# Explaining the Joint Behavior of Employment, Unemployment and Nonparticipation[*]

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This paper proposes a matching model in which the worker's labor force state is determined after matches take place. The model generates the direct transition from nonparticipation to employment and accounts for the features of the U.S. labor market with respect to the volatilities and correlations. This paper also shows that matching models with unemployment as active search and nonparticipation as inactive search predict counterfactual results: accounting for only 10 percent of the actual volatility of the unemployment rate, a perfect relationship between unemployment and nonparticipation, and a very weak relationship between vacancy and unemployment.

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# 1. INTRODUCTION

In the standard search and matching model, unemployment captures those who have not found employment or have separated as well as those who are currently looking for work. The current-period unemployment is then determined by the former, and it also determines the latter. According to the definition of unemployed persons of the U.S. Current Population Survey (CPS), the unemployed are persons aged 16 years and older who had no employment during the reference week and had made specific efforts to find employment sometime during the 4-week period ending with the reference week.[1] The CPS definition demonstrates that the unemployed are those who searched but did not find employment.

Recent studies about the labor force participation which attempt to distinguish between unemployment and nonparticipation have neglected the feature of unemployment that the unemployed are those who find no employment and have focused on the search feature of unemployment.[2] Many take one of the following classifications: unemployment as a searching state and nonparticipation as a non-searching state *or* unemployment as active search and nonparticipation as inactive search.

The searching versus non-searching classification gives rise to the problem that no one can move directly from nonparticipation to employment. Petrongolo and Pissarides (2001) argue that the direct flows from nonparticipation to employment are due to misclassification problems, the so called time aggregate bias: any person who now has a job must have made some effort, which cannot be detected by labor force surveys. Thus several authors who take this classification cannot but adjust the time period of the model.[3] The method used to adjust the time period of the model makes it

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[1] See http://www.bls.gov/bls/glossary.htm
[2] Among others, see Kim (2008), Pries and Rogerson (2009), Garibaldi and Wasmer (2005), Hæfke and Reiter (2006) for the Mortensen and Pissarides (1994) matching model, Veracierto (2008) for the Lucas and Prescott (1974) island model, and Tripier (2004) for the real business cycle model.
[3] See Garibaldi and Wasmer (2005) and Hæfke and Reiter (2006). For example, Hæfke and Reiter (2006) choose one week as a model period and match the monthly transition rates.

hard to interpret other transition rates.   One may ask that the monthly transition rate from unemployment to employment generated by a weekly model is consistent with the actual monthly transition rate.   Therefore, we need more sophisticated methods to explain the direct transition from nonparticipation to employment.

The active versus inactive search classification, on the other hand, helps produce the direct transition from nonparticipation to employment, but generates counterfactual implications.   In this paper, I evaluate the matching model with the active versus inactive search classification as in Kim (2008) and Pries and Rogerson (2009).   The models are called the ante-match model hereafter because labor force classifications are made before matches take place. Based on the reduced-form dynamics derived from the model, I show that the unemployment rate is much less volatile than the employment-population ratio and the labor market variables are highly correlated with each other.

For that reason, I modify the matching model in the following ways. First, I classify those who do not engage in job searches as out of the labor force.   Second, I classify both those who have been working and those who find employment through a job search as employed.   The transitions from nonparticipation to employment are generated between two consecutive periods, and it is not necessary to assume that nonparticipants are inactive searchers.   This model is called the post-match model because labor force classifications are made after matches take place.

The post-match model has a couple of novel features.   One is that the direct transition from nonparticipation to employment is generated with no assumption that nonparticipants are inactive searchers and without adjusting the model period.   Another novel feature is that, as in Cole and Rogerson (1999), the reduced-form dynamics are easily derived from the model, and identifying the parameters governing the model is straightforward.

The findings can be summarized as demonstrating that the post-match model accounts for the U.S. labor market quite well.   First, the model predicts that the unemployment rate is most volatile, and accounts for more

than 60% of the actual volatility of the unemployment rate. Second, the employment-population ratio is highly negatively correlated with both the unemployment-population ratio and the nonparticipation rate. Third, the model predicts a positive correlation between the unemployment-population ratio and the nonparticipation rate. Finally, the unemployment-population ratio is quite negatively correlated with the vacancy rate, –0.73, so that the model predicts a very strong Beveridge relationship.

This paper is organized as follows. Section 2 discusses and compares the different labor force classifications and introduces matching models. Section 3 quantifies both the ante- and post-match models, and section 4 states my conclusions.

## 2. MODEL

The model economy is a variant of the Mortensen and Pissarides (1994) matching model which consists of workers and firms (or entrepreneurs). Both workers and firms are homogeneous. Unlike the standard matching models in which all workers participate in the labor market, I assume that workers can be not only employed or unemployed, but also in or out of the labor force. The worker's decision making is not endogenous, but determined by exogenous probabilities. In this sense, the model discussed below has similar assumptions to Hansen (1985), Rogerson (1988) and Cole and Rogerson (1999).

