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X-inefficiency and Privatization in a Mixed Duopoly*

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The purpose of this study is two-fold: First, we investigate how X-inefficiency affects the behaviors of firms in a mixed oligopoly market. Second, we explore whether optimal subsidies can restore the social optimum like the previous literature on subsidization, with assuming that the efficiency of a public firm depends on the degree of privatization. The main results are as follows. First, despite higher marginal cost due to X-inefficiency, the price of a public firm tends to be lower than that of a private firm, when the degree of nationalization of the public firm is low enough. Second, if the marginal effect of nationalization on X-inefficiency measured at full privatization is relatively low and the elasticity of X-inefficiency measured at full nationalization is greater than the critical level, then the partial privatization is the optimal strategy from the social welfare viewpoint. Third, when an efficiency gap exists between firms, asymmetric subsidization can restore the social optimum, and the optimal subsidization level is not directly related to the degree of nationalization, but is affected by the efficiency gap. In addition, the subsidy for a private firm is larger than that for a public firm, which is relatively inefficient compared to the private firm.

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1. INTRODUCTION

Some researches on the mixed oligopoly model have proven that welfare maximization subsidies could restore the social optimum, regardless of the public firm's stance. These results, the so-called 'irrespective results', considered the stance as four cases, i.e., the nationalized public firm moves simultaneously with the private firms, or moves as a Stackelberg leader, and the privatized public firm simultaneously moves with private firms, or moves as a Stackelberg leader (see White, 1996; Poyago-Theptoky, 2001; Myles, 2002; Zikos, 2007). While Fjell and Heywood (2004) argue that welfare with subsidization will be reduced when a public firm is privatized and becomes a private leader, Zikos (2007) investigates the case of asymmetric subsidies between public firm and private firms, and insists that a firm's specific subsidies could restore first best allocation, though the public firm becomes the private leader.

On the other hand, the wide belief in privatization is that public firms cause X-inefficiency, and privatization could mitigate this inefficiency. Some empirical researches point out the inefficiency of public firms, for instance, Megginson and Netter (2001). Glazer and Niskanen (1997) argue that governmental facilities are often small, and of poor quality. Ishibashi and Kaneko (2008) point out the quality of services by a public firm is lower than that of private firms, and raise some examples, such as public hospitals, and express-delivery services in the United States. Moreover, Wang *et al.* (2009) and Wang and Chen (2011) take into account the efficiency gap in the mixed oligopoly model.

Nishimori and Ogawa (2002) and Cato (2008) insist that privatization can expect two effects: one is an improvement in allocation efficiency, the other is an improvement of productive efficiency in the public sector. Though productive efficiency is a significant issue when considering privatization, it may be not sufficient, which introduces this issue in the mixed oligopoly model.[1)]

[1)] Haskel and Sanchis (1995) handle privatization and X-inefficiency through a bargaining approach.

If privatization improves the productive efficiency of the public firm, it should be included in the model of privatization. In this paper, we assume that the fully nationalized public firm has an inefficient cost structure, compared with private firms. We also assume that the productive efficiency is improved according to the degree of privatization; and that when the firm is fully privatized, the inefficiency problem is solved.

Furthermore, the previous literature on subsidization does not pay attention to the possibility of partial privatization. Since De Fraja and Delbono (1989), a lot of studies have pointed out that partial privatization might be socially optimal, if some condition are satisfied, e.g., Matsumura (1998). Therefore, we focus in this paper on these two points when considering privatization and optimal subsidies.

The purpose of this study is two-fold: First, we investigate how X-inefficiency affects the behavior of firms in a mixed oligopoly market. Second, we explore whether optimal subsidies can restore the social optimum like the previous literature on subsidization, with assuming that the efficiency of a public firm depends on the degree of privatization. Also, in contrast to the previous literature, we assume price competition rather than quantity competition. Matsumura and Ogawa (2012) investigate the choice of a strategic variable (price or quantity) for a mixed duopoly market and find that firms choose the price contract in both substitutable and complementary goods markets. This result indicates that the Bertrand model should be used more in mixed oligopolies. Choi (2012) compares both price and quantity competition in a mixed oligopoly market, and shows that in the real world, the price by the public sector is lower than that by the private sector. Generally, it could be considered that the goods provided in the mixed oligopoly market have the property of public goods and we have more concern about the price rather than the quantity in these kinds of goods.

