less restricted method for testing the empirical relevance of the uncorrelatedness assumption. The starting point is to present the BQ model in a cointegration framework. This is possible because the model assumes that real output is difference-stationary and that unemployment rates are stationary in levels. One implication is that the two variables are cointegrated where the unemployment rate itself is the cointegration relation. The corresponding vector error correction model is structurally identified using a procedure that decomposes the shocks into those with permanent effects and those with transitory effects. The key feature is that this cointegration representation can be extended to relax the uncorrelatedness of structural shocks without entailing an additional identifying restriction. The AS and AD shocks are allowed to correlate with each other and their correlation is determined freely by data. Thus, a comparison of the results with those from the BQ model provides a test for evaluating the empirical relevancy of the uncorrelatedness assumption with respect to the data.

The remainder of this paper is organized as follows. Section 2 presents a cointegration representation of the BQ model with an extension to allow for correlation between AS and AD shocks. Empirical applications to the six G-7 countries are provided in section 3. Section 4 offers an alternative

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[2] This alternative assumption restricts the contemporaneous value of real output change in the inflation equation to have a coefficient of –1.

identification scheme, with its empirical results reported in section 5. Section 6 concludes the paper. Appendix briefly explains the causal relationships embodied in the BQ model, as well as in alternatively identified models with reference to sections 4 and 5.

2. TESTING PROCEDURE

Blanchard and Quah (BQ) developed a VAR model that identifies the effects of aggregate supply (AS) and aggregate demand (AD) shocks on real output and the unemployment rate. Two assumptions are employed for the identification of structural shocks. One is that an AD shock has no long-run effects on real output grounded on the long-run output neutrality condition. The other is that AS and AD shocks are mutually uncorrelated. The BQ model can be cast in a cointegated VAR framework. This is possible because the model assumes that real output is difference-stationary and that unemployment rates are stationary in levels. One implication is that the two variables are cointegrated where the unemployment rate itself is the cointegration relation, i.e. $\beta = [0,\ 1]'$. The error correction term is given as:

$$\beta' z_t = u_t,$$

where z_t is the vector $(\Delta y_t,\ \Delta u_t)'$, y_t is real output, u_t is the unemployment rate, and Δ is the first difference operator.

A vector error correction (VEC) representation may be written as:

$$\Delta y_t = \sum_{i=1}^{p} g_{yy,\,i} \Delta y_{t-i} + \sum_{i=1}^{p-1} g_{yu,\,i} \Delta u_{t-i} + \alpha_1 u_{t-p} + e_{1t}, \tag{1}$$

$$\Delta u_t = \sum_{i=1}^{p} g_{uy,\,i} \Delta y_{t-i} + \sum_{i=1}^{p-1} g_{uu,\,i} \Delta u_{t-i} + \alpha_2 u_{t-p} + e_{2t}, \tag{2}$$

where $\alpha = [\alpha_1,\ \alpha_2]'$ is a vector of error correction coefficients, $e_t = (e_{1t},\ e_{2t})'$ is a vector of reduced-form shocks and is i.i.d with a mean of zero and a covariance matrix of $E(e_t e_t') = \Omega$. [3] The Granger Representation Theorem shows that the VEC model in (1) and (2) can be inverted to obtain the vector moving average (VMA) representation (for details, see Engle and Granger, 1987):

$$z_t = D(L)e_t, \tag{3}$$

where $D(L) = D_0 + D_1 L + D_2 L^2 + \cdots,\ \ D_0 = I,$ and

$$D(1) = \sum_{i=0}^{\infty} D_i L^i = \beta_\perp \psi \alpha_\perp', \tag{4}$$

where $\alpha_\perp = [\alpha_{1\perp},\ \alpha_{2\perp}]'$ and $\beta_\perp = [\beta_{1\perp},\ \beta_{2\perp}]'$ are vectors orthogonal to α and β, respectively (i.e., $\alpha'\alpha_\perp = 0$ and $\beta'\beta_\perp = 0$), $\psi = (\alpha_\perp' G(1)\beta_\perp)^{-1}$, and G(1) is the short-run impact matrix of reduced-form shocks from (1) and (2), given by:

$$G(1) = \begin{bmatrix} 1 - \sum_{i=1}^{p} g_{yy,\,i} & -\sum_{i=1}^{p-1} g_{yu,\,i} \\ -\sum_{i=1}^{p} g_{uy,\,i} & 1 - \sum_{i=1}^{p-1} g_{uu,\,i} \end{bmatrix}.$$

A detailed derivation is found in Johansen (1991).

Subject to identification, a structural VMA representation corresponding to (3) is given as:

$$z_t = \Gamma(L)\varepsilon_t, \tag{5}$$

where $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + \cdots,\ \ \Gamma(1) = \sum_{i=0}^{\infty} \Gamma_i L^i$, and $\varepsilon_t = (\varepsilon_{1t},\ \varepsilon_{2t})'$ is a vector of structural shocks. Following BQ, ε_{1t} and ε_{2t} are denoted as an aggregate supply (AS) shock and an aggregate demand (AD) shock,

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[3] The constant term is suppressed for the sake of illustration.

respectively.   These are assumed to have a mean of zero and a covariance
matrix of the form:

$$E(\varepsilon_t \varepsilon_t') = \Sigma = \begin{pmatrix} 1 & \sigma_{12} \\ \sigma_{12} & 1 \end{pmatrix}, \tag{6}$$

where each structural shock is normalized to have unit variance without the
loss of generality.   The BQ model restricts the correlation parameter σ_{12}
to being zero by the uncorrelatedness assumption between AS and AD shocks.
From (3) and (5), the relationships between the reduced-form and structural
parameters are:

$$\Gamma(L) = D(L)\Gamma_0 \tag{7}$$

and

$$\varepsilon_t = \Gamma_0^{-1} e_t. \tag{8}$$

   The presence of one cointegrating relationship in the model implies that
while one shock exhibits permanent effects, the other shock demonstrates
only transitory effects.[4]   The first (ε_{1t}) is designated as an AS shock and
the second (ε_{2t}) as an AD shock.   This interpretation is plausible in that the
AS shock has permanent effects on real output while the AD shock has only
transitory effects.   To decompose the shocks into those with permanent
effects and those with transitory effects, we employ a type of identification
procedure proposed for VEC models by Gonzalo and Ng (2001) and Lettau
and Ludvigson (2004).   The model posits for the relationship between the
reduced-form and structural shocks of (8) that:

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[4] See King *et al*. (1991), Mellander *et al*. (1992), and Levtchenkova *et al*. (1996) for the
implications of cointegration in the structural identification of VEC models.

$$\Gamma_0^{-1} = \begin{bmatrix} \alpha'_\perp \\ \beta'\Omega^{-1} \end{bmatrix}. \tag{9}$$

Equation (9) is modified for each structural shock to have unit variance, which is given as:

$$\Gamma_0^{-1} = \begin{bmatrix} \Lambda_1 \alpha'_\perp \\ \Phi_1^{-1}\beta'\Omega^{-1} \end{bmatrix}, \tag{10}$$

where $\Lambda_1^{-1}\Lambda_1^{-T} = \alpha'_\perp\Omega\alpha_\perp$ and $\Phi_1\Phi'_1 = \beta'\Omega^{-1}\beta$. This model is denotes as BQ-CORR hereafter.