### 2.1. Environments

There is a continuum of infinitely-lived and risk-neutral workers with total mass equal to one. Each worker has preferences defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t, \tag{1}$$

where $0 < \beta < 0$ is the discount factor and $c_t$ is consumption, which takes the following values depending upon the worker's labor market status: wages $w_t$ if the worker is working, unemployment insurance benefits $b$ if the worker is searching for a job, and $0$ if the worker is out of the labor force.

There are also an infinite number of risk-neutral firms in this economy. Each firm has preferences defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t, \tag{2}$$

where $0 < \beta < 0$ is the discount factor and $d_t$ profits. In a certain period, firms can be active or vacant. An active firm is one that is matched with a worker and is currently producing output $z_t$, where $z_t$ is assumed to follow an AR(1) process in logs:

$$\ln z_{t+1} = \rho \ln z_{t+1} + \varepsilon_{t+1}, \tag{3}$$

where $\varepsilon_t$ follows a normal distribution with mean zero and variance $\sigma_\varepsilon^2$. The steady state productivity level is normalized to 1. All active firms confront exogenous separation with probability $\lambda$. A vacant firm is one that is posting a vacant position and looking for workers. All vacant firms find workers with probability $q$. I assume that firms pay $k$ units of the consumption good to post a vacancy.

A constant-returns-to-scale matching function is assumed:

$$m(s, v) = \omega s^\gamma v^{1-\gamma}, \tag{4}$$

where $s$ is the number of job-searchers expressed as an efficiency unit, $v$ the number of vacancies, $\gamma$ the elasticity of the matching function, and $\omega$ a matching function parameter. In what follows, I introduce two different job-finding probabilities. A worker who was in the labor force in the

previous period finds a job with probability $p$ in the current period, while a worker who was out of the labor force in the previous period finds a job with probability $f$ in the current period:

$$p \equiv \frac{m}{s} = \omega\theta^{1-\gamma},$$   (5)

$$f \equiv e\frac{m}{s} = e\omega\theta^{1-\gamma},$$   (6)

where $\theta$ is the vacancy-searcher ratio and $e$ the relative search intensity or efficiency.    The worker-finding probability is

$$q \equiv \frac{m}{v} = \omega\theta^{1-\gamma}.$$   (7)

In the following section, I introduce two different labor force classifications; one (ante-match model) made by Pries and Rogerson (2009) and the other (post-match model) by Moon (2011).

## 2.2. Labor Force Classifications of Ante- and Post-Match Models

In this section, I begin by reporting the labor flows from the U.S. Current Population Survey.  Table 1 shows the average monthly probabilities of transition between different labor force states for the U.S. labor market. The transition rate from unemployment to employment (hereafter the UE transition rate) is about 0.22 and the transition rate from OLF to employment about 0.03. On the other hand, the transition rate from employment to unemployment is 0.12 and from employment to OLF is 0.18.  Finally, the UO transition rate is 0.14, while the OU transition rate is 0.23.

**Table 1   The Transition Rates for the CPS, 1978-2005 (% Per Month)**

(1) Unadjusted

|       |            | To       |            |       |
|-------|------------|----------|------------|-------|
|       |            | Employed | Unemployed | OLF   |
| From  | Employed   | 95.62    | 1.49       | 2.89  |
|       | Unemployed | 26.66    | 51.23      | 22.11 |
|       | OLF        | 4.63     | 2.56       | 92.82 |

(2) Adjusted with the Abowd and Zellner (1985, Table 5) Correction

|       |            | To       |            |       |
|-------|------------|----------|------------|-------|
|       |            | Employed | Unemployed | OLF   |
| From  | Employed   | 97.05    | 1.19       | 1.76  |
|       | Unemployed | 22.33    | 63.42      | 14.25 |
|       | OLF        | 2.79     | 2.27       | 94.94 |

Source: Robert Shimer's tabulations of raw data from the CPS.

### 2.2.1. Ante-match model á la Pries and Rogerson (2009)

First, I describe the labor force classification used in the ante-match model to account for U.S. labor market flows.   Figure 1 presents the flow chart describing how workers move between labor force states within a given period.

At the beginning of a certain period, there are two types of workers: matched and unmatched.   Matched workers have employment opportunities, but unmatched workers do not.   Matched workers are to work on their current jobs with probability $1-\eta$ or not with probability $\eta$.   Matched workers who are not to work with probability $\eta$ become unmatched.   If a matched worker is to work with probability $1-\eta$, then (s)he is classified as employed.   At the end of the period, those who are classified as employed separate with probability $\lambda$.   At the beginning of the subsequent period, those who separate become unmatched but those who do not separate remain matched.