The rest of this paper is organized as follows. The basic model of our research is formulated in section 2. In section 3, we investigate the market equilibrium without subsidization to see how inefficiency affects a mixed duopoly market. Section 4 explores whether the government’s subsidies restore social optimum, or not. Section 5 concludes the paper.

2. THE MODEL

Suppose an economy that has two firms; one public firm (firm 0) and one private firm (firm 1). The two firms supply imperfectly substitutable goods, and all of these goods are consumed in the economy. The preference of a representative consumer is represented by a quasi-linear utility function as

$$U[q_0,\ q_1,\ x] = u[q_0,\ q_1] + x, \tag{1}$$

where q_0, q_1 and x are the consumption of good 0, good 1, and the numeraire good, respectively. For convenience, we assume that $u[q_0,\ q_1]$ takes the functional form

$$u[q_0,\ q_1] = a(q_0 + q_1) - \frac{q_0^2 + 2bq_0q_1 + q_1^2}{2},\ \ a,\ b > 0;\ b < 1, \tag{2}$$

where a refers to the potential market size, and b is the parameter measuring product substitutability. If the value of b is positive, zero, or negative, then the goods are respectively substitute, independent, or complements.[2)] The representative consumer will maximize equation (1) subject to the following budget constraint $\sum_{i=0,\ 1} p_i q_i + x \le I$, where p_i is the price of good i (=0, 1), and I is income. From the utility maximization, we obtain the following demand function for good i:

$$q_i[p_0,\ p_1] = \alpha - \beta p_i + \gamma p_j,\ \ i,\ j = 0,\ 1;\ i \ne j, \tag{3}$$

where $\alpha = a/(1+b)$, $\beta = 1/(1-b^2)$, $\gamma = b/(1-b^2)$.

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2) Note that $b<1$; because if $b=1$, the two goods are perfect substitutes, and the demand function becomes $p_i = a - (q_0 + q_1)$.

As is well-known, this demand function has some useful properties. First, since there is no income effect, the traditional methods of measuring consumer surplus (i.e., above the price and below the demand curve) are still valid, and second, the total consumer surplus from consuming q_0 and q_1 at prices p_0 and p_1 can be written as

$$\begin{aligned} CS[p_0,\ p_1] &= u[q_0,\ q_1] - \sum p_i q_i \\ &= a(q_0+q_1) - \frac{q_0^2 + 2bq_0q_1 + q_1^2}{2} - p_0q_0 - p_1q_1. \end{aligned} \tag{4}$$

We assume that the production cost of each firm is marginally increasing, and the technology levels of the firms are asymmetric, that is, the marginal cost of the public firm is higher than that of the private firm.[3] This means that the public firm produces the good with less efficiency than the private firm. The efficiency gap is lessened according to the degree of privatization. For instance, if the government chooses full privatization, this gap vanishes, so that the cost functions between the two firms become identical. Accordingly, we can define the cost functions of firm i (=0, 1) as $c_1(q_1) = 1/2q_1{}^2$ and $c_0(q_0) = 1/2\{1+\mu(\theta)\}q_0{}^2$, where θ represents the degree of nationalization; i.e., $\theta = 0$ means full privatization and $\theta = 1$ means full nationalization, respectively. Though Wang *et al*. (2009) and Wang and Chen (2011) only consider the extreme cases (μ is an exogenous variable and θ is zero or one), we assume $\theta \in [0,\ 1]$ to incorporate the possibility of partial privatization.

Here, $\mu(\theta)$ indicates the efficiency gap in the production technology between the public and private firms and depends on θ. We consider that the higher the degree of privatization, the more efficient the public firm becomes so that we assume that $\mu(0) = 0,\ \mu' > 0$ and $\mu'' \geq 0$[4] (hereafter,

[3] The assumption of increasing marginal cost is standard in the mixed oligopoly model. See Zikos (2007) and Wang *et al*. (2009).