Combining (8) and (10) reveals that $\Lambda_1\alpha'_\perp e_t$ and $\Phi_1^{-1}\beta'\Omega^{-1}e_t$ are permanent AS (ε_{1t}) and transitory AD (ε_{2t}) shocks, respectively. The covariance matrix of the structural shocks is

$$E(\varepsilon_t\varepsilon'_t) = \Gamma_0^{-1}\Omega\Gamma_0^{-T} = \begin{bmatrix} 1 & \Lambda_1\alpha'_\perp\beta\Phi_1^{-T} \\ \Lambda_1\alpha'_\perp\beta\Phi_1^{-T} & 1 \end{bmatrix}. \tag{11}$$

The inverse matrix of Γ_0^{-1} is calculated as:

$$\Gamma_0 = [\Omega\beta_\perp (\Lambda_1\alpha'_\perp\Omega \beta_\perp)^1 \qquad \alpha\Phi_1^{-1}('\beta \alpha)^{-1} \quad \alpha^{-1} \tag{12}$$

The application of (4), (7), and (12) yields the long-run impact matrix of the structural shocks as:

$$\Gamma(1) = D(1)\Gamma_0 = [\beta_\perp\psi\Lambda_1^{-1} \quad 0 \ ]. \tag{13}$$

As the implied cointegrating vector of $\beta = [0, 1]'$ gives $\beta_\perp = [1, 0]'$, $\Gamma(1)$ of dimension (2x2) becomes:

$$\Gamma(1) = \begin{bmatrix} \psi\Lambda_1^{-1} & 0 \\ 0 & 0 \end{bmatrix}. \tag{14}$$

Equation (14) confirms that the AS shock has permanent effects on real output, while the AD shock has only transitory effects. Both shocks have transitory effects on the unemployment rate, which is consistent with the assumption that the rate is stationary in levels. This representation matches the BQ model. The key difference is that the BQ-CORR model in (11) allows the AS and AD shocks to correlate with each other. The correlation between two structural shocks is not restricted to being zero but is determined freely by the data. This provides one way of testing the empirical relevance of the uncorrelatedness assumption underlying the BQ model. If results from the BQ model are different from those from the BQ-CORR model, this can be taken as evidence that the uncorrelatedness assumption between the two shocks is inconsistent with the actual data. If the results do not differ, the uncorrelatedness assumption is empirically supported, justifying its use in the BQ model. Further explanatory remarks will be found in the section to follow.

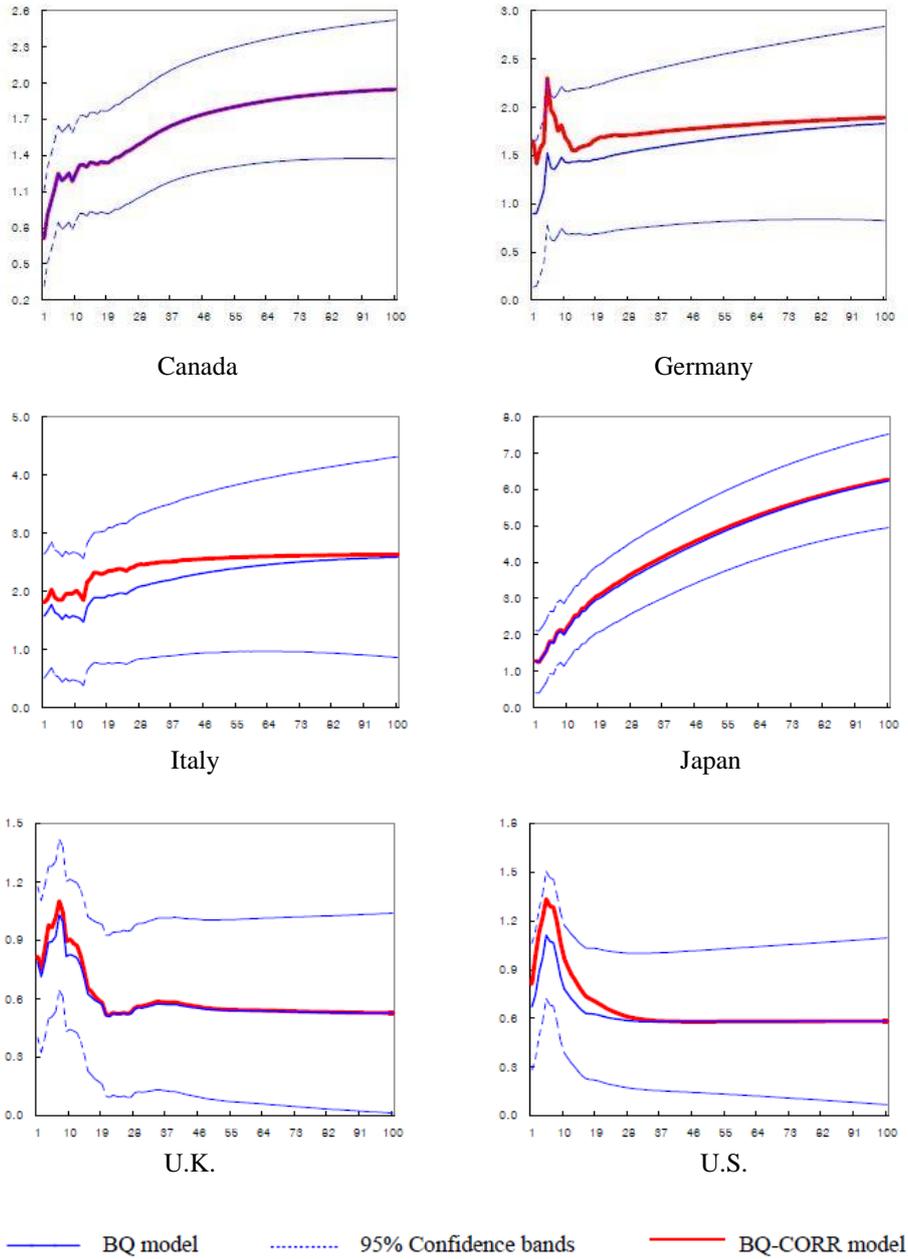## 3. TESTING THE RESULTS OF THE NCORRELATEDNESS BETWEEN AS AND AS SHOCKS

The analysis outlined above is applied to quarterly observations of real GDP (y_t) and unemployment rates (u_t) in six G-7 countries. Due to data availability, France is excluded. Data were obtained from the IMF *International Financial Statistics*. The sample period is 1960:Q1 to 2006:Q4. The real GDP series was transformed using natural logarithms and differenced once to achieve stationarity. For estimation of the VEC models in (1) and (2), the lag length p was chosen on the basis of the Sims likelihood ratio test. Namely, $p=9$ for Canada, $p=8$ for Germany, $p=12$ for Italy, and $p=11$ for Japan and the U.K. With respect to the U.S., $p=8$ is used