## Figure 1    Ante-Match Model: Classifications Are
## Made before Matches Take Place
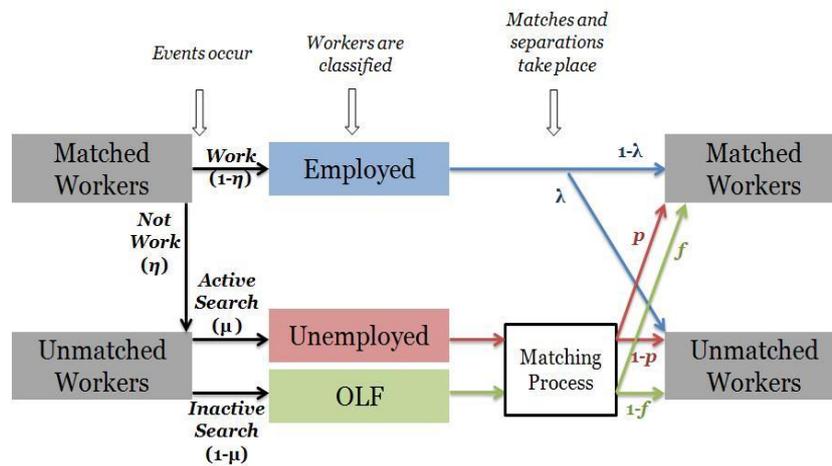


Unmatched workers are to look for work actively with probability $\mu$ or inactively with probability $1-\mu$. Unmatched workers who are to search actively are classified as unemployed, and find jobs with probability $p$. At the beginning of the subsequent period, those who find employment become matched, but those who do not find employment become unmatched. If an unmatched worker is to search inactively with probability $\mu$, then (s)he is classified as out of the labor force (OLF) and finds employment with probability $f$.

From table 1, one can compute the average job-finding probabilities for the unemployed and nonparticipant. The average probability of the unemployed finding a job is 0.22 and the average probability of a nonparticipant finding a job to 0.03, respectively. With unemployed workers' search intensity normalized to unity, the relative search intensity of nonparticipants, denoted by $x$, is about 1/8 which comes from 0.03 divided by 0.22. The efficiency unit of job-searchers can be expressed as $s = U + xO$, where $U$ denotes the number of unemployed workers and $O$ the number of nonparticipants.[4]

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[4] $x$ corresponds to $e$ in equation (6).

### 2.2.2. Post-match model á la Moon (2011)

In this subsection, I introduce another way of classifying workers in which workers are classified after matches take place.    At the beginning of each period, there are three types of workers (based on the classifications made one period before): employed, unemployed, and out of the labor force (OLF).    Employed workers are to work on their current jobs with probability $1-\eta$ or not with probability $\eta$.    If an employed worker is not to work with probability $\eta$, then (s)he is classified as OLF.    If an employed worker is to work, (s)he separates with probability $\lambda$ at the end of that period.    Those who separate are classified as unemployed, but those who survive a separation are classified as employed.

Unemployed workers are to look for work with probability $\pi$ or not with probability $1-\pi$.    If an unemployed worker is not to search, then (s)he is classified as OLF.    If an unemployed worker is to search with probability $\pi$, (s)he finds a job with probability $p$, and then is classified as employed. Those who do not find employment, otherwise, are classified as unemployed.

Nonparticipants are to look for work with probability $\xi$ or not with probability $1-\xi$.    If a nonparticipant is not to search, then (s)he is classified

**Figure 2    Post-Match Model: Classifications Are Made after Matches Take Place**
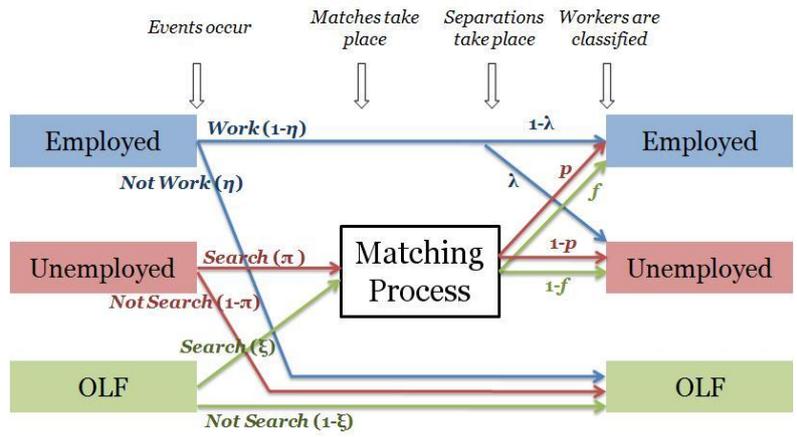
as OLF.  If a nonparticipant is to search, (s)he finds employment with probability *f*, and then is classified as employed.  Those who do not find employment, otherwise, are classified as unemployed.  Figure 2 summarizes the worker flows.

To be consistent with the model classification, the nonparticipant's job-finding probability, *f*, can be expressed as the ratio of those who make the transition from OLF to employment (*OE*) to those who make the transition from OLF to the labor force (*OE+OU*).  Similarly, the unemployed worker's job-finding probability, *p*, can be expressed as the ratio of those who make the transition from unemployment to employment (*UE*) to those who make the transition from unemployment to the labor force (*UE+UU*).  The average job-finding probabilities are then given by about 26% for the unemployed and 55% for the nonparticipants:

$$p \equiv \frac{T(UE)}{T(UE) + T(UU)} = 0.26, \tag{8}$$

$$p \equiv \frac{T(OE)}{T(OE) + T(OU)} = 0.55, \tag{9}$$

where $T(UE)$ is the transition rate from unemployment to employment, for instance.  Let $y$ denote the relative search efficiency:

$$y \equiv \frac{f}{p} = 2.12. \tag{10}$$

The number of searchers is then $s \equiv \pi U + y \xi O$, where $U$ and $O$ are the number of unemployed workers and the number of nonparticipants, respectively.[5]  Intuitively speaking, if a nonparticipant is to search with a certain probability, then (s)he will have about two times higher job-finding probability that an unemployed worker will.