[4] Wang *et al*. (2009) and Wang and Chen (2011) assume the inefficiency as an exogenous variable which does not depend on the degree of privatization; and Nishimori and Ogawa (2002) and Cato (2008) introduce the investment function for the improvement of

$\mu(\theta) \equiv \mu$). In addition, we focus on the case where two goods are substitute and assume $b = 1/2$ without loss of generality.[5] The profit function of firm i, π_i, can be represented by

$$\pi_0[p_0,\ p_1;\ \mu] = p_0 q_0 - \tfrac{1}{2}(1+\mu)q_0{}^2, \tag{5.1}$$

$$\pi_1[p_0,\ p_1] = p_1 q_1 - \tfrac{1}{2} q_1{}^2. \tag{5.2}$$

From equations (3) and (4), the consumer surplus (CS) is given by

$$CS[p_0,\ p_1] = \frac{2}{3}\left\{a^2 + p_0{}^2 - p_0 p_1 + p_1{}^2 - a(p_0 + p_1)\right\}.$$

At first, we examine the optimal allocation as a benchmark. The social welfare can be defined as the sum of consumer surplus and profits of firms

$$W[p_0,\ p_1;\ \mu] = CS[p_0,\ p_1] + \pi_0 + \pi_1. \tag{6}$$

From the first order condition of welfare maximization, the optimal price of each good and the price gap between the goods are given by

$$p_0^*[\mu] = \frac{6a(1+\mu)}{15+8\mu},\ \ p_1^*[\mu] = \frac{2a(3+2\mu)}{15+8\mu},\ \ p_0^*[\mu] - p_1^*[\mu] = \frac{4a\mu}{15+8\mu} \geq 0. \tag{7}$$

Substituting equations (7) into (3) yields the optimal output of good i and the output gap between the goods

$$q_0^*[\mu] = \frac{6a}{15+8\mu},\ \ q_1^*[\mu] = \frac{2a(3+2\mu)}{15+8\mu},\ \ q_0^*[\mu] - q_1^*[\mu] = \frac{-4a\mu}{15+8\mu} \leq 0. \tag{8}$$

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inefficiency.

5) The value of b does not affect our main results.

Here, it is noteworthy that the optimal price level of good i (=0, 1) equals its marginal cost, that is, $p_0^*[\mu]=(1+\mu)q_0^*[\mu]$ and $p_1^*[\mu]=q_1^*[\mu]$ hold. Equations (7) and (8) indicate that it is socially desirable that the public firm produces less and charges a higher price than the output level and price of the private firm, i.e., $p_0^* \geq p_1^*$ and $q_0^* \leq q_1^*$. Moreover, through comparative statics we can confirm the following.

$$\frac{dp_0^*}{d\mu}>0,\quad \frac{dp_1^*}{d\mu}>0,\quad \frac{dq_0^*}{d\mu}<0,\quad \frac{dq_1^*}{d\mu}>0,\quad \frac{d\pi_0^*}{d\mu}<0,\quad \frac{d\pi_1^*}{d\mu}>0.$$

Equation (8) indicates that the output gap between firm 0 and firm 1 depends on the efficiency gap, μ. If μ becomes high (i.e., the less efficiency of firm 0), it is socially desirable that the private firm, which has relatively efficient technology, expands its output while the public firm reduces its output. Therefore, the production gap between firms expands as μ increases, i.e., $d\left|q_0^*-q_1^*\right|/d\mu>0$. The total output of firms decreases as μ increases, i.e., $d(q_0^*+q_1^*)/d\mu<0$, implying that a decrease in the output of firm 0 dominates the increase in the output of firm 1. An increase in q_1^* results in the rise of p_1^*, because the socially optimal output charges the price equal to the marginal cost.