to match the lag structure in the original BQ model.   All lag lengths are also consistent with the results of Breusch-Godfrey Lagrange Multiplier tests, which indicate the absence of serial correlation in both real output growth and unemployment rate equations at the 5% significance level.
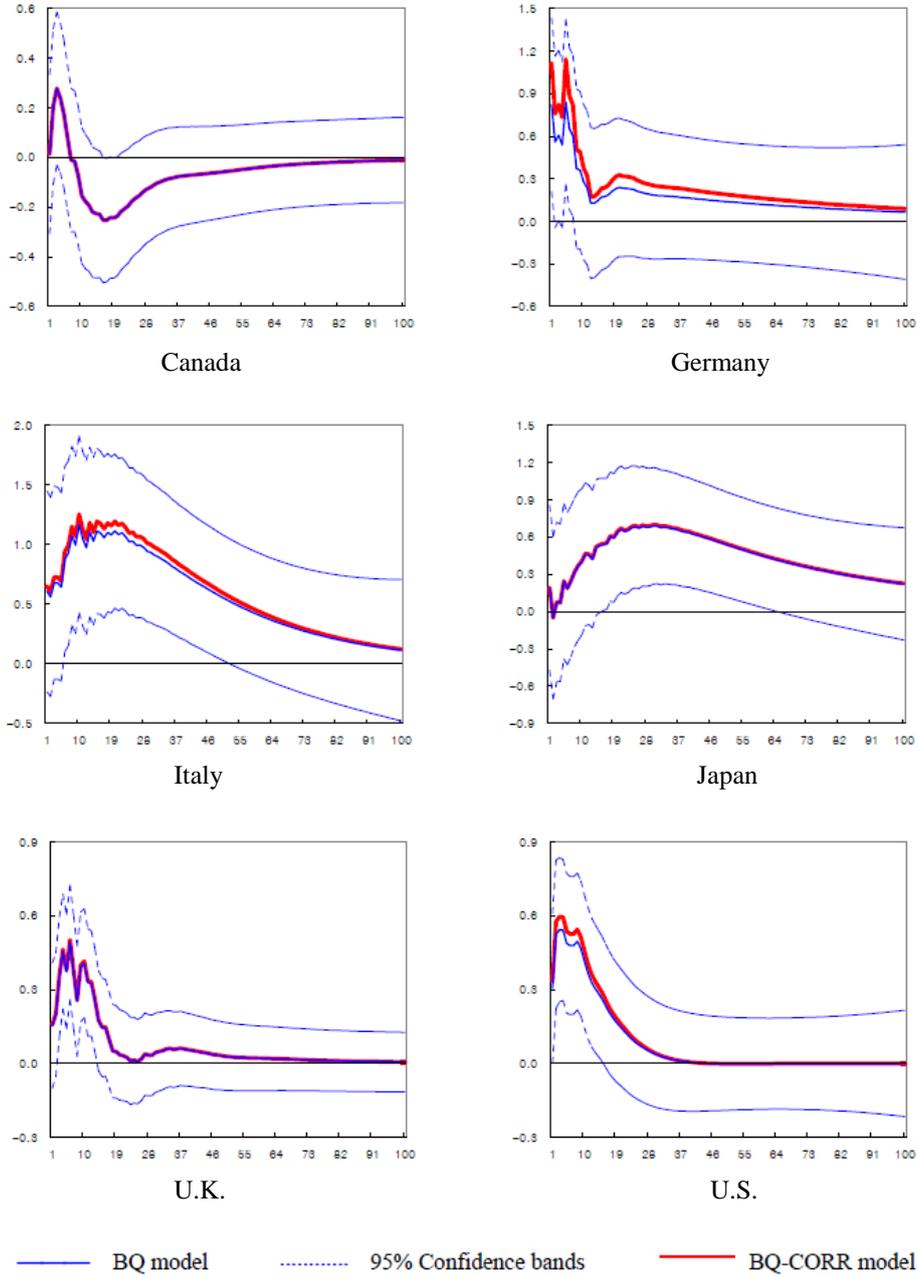
The estimated VEC models are expanded to models in the levels of the series.   They are then inverted numerically to generate estimates of the reduced-form shocks.   After these estimates have been obtained, the structural shocks are identified and their impacts on the series are calculated utilizing the procedures outlined in section 2.   Figure 1 depicts the responses of the levels of the series to a one-standard-deviation shock in each structural disturbance.   The corresponding figures from the BQ model are reported together for comparison.   Both the BQ and BQ-CORR models produce responses that are consistent with standard economic theory.   A favorable AS shock causes real output to increase across the horizons.   This shock lowers the unemployment rate at short horizons.   In response to a favorable AD shock, real output increases and the unemployment rate declines.   Real output eventually returns to its pre-shock levels as a consequence of the long-run output neutrality assumption.   For the unemployment rate, the long-run responses to AS and AD shocks are all zero, reflecting the assumption that it is a stationary process.

The issue is how significantly the responses in the BQ model differ from those of the BQ-CORR model.   To evaluate this, depicted together in Figure 1 are 95% confidence bands generated using 500 bootstrap replications of the BQ model.   It appears that the responses from the BQ-CORR models reside within the 95% confidence bands of the responses in the BQ models. Evidence is particularly strong with respect to Canada and Japan where both the BQ and BQ-CORR models produce impulses of almost identical shape and magnitude.   The other countries also indicate that the responses from the BQ-CORR model are not statistically distinguishable from those of the BQ model.   There are a few exceptions in Germany and Italy, but the effects are marginal.   Thus, through analysis of the data from all six G-7 countries, it can be said that the uncorrelatedness assumption of the BQ model is acceptable.

Figure 1A   Response of Real Output to an Aggregate Supply Shock

Canada

Germany

Italy

Japan

U.K.

U.S.

——— BQ model   ·········· 95% Confidence bands   ——— BQ-CORR model

Figure 1B   Response of Real Output to an Aggregate Demand Shock

Canada

Germany

Italy

Japan

U.K.

U.S.

——— BQ model   ·········· 95% Confidence bands   ——— BQ-CORR model

**Figure 1C   Response of the Unemployment Rate
to an Aggregate Supply Shock**

Canada

Germany

Italy

Japan

U.K.

U.S.

———— BQ model   ·········· 95% Confidence bands   ———— BQ-CORR model
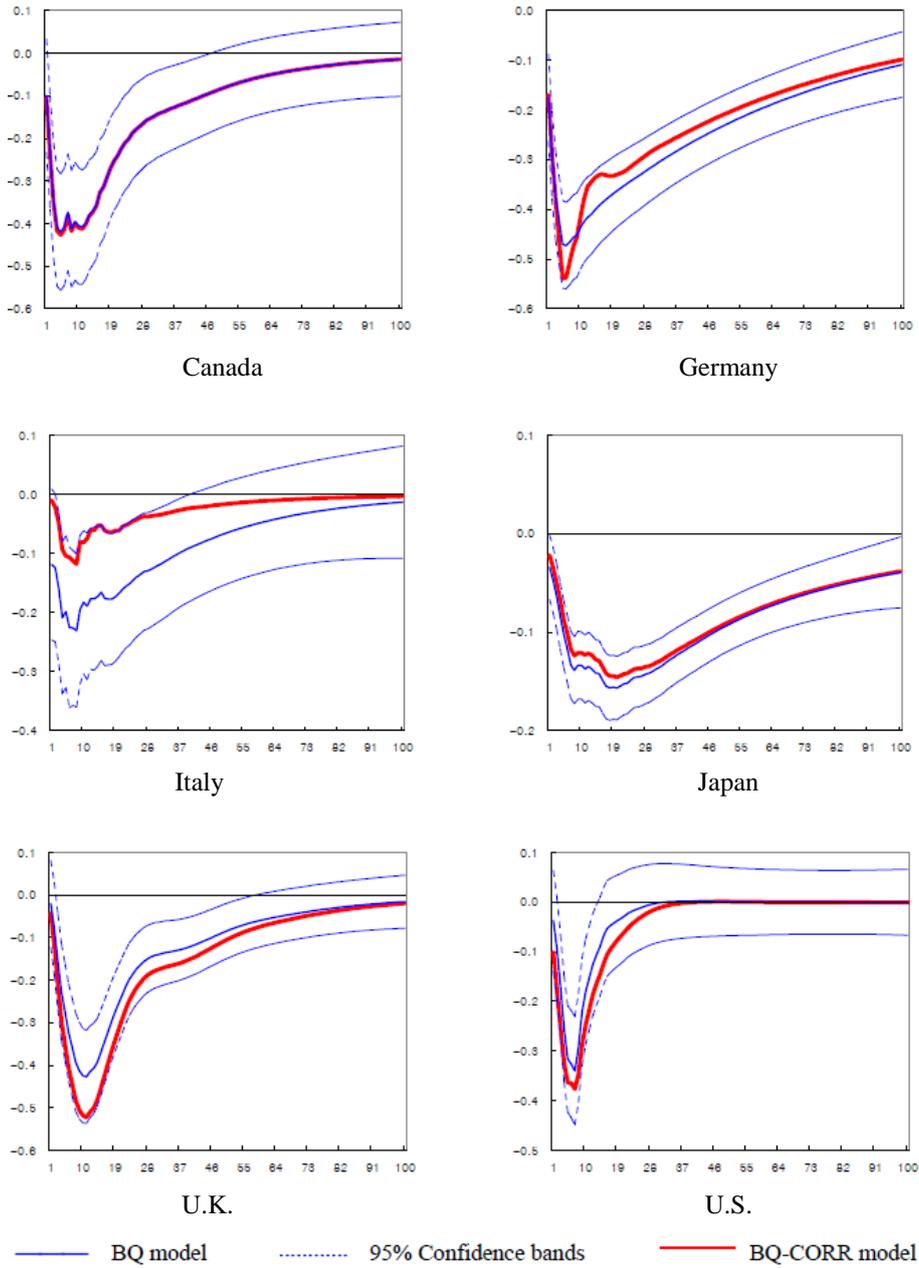
**Figure 1D   Response of the Unemployment Rate
to an Aggregate Demand Shock**

Canada

Germany

Italy

Japan

U.K.