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[5] $y$ corresponds to $e$ in equation (6).

## 2.3. Recursive Equilibrium

I will now describe the equilibrium of the post-match model when workers are classified after matches take place.[6]   The individual worker's and firm's problems can be formulated recursively.   The state of the economy is described by $(z, \varphi)$, where $z$ is aggregate productivity and $\varphi$ is the distribution of workers.   Let $V^w(z, \varphi)$ denote the value function of a worker who is actually working, $V_p^s(z, \varphi)$ the value function of a worker who was classified as unemployed in the previous period and who searches in the current period, $V_f^s(z, \varphi)$ the value function of a worker who was classified as OLF in the previous period and who searches in the current period, and $V^0(z, \varphi)$ the value function of a worker who does not search in the current period.

The value function of a worker who actually works is given by

$$
\begin{aligned}
V^w(z, \varphi) = w(z, \varphi) \\
+ \beta(1-\lambda)E[(1-\eta)V^w(z', \varphi') + \eta V^o(z', \varphi')] \quad (11) \\
+ \beta\lambda E[\pi V_p^s(z', \varphi') + (1-\pi)V^o(z', \varphi')],
\end{aligned}
$$

where $w(z, \varphi)$ is a Nash bargaining wage and $\lambda$ the exogenous separation rate.   A worker earns wages in the current period, and in the subsequent period if the match survives with probability $1-\lambda$, then the worker will continue or terminate the match, depending on probability $\eta$.   However, if the match is dissolved exogenously with probability $\lambda$, then the worker will search with probability $\pi$.

The value function of a worker who was classified as unemployed in the previous period and searches in the current period is given by

$$
\begin{aligned}
V_p^s(z, \varphi) = b + \beta p(z, \varphi)E[(1-\eta)V^w(z', \varphi') + \eta V^o(z', \varphi')] \\
+ \beta(1 - p(z, \varphi))E[\pi V_p^s(z', \varphi') + (1-\pi)V^o(z', \varphi')],
\end{aligned} \quad (12)
$$

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[6] See Appendix for the recursive equilibrium of the ante-match model.

where $p(z, \varphi)$ is the job-finding probability. A worker who looks for work receives unemployment insurance benefits $b$ in the current period, and in the subsequent period finds a job with probability $p(z, \varphi)$. If a worker finds a job, then (s)he will take that job or not, depending on probability $\eta$. Otherwise, (s)he will search again or not, with probability $\pi$.

The value function of a worker who was classified as OLF in the previous period and searches in the current period is given by

$$V_f^s(z, \varphi) = \beta f(z, \varphi)E[(1-\eta)V^w(z', \varphi') + \eta V^o(z', \varphi')]$$
$$+ \beta(1 - f(z, \varphi))E[\pi V_p^s(z', \varphi') + (1-\pi)V^o(z', \varphi')], \tag{13}$$

where $f(z, \varphi)$ is the job-finding probability with $f(z, \varphi) \equiv yp(z, \varphi)$. Note that a worker who was out of the labor force in the previous period and is looking for work in the current period cannot receive unemployment insurance benefits.

The value function of a worker who does not search is given by

$$V^o(z, \varphi) = \beta E[\xi V_f^s(z', \varphi') + (1-\xi)V^o(z', \varphi')]. \tag{14}$$

A worker does not search in the current period is to search or not in the subsequent period with probability $\xi$.

Let $J(z, \varphi)$ denote the value function of a firm matched with a worker. The value function of this matched firm is given by

$$J(z, \varphi) = z - w(z, \varphi) + (1-\lambda)E[(1-\eta)J(z', \varphi')], \tag{15}$$

where $z$ is output, $w(z, \varphi)$ a Nash bargaining wage, and the remaining term the discounted expected values of the match weighted by the probability that the match survives, $1-\lambda$.

The equilibrium number of job vacancies is determined by the following free-entry condition which states that vacancies earn zero profits:

$$k = \beta q(z, \varphi) E[(1-\eta) J(z', \varphi')], \qquad (16)$$

where $k$ is the job posting cost, and $q(z, \varphi)$ the worker-finding probability.

Let $S(z, \varphi)$ denote the match surplus between a worker and a firm. The match surplus is defined to be the difference in the sum of the payoffs of the worker and the firm:

$$S(z, \varphi) = V^W(z, \varphi) - V^o(z, \varphi) + J(z, \varphi). \qquad (17)$$

Note that the threat point of the worker is the value from being out of the labor force.    By construction, a worker who breaks up the match is then out of the labor force, but never becomes a job-seeker.    This can be supported because workers cannot find a better wage through a search in this framework even though all decisions are endogenous.

The wage is derived by assuming that fixed fractions of the surplus accrue to the worker and the firm.    The total match surplus is shared in accordance with the following Nash product:

$$w(z, \varphi) = \arg\max(V^w(z, \varphi) - V^o(z, \varphi))^\alpha (J(z, \varphi))^{1-\alpha}, \qquad (18)$$

where $\alpha$ is the worker's bargaining power, which is set to equal the elasticity of the matching function with respect to search, $\alpha = \gamma$, so that the Hosios (1990) rule is satisfied.    Following Hall (2005), I assume that wages are rigid over the business cycle so that they are given by $w(z, \varphi) = w(z^*, \varphi^*)$ for all $(z, \varphi)$ where $z^*$ is the steady state productivity, $\varphi^*$ the steady state distribution of workers, and $w(z^*, \varphi^*)$ the Nash bargaining wage at $(z^*, \varphi^*)$.

Finally, the evolution of the aggregate state is described by the function $\Theta(z, \varphi)$, where for each $(z, \varphi)$ this function specifies a distribution over the next period's values of the state variables.

The recursive equilibrium is a list of value functions, job- and worker-finding probabilities and wages such that:

a. Taking the probabilities and the wages as given, workers and firms solve their value functions (11)-(15),

b. The free-entry condition (16) is satisfied,

c. Wages are determined by Nash bargaining (18), and

d. For each $(z, \varphi)$, probabilistic decisions generate a distribution over the next period's state that is consistent with the distribution given by $\Theta(z, \varphi)$.

## 2.4. Reduced-Form Labor Market Dynamics

Following Cole and Rogerson (1999), I characterize the implications of the post-match model for the time series. For simplicity, the population size is normalized to unity. Let $E_t$, $U_t$ and $O_t$ denote the number of employed workers (employment-population ratio), the number of unemployed workers (unemployment-population ratio), and the number of nonparticipants (nonparticipation rate), respectively.

The model-implied times series of the employment-population ratio, the unemployment-population ratio and the nonparticipation rate are as follows:

$$E_t = (1-\eta)(1-\lambda)E_{t-1} + \pi p_{t-1}U_{t-1} + \xi f_{t-1}O_{t-1}, \tag{19}$$

$$U_t = (1-\eta)\lambda E_{t-1} + \pi(1-p_{t-1})U_{t-1} + \xi(1-f_{t-1})O_{t-1}, \tag{20}$$

$$O_t = \eta E_{t-1} + (1-\pi)U_{t-1} + (1-\xi)O_{t-1}. \tag{21}$$

Note that the nonparticipation rate $O_t$ is independent of the job-finding probabilities, $p_{t-1}$ and $f_{t-1}$. One can notice that the average *UE* and average *OE* transition rates, $\pi p$ and $\xi f$, are directly identified from the data. In particular, the average OE transition rate, $\xi f$, consists of two parts: the probability of a nonparticipant entering the labor force $\xi$, and the job-finding probability $f$. Although we observe that the average *UE* transition rate is higher than the average OE transition rate, this does not

necessarily imply that $p$ is greater than $f$.   Suppose that $p$ is less than $f$. If the probability of remaining unemployed, $\pi$, is much higher than the probability of entering the labor force, $\xi$, then a higher average $UE$ transition rate can be observed.

As I will mention in the following section, the different labor market classifications make the dynamics given in equation (19) to (21) different. In the ante-match model, the key parameter which distinguishes between unemployment and nonparticipation is $\mu$.   More specifically, the number of unemployed workers is fraction $\mu$ of all the unmatched workers, and the number of nonparticipants is fraction of $1-\mu$ of all the unmatched workers. This can imply that there is perfect correlation between unemployment and nonparticipation.

# 3. QUANTITATIVE ANALYSIS

## 3.1. Calibration

I calibrate the model to the U.S. data and evaluate it quantitatively.   The model operates at a monthly frequency, and therefore the discount factor, $\beta$, is set to 0.9967, equivalent to an annual interest rate of 4%.  Workers' bargaining power $\alpha$ and the matching function elasticity with respect to search $\gamma$ are set to 0.5.   The level of unemployment benefits (or the replacement ratio) is set to 40% of the steady-state wage as in Shimer (2005). Following Andolfatto (1996), I set the worker-finding probability to 0.5, which is consistent with the average vacancy duration of about 45 days.[7] Accordingly, the steady state vacancy-searcher ratio $\theta^*$ and matching function parameter $\omega$ are pinned down.   That is, $\theta^* = p^* / q^*$ and $\omega = p^*(\theta^*)^{\gamma-1}$.   The persistence parameter of the productivity shock, $\rho_z$, is set to 0.97 and the standard deviations of the innovation to the productivity

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[7] Andolfatto (1996) sets the worker-finding probability to 0.9 at quarterly frequencies.   At monthly frequencies, it is about 0.5.

shock, $\sigma_\varepsilon$, to 0.01. The average probability that an unemployed worker remains in the labor force, $\pi$, is set to $T(UE) + T(UU)$. The steady-state probability that an unemployed worker finds a job is $p^* = T(UE) /$