U.S.

——— BQ model   ·········· 95% Confidence bands   ——— BQ-CORR model
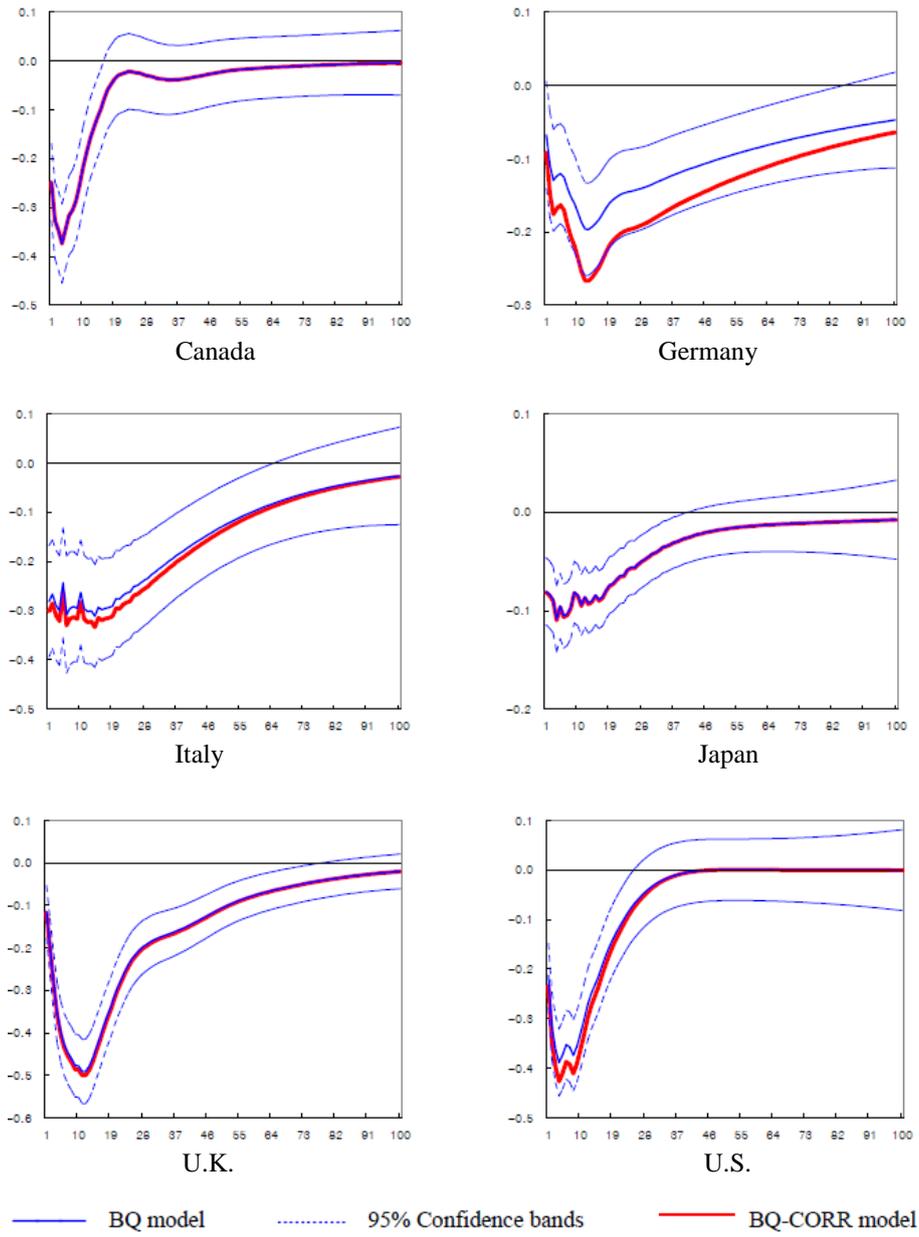
We check the correlation coefficients of AS and AD shocks that are estimated from the BQ-CORR models.   These estimates are low, ranging from 0.02 to 0.28: 0.02 (Canada), 0.28 (Germany), 0.27 (Italy), 0.11 (Japan), 0.19 (U.K.), and 0.15 (U.S.).   This is in line with the finding of no statistical disparity between the BQ and BQ-CORR models.

4. AN ALTERNATIVE IDENTIFICATION SCHEME

   Testing results are conditional on the model specification in use. Different model specifications can lead to different implications on the uncorrelatedness of structural shocks.   To explore this possibility, we consider an alternative identification scheme.   One interesting case is due to a recent study by Cover *et al*. (2006).   As they demonstrated, an implicit assumption in the BQ model is that the AS shock is causally prior to the AD shock.   This causal ordering is consistent with some economic theories. For example, a rise in oil prices and real import prices are reflected as AS shocks and are fed into consumer price inflation.   The monetary authority may react to the inflationary consequences of these shocks via the policy rate. Another example is that a permanent rise in productivity raises not only potential output but also output demand due to the impact on permanent income (Clarida *et al*., 1999; McCallum and Nelson, 1999).

   Cover *et al*. suggest, however, that there are other theories positing the reverse causality running from the AD to AS shocks.   The intertemporal substitution of labor predicts that a temporary increase in demand raises the level of output, as agents work more in response to a temporary increase in real wages.   The monopolistic competitive model implies that firms may be reluctant to raise prices in order to maintain market shares and hence, they react to a positive demand shock with muted price changes and output increases.   Appendix provides an exposition on the causal relationships embodied in the BQ and reverse causality models.

   We investigate the reverse causality specification, denoted as BQX.   One

interesting feature of the BQX model is that as the AD shock is causally prior to the AS shock, it is allowed to have potentially long-run effects on real output.   Whether the long-run output neutrality condition holds is determined empirically by data.[5)]   To take account of this possibility, the cointegrated models of section 2 need to be modified.   The BQX model is replicated identically by assuming, for the relationship between the reduced-form and structural shocks of (8), that

$$\Gamma_0^{-1} = \begin{bmatrix} \Lambda_2 \beta_\perp' \Omega^{-1} \\ \Phi_2^{-1} \beta' \end{bmatrix}, \tag{15}$$

where $\Lambda_2^{-1} \Lambda_2^{-T} = \beta_\perp' \Omega^{-1} \beta_\perp$ and $\Phi_2 \Phi_2' = \beta' \Omega \beta$.   Previously, Escribano and *Peña* (1994) proposed a similar decomposition procedure but did not pursue the identification of structural shocks.   Here, $\Lambda_2 \beta_\perp' \Omega^{-1} e_t$ and $\Phi_2^{-1} \beta' e_t$ correspond to AS and AD shocks, respectively.