**Table 2    Calibration to the U.S. Labor Market**

| Parameters | | Ante-Match Model | Post-Match Model |
|---|---|---|---|
| discount factor | $\beta$ | 0.9967 | 0.99678 |
| workers' bargaining power | $\alpha$ | 0.5 | 0.5 |
| matching function elasticity | $\gamma$ | 0.5 | 0.5 |
| unemployment insurance benefits | $b$ | 0.265 | 0.264 |
| probability of the unemployed being in the labor force | $\pi$ | | 0.8575 |
| probability of a nonparticipant entering the labor force | $\xi$ | | 0.0506 |
| job-finding probability for unemployed workers | $p$ | 0.2233 | 0.2604 |
| job-finding probability for nonparticipants | $f$ | 0.0279 | 0.5517 |
| probability of a matched worker leaving the labor force | $\eta$ | 0.0187 | 0.0187 |
| probability of being an active job-searcher | $\mu$ | 0.1059 | |
| probability of an employed worker being laid off | $\lambda$ | 0.0302 | 0.0117 |
| worker-finding probability | $q$ | 0.5 | 0.5 |
| vacancy-searcher ratio | $\theta$ | 0.4466 | 0.5209 |
| matching function parameter | $\omega$ | 0.3342 | 0.3609 |
| job-posting cost | $k$ | 5.0264 | 4.9879 |
| persistence of productivity shock | $\rho_z$ | 0.97 | 0.97 |
| standard deviation of shock | $\rho_\varepsilon$ | 0.01 | 0.01 |

Note: See Appendix for the calibration of the Ante-Match model.

$(T(UE) + T(UU))$.   The average probability that a nonparticipant enters the labor force is set to $\xi = T(OE) + T(OU)$.   The steady-state probability that a nonparticipant finds a job is $f^* = T(OE) / T(OE) + T(OU))$.   The average probabilities of a worker leaving the labor force and of being laid off are calculated from the steady-state condition of the reduced-form labor market dynamics:

$$\eta = \frac{\xi O^* - (1 - \pi)U^*}{E^*}, \tag{22}$$

$$\eta = \frac{(1 - \pi + \pi p)U^* - \xi(1 - f^*)O^*}{(1 - \eta)E^*}, \tag{23}$$

where $E_t$, $U_t$, and $O_t$ are the steady-state employment-population ratio, the unemployment-population ratio and the nonparticipation rate, respectively. The relative search efficiency is given by $y = f^* / p^*$.   All parameters are summarized in table 2.

### 3.2. Simulation

The time period of the models is 336 months, and the associated statistics are obtained by 100 simulations.   I report both variables with a trend and cyclical (or detrended) ones.   As in Shimer (2005), the cyclical variables are obtained by a Hodrick-Prescott filter with smoothing parameter 900,000. The main purpose of the paper is to investigate to what extent both models are able to explain the volatilities of the employment-population ratio, the unemployment-population ratio and the nonparticipation rate as well as correlations between them.   Table 3 describes the U.S. data.

**Table 3    Descriptive Statistics for the CPS, 1978-2005**

(1) Original Variables

| Statistics | E | U | O | u |
|---|---|---|---|---|
| Mean (%) | 61.67 | 4.06 | 34.28 | 6.19 |
| Standard Deviation (%) | 1.90 (1.00) | 0.87 (0.46) | 1.23 (0.65) | 1.41 (0.74) |
| Correlations | | | | |
| E | 1 | −0.86 | −0.93 | −0.89 |
| U | | 1 | 0.61 | 0.99 |
| O | | | 1 | 0.66 |

(2) Detrended Variables

| Statistics | E | U | O | u |
|---|---|---|---|---|
| Standard Deviation (%) | 1.21 (1.00) | 12.99 (10.75) | 0.72 (0.60) | 13.22 (10.94) |
| Correlations | | | | |
| E | 1 | −0.96 | −0.78 | −0.96 |
| U | | 1 | 0.62 | 0.99 |
| O | | | 1 | 0.64 |

Notes: 1) Sample period: Jan 1978-Dec 2005.    The seasonally adjusted BLS series are used: employment-population ratio ($E$), unemployment-population ratio ($U$) nonparticipation rate ($O$), and unemployment rate ($u$) for those aged 16 years and over as provided by the BLS.    The relative standard deviations, as expressed a ratio to the standard deviation of employment, are given in the parentheses.    Cyclical variables are obtained by a Hodrick-Prescott filter with smoothing parameter 900,000.

### 3.2.1. Simulation results of the ante-match model

In this subsection I evaluate the ante-match model quantitatively in which unemployment is active search and nonparticipation is inactive search. Table 4 (Ante-Match Model) shows the simulation results.    The relative standard deviation of the unemployment-population ratio, expressed as the ratio to the standard deviation of the employment-population ratio, is 10 times less than that of the nonparticipation rate for the variables with a trend. For the detrended variables, the relative volatility of the unemployment-population ratio, 0.93, is the same as one of the nonparticipation rate. Therefore, we find that the ante-match model accounts for only 10% of the actual volatility of the unemployment rate.