The inverse matrix of Γ_0^{-1} is calculated as:

$$\Gamma_0 = [\beta_\perp \Lambda_2' \quad \Omega \beta \Phi_2^{-T}] \tag{16}$$

Imposition of $\beta_\perp = [1, \ 0]'$ on (16) shows that the AS shock does not have a contemporaneous effect on the unemployment rate.   This is, of course, due to the presumed causality that runs from the AD to AS shocks. The covariance matrix of the structural shocks is given as:

$$E(\varepsilon_t \varepsilon_t') = \Gamma_0^{-1} \Omega \Gamma_0^{-T} = I. \tag{17}$$

The long-run impact matrix of the structural shocks is obtained from (4) and (16) as:

[5)] Campbell and Mankiw (1987) and Keating (2005) offer a summary list of economic reasons that support long-run effects of AD shocks on real output.   In fact, BQ acknowledged some of these effects, such as hysteresis (*à la* Blanchard and Summers, 1986) and endogenous models of growth.

$$\Gamma(1) = D(1)\Gamma_0 = [\beta_\perp \psi \alpha'_\perp \beta_\perp \Lambda'_2 \quad \beta_\perp \psi \alpha'_\perp \Omega \beta \Phi_2^{-T}], \tag{18}$$

and imposing $\beta_\perp = [1,\ 0]'$ gives:

$$\Gamma(1) = D(1)\Gamma_0 = \begin{bmatrix} \psi \alpha'_\perp \beta_\perp \Lambda'_2 & \psi \alpha'_\perp \Omega \beta \Phi_2^{-T} \\ 0 & 0 \end{bmatrix}. \tag{19}$$

The AD shock is allowed to have long-run effects on real output.

We consider the following model, denoted as BQX-CORR, for evaluating the empirical relevance of the uncorrelatedness assumption in the BQX model:

$$\Gamma_0^{-1} = \begin{bmatrix} \Lambda_3 \beta'_\perp \Omega^{-1} \\ \Phi_3^{-1} \alpha' \end{bmatrix}. \tag{20}$$

Its inverse is given as:

$$\Gamma_0 = [\alpha_\perp (\Lambda_3 \beta'_\perp \Omega^{-1} \alpha_\perp)^{-1} \quad \Omega \beta (\Phi_3^{-1} \alpha' \Omega \beta)^{-1}], \tag{21}$$

where $\Lambda_3^{-1} \Lambda_3^{-T} = \beta'_\perp \Omega^{-1} \beta_\perp$ and $\Phi_3 \Phi'_3 = \alpha' \Omega \alpha$. $\Lambda_3 \beta'_\perp \Omega^{-1} e_t$ and $\Phi_3^{-1} \alpha' e_t$ correspond to AS and AD shocks, respectively. The BQX-CORR model has a covariance matrix of the structural shocks given as:

$$E(\varepsilon_t \varepsilon'_t) = \Gamma_0^{-1} \Omega \Gamma_0^{-T} = \begin{bmatrix} 1 & \Lambda_3 \beta'_\perp \alpha \Phi_3^{-T} \\ \Lambda_3 \beta'_\perp \alpha \Phi_3^{-T} & 1 \end{bmatrix}. \tag{22}$$

The correlation between AS and AD shocks is not restricted to zero, but rather it is determined by the data. The long-run impact matrix is calculated as:

$$\Gamma(1) = D(1)\Gamma_0$$

$$= [\beta_{\perp}\psi\alpha_{\perp}'\alpha_{\perp}(\Lambda_3\beta_{\perp}'\Omega^{-1}\alpha_{\perp})^{-1} \ \ \beta_{\perp}\psi\alpha_{\perp}'\Omega\beta(\Phi_3^{-1}\alpha'\Omega\beta)^{-1}] \qquad (23)$$

and

$$\Gamma(1) = D(1)\Gamma_0 = \begin{bmatrix} \psi\alpha_{\perp}'\alpha_{\perp}(\Lambda_3\beta_{\perp}'\Omega^{-1}\alpha_{\perp})^{-1} & \psi\alpha_{\perp}'\Omega\beta(\Phi_3^{-}\alpha'\Omega\beta)^{-} \\ 0 & 0 \end{bmatrix}, \quad (24)$$

with imposition of $\beta_{\perp} = [1, \ 0]'$. Like the BQX model, the AD shock is allowed to have long-run effects on real output.

5. TESTING RESULTS FOR THE REVERSE CAUSALITY MODEL

We first compare the BQ and BQX models. Table 1 reports the forecast error variance decompositions at various horizons. The results appear to differ considerably. A key difference is that the BQX model exhibits a larger contribution of the AD shock to explaining the variability of the variables in virtually all cases. The evidence is particularly pronounced with respect to the unemployment rate. The AD shock is most important and accounts for at least 51% of the forecast error variance across the six G-7 countries. One reason may be associated with the underlying assumption that the AD shock is causally prior to the AS shock. This causal ordering implies that the uncorrelatedness assumption in the BQX model forces any variation of the unemployment rate, resulting from common shifts in the AS and AD curves, to be attributed entirely to the AD shock. In contrast, the BQ model assumes causality running from the AS to AD shocks. The uncorrelatedness assumption forces the AS shock to be responsible for any variation of real output that occurs due to common shifts in AS and AD curves. This may have partly contributed to the finding that the AS shock

accounts for a larger portion of real output fluctuations in all six G-7 countries than implied by the BQX model.

We proceed to examine the empirical relevance of the uncorrelatedness assumption between AS and AD shocks in the BQX model. Figure 2 presents the impulse responses of a series generated from the BQX and BQX-CORR models, together with 95% confidence bands generated using 500 bootstrap replications of the BQX model. Both models indicate that a favorable AS shock increases real output across horizons. In response to a favorable AD shock, real output increases. The BQX model indicates that the increases in the long run are also statistically significant for Canada, Germany, Italy, and Japan. They are not for the U.K. and the U.S., while the model allows the AD shock to have long-run effects on real output. The U.K. and the U.S. are the only countries that empirically support the long-run output neutrality condition. The AS and AD shocks cause the unemployment rate to fall and the long-run responses converge to zero as a consequence of the stationarity assumption.
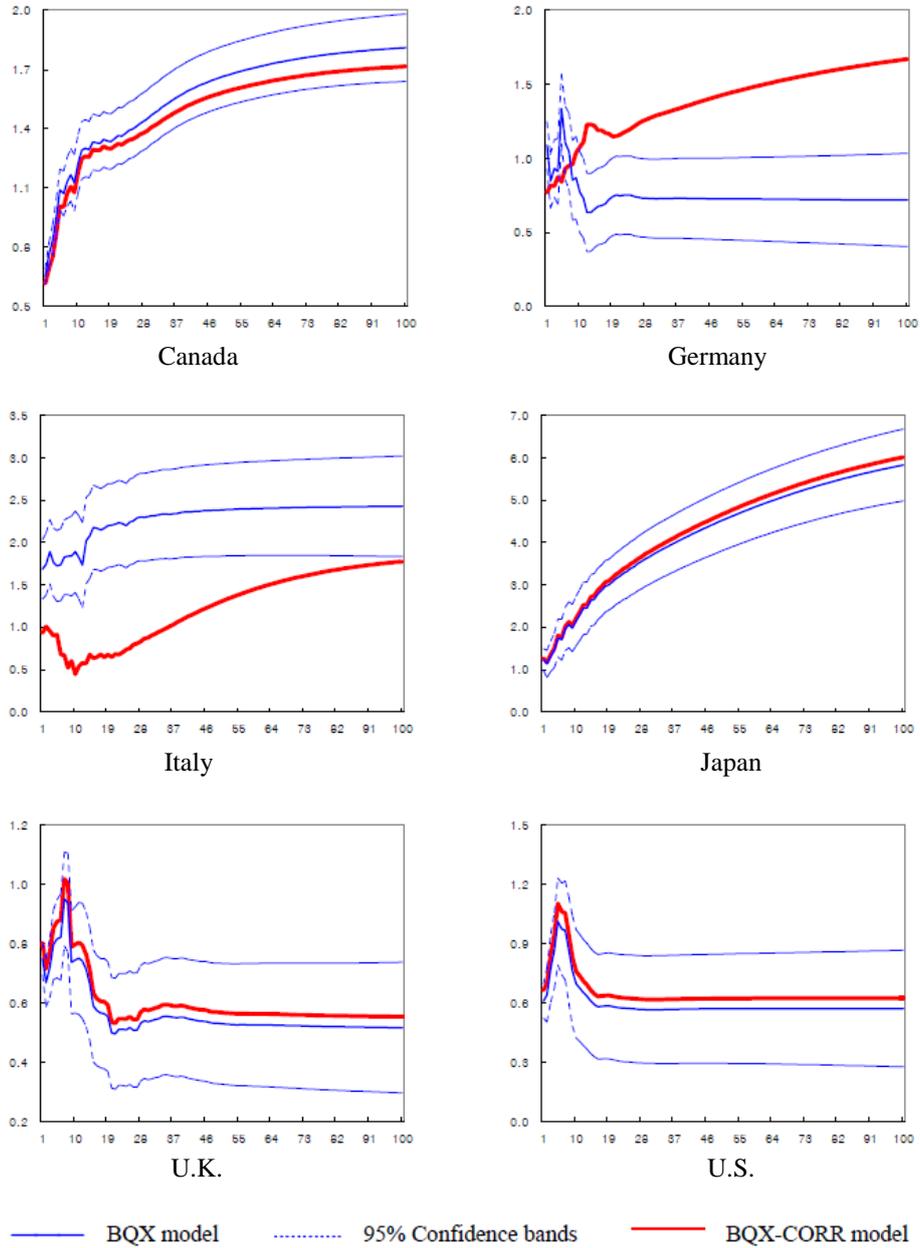
In Canada, Japan, the U.K., and the U.S., the responses from the BQX-CORR models reside within the 95% confidence bands of the responses from the BQX models. There are a few exceptions, but their effects are marginal. An implication is that the uncorrelatedness assumption of AS and AD shocks in the BQX model is consistent with the actual data. Not surprisingly, the BOX-CORR model reports that the estimated correlation coefficients of the two shocks are low in the range between 0.13 and 0.34: 0.13 (Canada), 0.34 (Japan), 0.26 (U.K.), and 0.19 (U.S.). Germany and Italy produce different results, however. In the former, the 95% confidence bands in the BQX model barely include the responses of the BQX-CORR model. The latter has similar results. Although the evidence weakens slightly, many of the responses in the BQX-CORR model lie outside of the 95% confidence bands that are calculated from the BQX model. Thus, the uncorrelatedness assumption underlying the BQX model cannot be accepted based on the data. This suggests that the correlation between AS and AD shocks needs to be taken into account explicitly in order to capture their actual effects on the

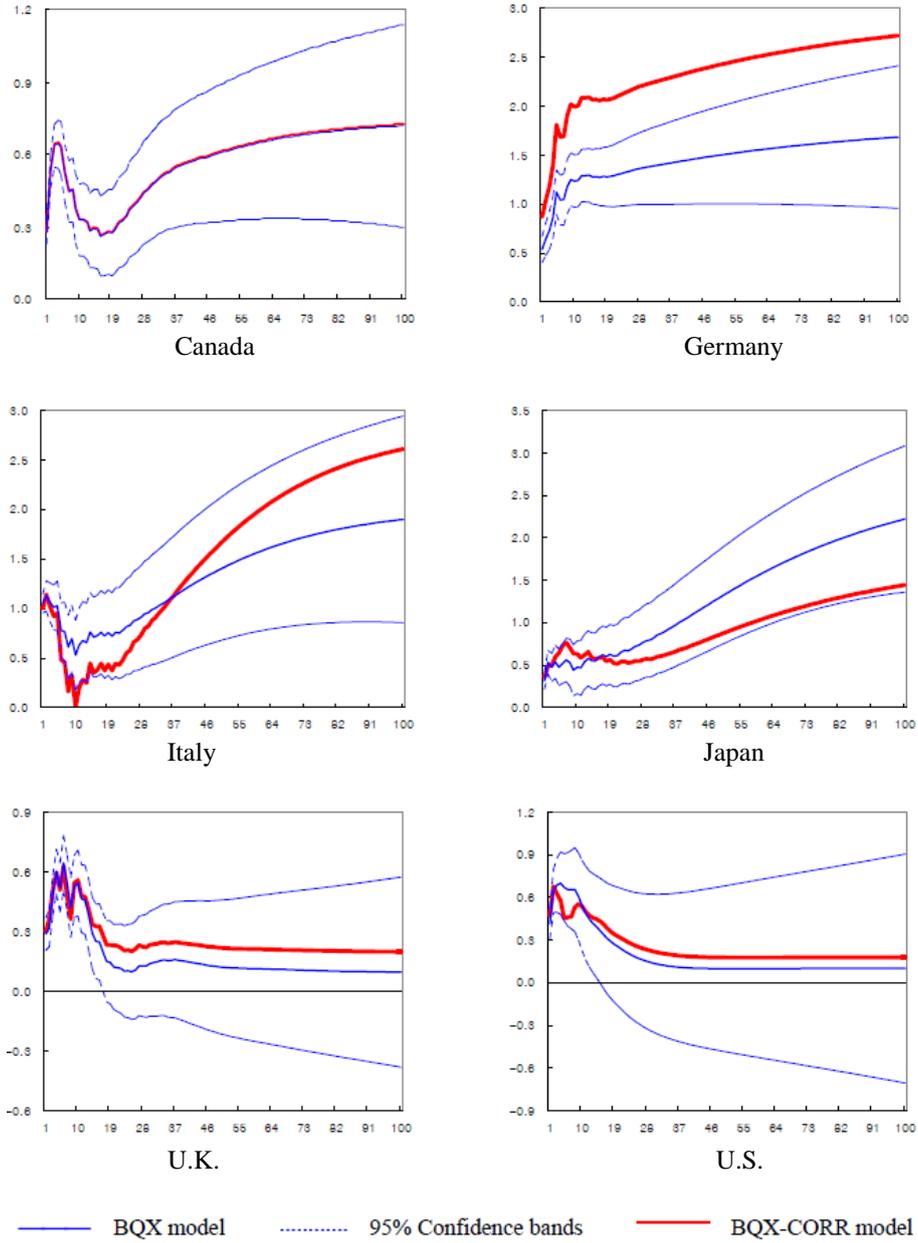**Table 1   Forecast Error Variance Decompositions**

BQ Model

Qtrs	Canada		Germany		Italy		Japan		U.K.		U.S.	
	y_t	u_t	y_t	u_t	y_t	u_t	y_t	u_t	y_t	u_t	y_t	u_t
1	0.1	86.1	45.7	12.9	12.9	84.8	2.2	84.8	3.6	97.0	17.1	97.0
4	4.7	54.8	29.5	10.1	12.7	76.5	0.6	69.5	13.3	75.4	25.9	80.3
8	2.2	45.0	21.5	8.5	18.7	68.8	1.3	49.2	13.4	64.9	20.8	65.1
16	2.2	32.5	11.3	12.8	24.1	70.9	2.8	36.9	13.2	60.0	19.9	70.1
24	2.4	28.2	8.1	13.6	24.0	71.9	3.4	29.4	11.2	60.1	17.2	71.5
36	1.6	26.4	5.6	14.2	20.8	73.1	3.3	23.5	9.1	60.3	14.1	71.7
50	1.1	25.7	5.6	14.4	16.6	73.7	2.6	20.3	7.5	60.4	11.5	71.7
60	0.8	25.4	3.4	14.5	14.0	73.9	2.1	19.1	6.6	60.4	10.2	71.7
70	0.7	25.3	2.9	14.6	12.0	74.1	1.7	18.3	6.0	60.4	9.2	71.7
80	0.6	25.3	2.5	14.6	10.3	74.1	1.4	17.9	5.5	60.4	8.3	71.7
100	0.4	25.3	2.0	14.7	8.0	74.2	1.1	17.3	4.7	60.4	7.0	71.7