## Table 4   Simulation Results

### (1) Original Variables

| | U.S. Data | Ante-Match Model | Post-Match Model |
|---|---|---|---|
| Standard Deviations | | | |
| $Std(E)$ (%) | 1.90 | 3.34 | 0.58 |
| $Std(U)$ / $Std(E)$ | 0.46 | 0.11 | 0.44 |
| $Std(O)$ / $Std(E)$ | 0.65 | 0.89 | 0.62 |
| $Std(u)$ / $Std(E)$ | 0.74 | 0.42 | 0.72 |
| $Std(V)$ / $Std(E)$ | | 0.09 | 0.48 |
| Correlations | | | |
| $Corr(E, U)$ | −0.86 | −1.00 | −0.92 |
| $Corr(E, O)$ | −0.93 | −1.00 | −0.96 |
| $Corr(U, O)$ | 0.61 | 1.00 | 0.78 |
| $Corr(U, V)$ | | 0.03 | −0.83 |

### (2) Detrended Variables

| | U.S. Data | Ante-Match Model | Post-Match Model |
|---|---|---|---|
| Standard Deviations | | | |
| $Std(E)$ (%) | 1.21 | 3.76 | 0.55 |
| $Std(U)$ / $Std(E)$ | 10.75 | 0.93 | 7.85 |
| $Std(O)$ / $Std(E)$ | 0.60 | 0.93 | 1.06 |
| $Std(u)$ / $Std(E)$ | 10.94 | 1.71 | 8.21 |
| $Std(V)$ / $Std(E)$ | | 2.60 | 10.60 |
| Correlations | | | |
| $Corr(E, U)$ | −0.96 | −0.98 | −0.88 |
| $Corr(E, O)$ | −0.78 | −0.98 | −0.91 |
| $Corr(U, O)$ | 0.62 | 1.00 | 0.61 |
| $Corr(U, V)$ | −0.91 | −0.02 | −0.73 |

Notes: Averages over 100 simulations of length 336 months.   $u$ denotes the unemployment rate.   Cyclical variables are obtained by an HP filter with smoothing parameter 900,000.   The seasonally adjusted monthly help-wanted advertising index $V$ is constructed by the Conference Board.

The correlations of the employment-population ratio with both the unemployment-population ratio and the nonparticipation rate are –0.98 and the correlation between the unemployment-population ratio and the nonparticipation rate is 1. The ante-match model fails to predict a strong negative relationship between the unemployment-population ratio and the vacancy rate with correlation of –0.04.

To better understand the above counterfactual results, it is instructive to investigate the system of equations for the reduced-form dynamics derived from the ante-match model and its classification. Time $t$ matched workers, denoted by $M_t$, and unmatched workers, denoted by $N_t$, are given by

$$M_t = (1-\lambda)E_{t-1} + p_{t-1}U_{t-1} + f_{t-1}O_{t-1}, \tag{24}$$

$$N_t = \lambda E_{t-1} + (1-p_{t-1})U_{t-1} + (1-f_{t-1})O_{t-1}. \tag{25}$$

Time $t$ employment-population ratio, unemployment-population ratio and nonparticipation rate can be expressed in terms of time $t$ matched workers and unmatched workers:

$$E_t = (1-\eta)M_t, \tag{26}$$

$$U_t = \mu(\eta M_t + N_t), \tag{27}$$

$$O_t = (1-\mu)(\eta M_t + N_t). \tag{28}$$

Note that the nonparticipation rate $O_t$ depends on the job-finding probabilities, $p_{t-1}$ and $f_{t-1}$. Using that $M_t + N_t = 1$, we have the law of motion for the unemployment-population ratio expressed in terms of $E_t$:

$$U_t = \mu(1-E_t), \tag{29}$$

which implies that $Corr(E_t, U_t) = -1$ and $Std(U_t) = \mu Std(E_t)$. The relative standard deviation of the unemployment-population ratio, expressed as a ratio to the standard deviation of the employment-population ratio, is simply $\mu$. We also find the relationship between the unemployment-population ratio and the nonparticipation rate:

$$O_t = \frac{1-\mu}{\mu} U_t. \tag{30}$$

which implies that $Corr(O_t, U_t) = 1$ and $Std(O_t) = ((1-\mu)/\mu)) Std(U_t)$. The relative standard deviation of the nonparticipation rate, expressed as a ratio to the standard deviation of the employment-population ratio, is $1-\mu$.

Suppose, for example, that in a steady state the unemployment-population ratio is 4% and the nonparticipation rate 36%. The probability of an active job search, $\mu$, is given by 0.1 (4% divided by 40%). Then, the relative standard deviation of the unemployment-population ratio is 0.1 and the relative standard deviation of the nonparticipation rate is 0.9, respectively. Therefore, the model predicts that the nonparticipation rate has nine times more variations than unemployment-population ratio does.

### 3.2.2. Simulation results of the post-match model

I return to the post-match model in which nonparticipation is not searching and unemployment captures both those who search but do not find employment and those who work but separate. Table 4 (Post-Match Model) shows the moments of the model-generated data. The post-match model gives better results than the ante-match model does. For the detrended variables, the relative volatilities of the unemployment-population ratio and the nonparticipation rate are 7.85 and 1.06, respectively. Compared with the ante-match model, the relative volatility of the unemployment-population ratio increases about five times, and the relative volatility of the vacancy rate increases fourfold. The post-match model predicts a much more volatile unemployment rate than the ante-match model does, so that it accounts for

three-quarters of the actual volatility of the unemployment rate.