BQX Model

Qtrs	Canada		Germany		Italy		Japan		U.K.		U.S.	
	y_t	u_t	y_t	u_t	y_t	u_t	y_t	u_t	y_t	u_t	y_t	u_t
1	15.3	100	19.7	100	30.1	100	6.1	100	12.8	100	31.8	100
4	31.0	86.6	35.5	99.7	28.2	98.3	11.9	95.9	26.5	88.2	42.1	91.2
8	22.8	79.7	44.7	99.3	21.1	95.6	8.7	83.5	26.8	80.0	36.3	79.5
16	12.7	66.8	61.3	99.4	29.2	96.7	5.8	73.9	26.6	76.0	35.1	83.6
24	9.6	60.9	66.3	99.6	26.7	97.1	4.8	66.5	23.3	76.1	31.5	84.5
36	9.5	58.7	70.6	99.6	31.1	97.5	4.8	59.4	19.9	76.3	26.6	84.6
50	10.2	57.8	73.6	99.7	38.2	97.7	5.7	54.9	17.3	76.4	22.4	84.6
60	10.6	57.6	75.2	99.7	32.8	97.8	6.5	53.2	15.8	76.4	20.2	84.6
70	11.1	57.4	76.4	99.7	35.6	97.8	7.5	52.1	14.7	76.4	18.4	84.6
80	11.4	57.4	77.5	99.7	40.5	97.9	8.3	51.4	13.7	76.4	17.0	84.6
100	11.9	57.3	79.1	99.7	39.6	97.9	9.6	50.7	12.2	76.4	14.8	84.6

Notes: The figures are the fractions of the forecast error variance of the series attributable to
an aggregate demand shock. Subtracting them from 100 gives the fraction that is
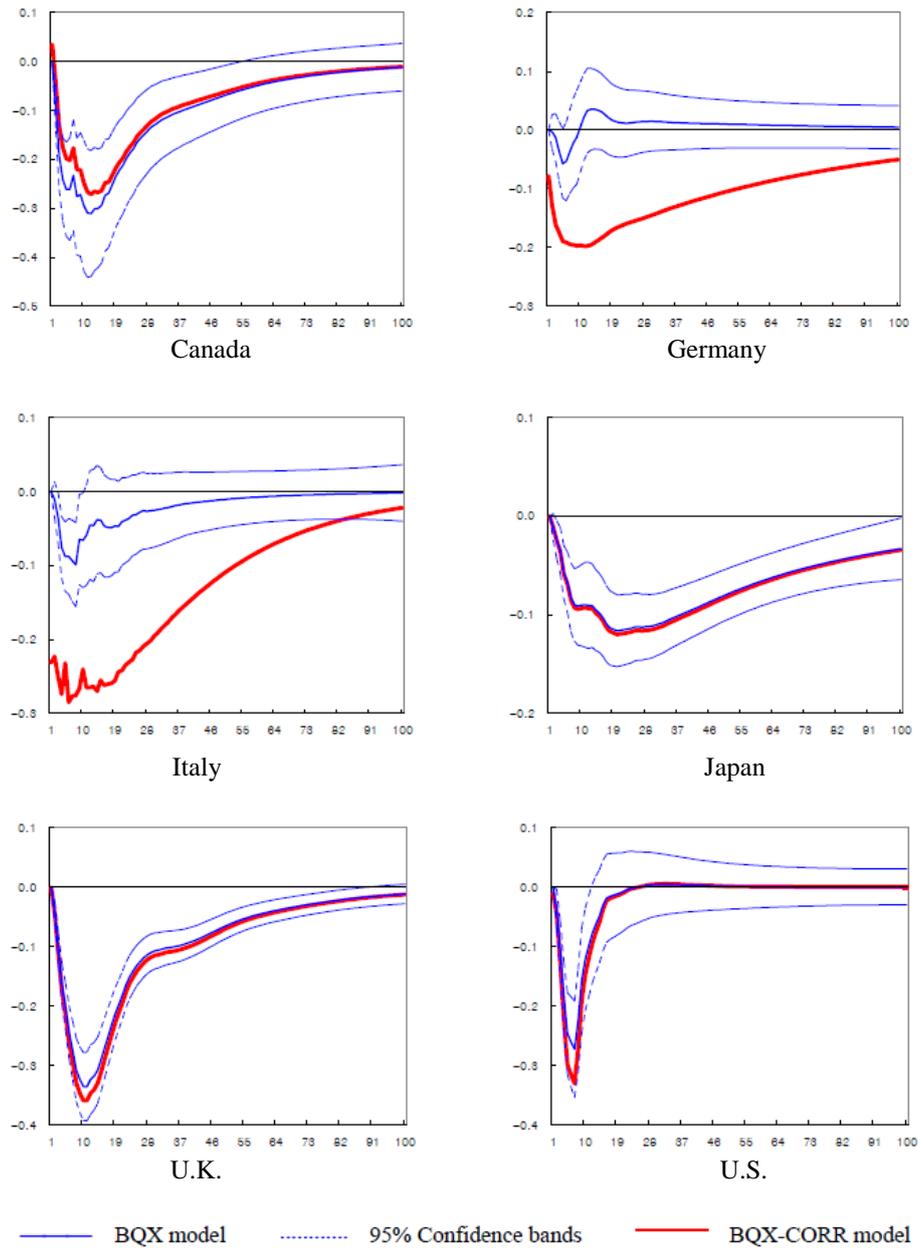attributable to an aggregate supply shock.