The correlation between the unemployment-population ratio and the nonparticipation rate (0.61) is very close to the actual number (0.62). Moreover the Beveridge relationship between the unemployment-population ratio and the vacancy rate is much stronger in the post-match model than in the ante-match model, and is also very close to the actual data. In spite of all the advantages, however, the post-match model predicts that the nonparticipation rate is more highly negatively correlated with the employment-population ratio than the data shows.

# 4. CONCLUDING REMARKS

This paper proposes a matching model in which workers are classified after matches take place and the distinction between unemployment and nonparticipation becomes clear. That is, those who do not engage in job search are classified as out of the labor force, and those who search and find no employment are classified as unemployed, consistent with the definition of the U.S. Current Population Survey.

As a result, the model generates the direct transition from nonparticipation to employment with no assumption that nonparticipation is inactive search and without adjusting the time period of the model. In addition, some parameters which govern the model are easily identified by the reduced-form dynamics derived from the model.

The model accounts for the U.S. labor market much better than the existing models with unemployment as active search and nonparticipation as inactive search do. It explains three-quarters of the actual volatility of the unemployment rate, realistic contemporaneous correlations between the labor market variables, and the Beveridge relationship.

# APPENDIX

## A1. Recursive Equilibrium of the Ante-Match Model

Let $V^w(z, \varphi)$ denote the value function of a worker who works, $V^S(z, \varphi)$ the value function of a worker who searches actively, $V^O(z, \varphi)$ the value function of a worker who searches inactively, $V^m(z, \varphi)$ the value function of a matched worker, and $V^n(z, \varphi)$ the value function of an unmatched worker.

The value function of a worker who works (or an employed worker) is given by

$$V^w(z, \varphi) = w(z, \varphi)$$
$$+ \beta\{(1-\lambda)E[V^m(z', \varphi')] + \lambda E[V^n(z', \varphi')]\}, \tag{A1}$$

where $w(z, \varphi)$ is a Nash bargaining wage and $\lambda$ the exogenous separation rate.   An employed worker earns wages in the current period, and in the subsequent period if the match survives with probability $(1-\lambda)$, then the worker becomes matched.   If, however, the match is dissolved exogenously with probability $\lambda$, the worker becomes unmatched.

The value functions of a matched worker and of an unmatched worker are given as follows

$$V^m(z, \varphi) = (1-\eta)V^w(z, \varphi) + \eta V^o(z, \varphi), \tag{A2}$$

$$V^n(z, \varphi) = \eta V^s(z, \varphi) + (1-\mu)V^o(z, \varphi). \tag{A3}$$

An unmatched worker who has no employment opportunities is to search actively or inactively, depending on probability $\mu$.   A matched worker who has employment opportunities is to work on his current job or not, depending on probability $\eta$.

The value function of a worker who becomes an active job-searcher (or an unemployed worker) is given by

$$
\begin{aligned}
V^s(z, \varphi) = b + \beta\{ p(z, \varphi)E[V^m(z', \varphi')] \\
+ (1 - p(z, \varphi))E[V^n(z', \varphi')]\},
\end{aligned}
\tag{A4}
$$

where $p(z, \varphi)$ is the job-finding probability. A worker who looks for work actively receives unemployment insurance benefits $b$ in the current period, and in the subsequent period finds a job with probability $p(z, \varphi)$. If a worker finds a job, then (s)he becomes matched. Otherwise, (s)he becomes unmatched.

The value function of a worker who becomes an inactive job-searcher (or a nonparticipant) is given by

$$
\begin{aligned}
V^o(z, \varphi) = \beta\{ f(z, \varphi)E[V^m(z', \varphi')] \\
+ (1 - f(z, \varphi))E[V^n(z', \varphi')]\},
\end{aligned}
\tag{A5}
$$

where $f(z, \varphi)$ is the job-finding probability given by $xp(z, \varphi)$, where $x$ captures the relative search intensity. A worker who looks for work inactively finds a job with probability $f(z, \varphi)$. If a worker finds a job, then (s)he becomes matched. Otherwise, (s)he becomes unmatched.

Finally, the evolution of the aggregate state is described by the function $\Gamma(z, \varphi)$, where for each $(z, \varphi)$ this function specifies a distribution over the next period's values of the state variables.

The recursive equilibrium is a list of value functions, job- and worker-finding probabilities and wages such that: (a) Taking the probabilities and the wages as given, workers and firms solve their value functions (A1)-(A5) and (15); (b) The free-entry condition (16) is satisfied; (c) Wages are determined by Nash bargaining (18), and; (d) For each $(z, \varphi)$, probabilistic decisions generate a distribution over the next period's state that is consistent with the distribution given by $\Gamma(z, \varphi)$.

### A2. Calibration for the Ante-Match Model

The steady-state probability that an unemployed worker finds a job, $p^*$, is set to the *UE* transition rate, 0.22, and the steady-state probability that a nonparticipant finds a job, $f^*$, is set to the *OE* transition rate, 0.028. The relative search intensity is then given by $x = f^* / p^*$. The probabilities of a worker being laid off and of an unmatched worker searching for a job actively are calculated from the steady-state condition of the reduced-form labor market dynamics:

$$\lambda = \frac{p^* U^* + f^* O^*}{E^*}, \tag{A6}$$

$$\mu = \frac{U^*}{U^* + O^*}. \tag{A7}$$

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