Figure 2A   Response of Real Output to an Aggregate Supply Shock
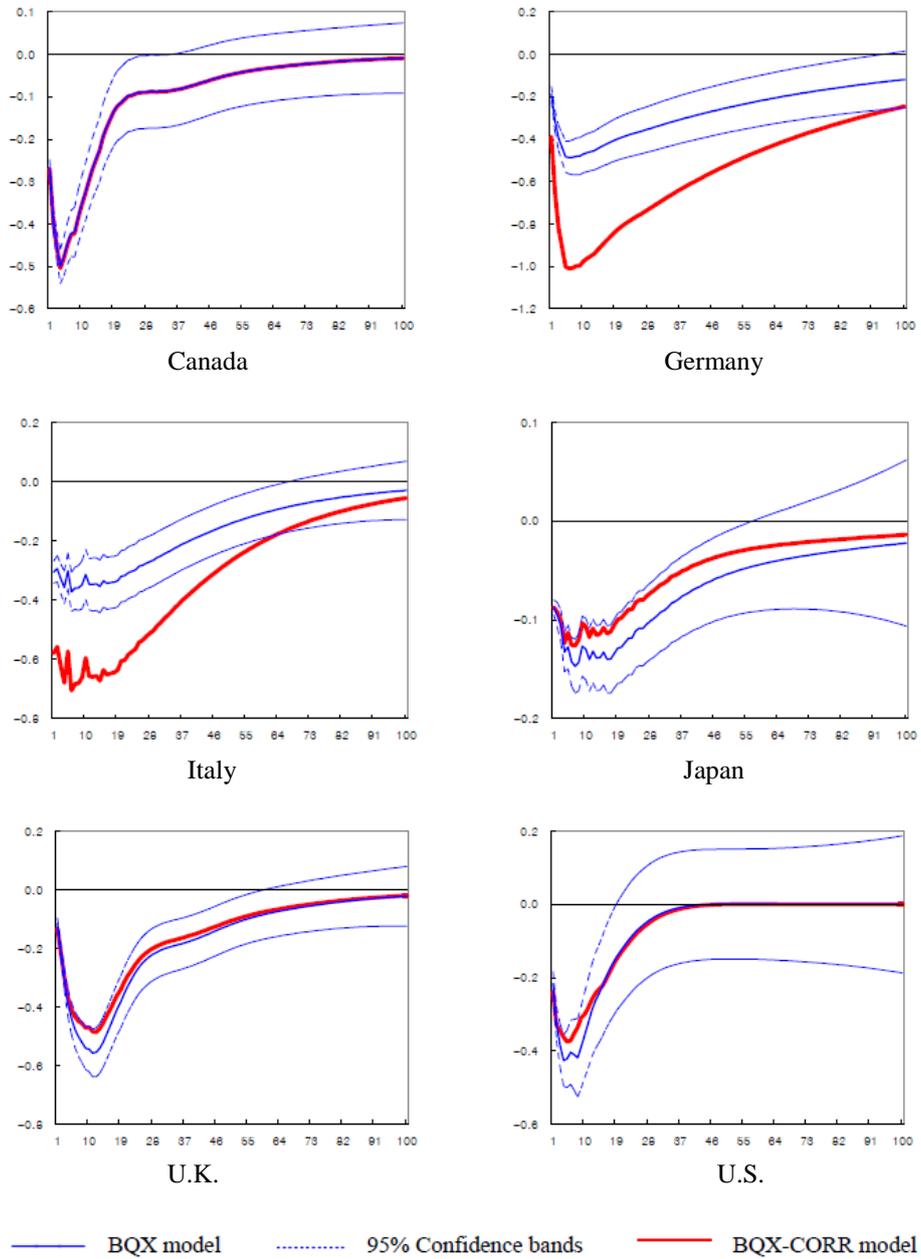

Canada

Germany

Italy

Japan

U.K.

U.S.

————— BQX model   ·········· 95% Confidence bands   ————— BQX-CORR model

Figure 2B   Response of Real Output to an Aggregate Demand Shock

Canada

Germany

Italy

Japan

U.K.

U.S.

———— BQX model   ·········· 95% Confidence bands   ———— BQX-CORR model

**Figure 2C   Response of the Unemployment Rate
to an Aggregate Supply Shock**

Canada

Germany

Italy

Japan

U.K.

U.S.

BQX model   95% Confidence bands   BQX-CORR model

**Figure 2D   Response of the Unemployment Rate
to an Aggregate Demand Shock**

Canada

Germany

Italy

Japan

U.K.

U.S.

———— BQX model   ·········· 95% Confidence bands   ———— BQX-CORR model

variables. In regard to Germany and Italy, the correlation coefficients of two shocks are estimated at 0.73 and 0.69, respectively.

6. CONCLUDING REMARKS

Most structural vector autoregression studies employ the uncorrelatedness between underlying shocks as an identifying assumption. We examine the empirical consistency of this assumption in the model of Blanchard and Quah (BQ, 1989). They studied the effects of aggregate supply and aggregate demand shocks on real output and unemployment rates. The two structural shocks are assumed to be mutually uncorrelated for identification. Presumably, the model implications would be dependent on the adequacy of the assumptions in use. Where the assumptions are inconsistent with the data, their imposition may result in misrepresentation of the true dynamic structure of the model. Nevertheless, the uncorrelatedness assumption of structural shocks has rarely been tested for empirical relevance. Additionally, several studies have also provided economic reasons against this assumption.

To derive a testable form, the BQ model is transformed into a cointegration representation. This alternative setup is extended to allow for the possibility that aggregate supply and aggregate demand shocks are correlated with each other. Empirical evidence shows that the uncorrelatedness between the two shocks is consistent with actual data in all six G-7 countries under study. Thus, its imposition in the BQ model may not be as inappropriate as some previous literature has suggested. Yet, results vary when an alternative identification scheme is adopted. Two cases among the six G-7 countries provide evidence, showing that the data reject the assumption of uncorrelatedness between aggregate supply and aggregate demand shocks. This suggests that the validity of uncorrelatedness between structural shocks may not be taken as granted, but rather should remain as an empirical issue.

APPENDIX

To see the causal relationships embodied in the BQ model and the BQX model of Cover *et al*. (2006), assume the structural equation for real output given as:

$$\Delta y_t = \sum_{i=1}^{p} h_{yy,i} \Delta y_{t-i} + \sum_{i=0}^{p} h_{yu,i} u_{t-i} + \varepsilon_{1t}, \tag{A1}$$

and the reduced-form equation of the unemployment rate given as:

$$u_t = \sum_{i=1}^{p} f_{uy,i} \Delta y_{t-i} + \sum_{i=1}^{p} f_{uu,i} u_{t-i} + e_{2t}, \tag{A2}$$

Following Shapiro and Watson (1988), the long-run output neutrality assumption can be imposed by restricting the sum of the coefficients on u_t in (A-1) to zero (i.e., $\sum_{i=0}^{p} h_{yu,i} = 0$), which yields

$$\Delta y_t = \sum_{i=1}^{p} h_{yy,i} \Delta y_{t-i} + \sum_{i=0}^{p-1} h'_{yu,i} \Delta u_{t-i} + \varepsilon_{1t}, \tag{A1a}$$

where $h'_{yu,i} = \sum_{j=0}^{i} h_{yu,j}$.

Equation (A1a) will not be estimated by OLS due to the contemporaneous value of Δu_t in the right-hand variables. However, this can be estimated consistently using an instrumental variable (IV) procedure. The long-run output neutrality assumption generates sufficient instruments to estimate the parameters of (A1a) including the AS shock ε_{1t}. An appropriate set of instruments is lags 1 through p of Δy_t and u_t. Equation (A2) can be estimated by OLS. Once (A1a) and (A2) are estimated, taking a Choleski decomposition produces exactly identical results as the BQ model. This procedure reveals an implicit assumption underlying the BQ model that the AS shock is causally prior to the AD shock. In contrast, the BQX model

assumes the reverse causality that runs from the AD to AS shocks.   This model can be identified by estimating (A2) and (A1a) sequentially, followed by an application of the Cholesky decomposition.

While not addressed in Cover *et al.*, both BQ and BQX models can also be estimated identically in the context of an IV framework.   To begin with the BQ model, assume the structural equation of the unemployment rate given as:

$$u_t \quad = \sum_{i=0}^{p} h_{uy,\,i} \Delta y_{t-i} + \sum_{i=1}^{p} h_{uu,\,i} u_{t-i} + \varepsilon_{2t}. \tag{A3}$$

Upon estimating (A1a), (A3) can be estimated using the same set of instruments plus $\hat{\varepsilon}_{1t}$, the estimated residual from (A1a).[6]   This matches the causal ordering running from AS to AD shocks.   To implement the BQX model, (A1a) is modified as:

$$\Delta y_t = \sum_{i=1}^{p} s_{yy,i} \Delta y_{t-i} + \sum_{i=0}^{p-1} s_{yu,i} \Delta u_{t-i} + s^* u_t + \varepsilon_{1t}. \tag{A4}$$

The model is identified by estimating (A2) and (A4) sequentially in a way such that the AD shock is causally prior to the AS shock.   Under the uncorrelatedness assumption of structural shocks, (A4) can be consistently estimated using lags 1 through p of Δy_t and u_t, and $\hat{e}_{2t}$, the estimated residual from (A2) as instruments.

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[6] See Fry and Pagan (2005) for an IV estimation of the BQ model.

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