When μ changes, the price of public firm, p_0^*, is affected by the two channels. To look at this, note that the marginal production cost of good 0 equals $(1+\mu)q_0^*$. An increase in μ implies higher inefficiency level of the public firm, which results in an increase in the price level of good 0, if the output of good 0 is equal (X-inefficiency effects). However, an increase in μ implies bringing about a decrease in the output of good 0 (i.e., $dq_0^*/d\mu<0$), which has the effect of reducing the socially optimal price level of good 0(output effects). Because the former dominates the latter, the price of the public firm, p_0^*, rises as μ increases.

The consumer surplus and social welfare under optimal allocation are given by

$$CS^*[\mu] = \frac{2a^2(27+18\mu+4\mu^2)}{(15+8\mu)^2}, \tag{9}$$

$$W^*[\mu] = \frac{2a^2(3+\mu)}{15+8\mu}. \tag{10}$$

Because the total output of firm 0 and firm 1 decreases as μ increases, the consumer surplus also decreases, i.e., $dCS^*/d\mu < 0$. In addition, from equation (10) we can confirm that $dW^*/d\mu < 0.$ This implies that social welfare becomes the highest when $\mu = 0.$ Because μ is an increasing function in θ and is zero when $\theta = 0,$ the above argument implies that full privatization is socially preferable.

3. MARKET EQUILIBRIUM

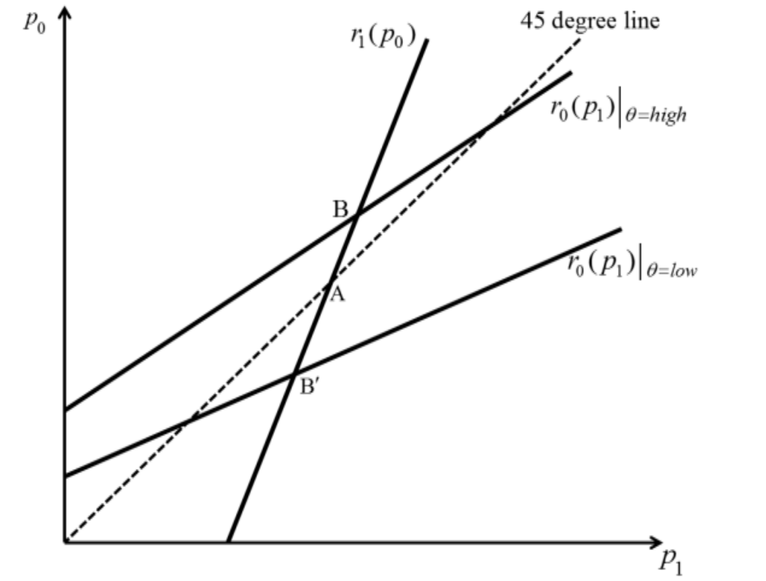
In this section, we investigate market equilibrium to see how the efficiency gap affects the market. As in the previous literature on mixed oligopoly, the public firm decides its output level to maximize the weighted sum of social welfare and its profits. The object function of the public firm is formally defined as

$$V = \theta W + (1-\theta)\pi_0, \tag{11}$$

where the profits of firm 0, π_0, and the social welfare, W, are defined in the previous section. Note that if $\theta = 0$ (i.e., fully privatized case), then the firm maximizes its own profits; while if $\theta = 1$ (i.e., fully nationalized case), then the firm maximizes social welfare.

Here, we consider a two-stage game. In the first stage, the government chooses the level of privatization, $\theta,$ to maximize the social welfare W of equation (6); and in the second stage, the two firms engage in Bertrand competition in the product market. By following the concept of backward

Figure 1 Price Determination in Mixed Duopoly



induction, we solve the equilibrium from the second stage. From the object functions, the reaction function of firm $i(=0,\ 1)$, r_i, is given as

$$p_0 = r_0[\mu,\ \theta;\ p_1] = \frac{a(7-5\theta+4\mu)}{20-4\theta+8\mu} + \frac{7+4\theta+4\mu}{20-4\theta+8\mu}p_1, \tag{11.1}$$

$$p_1 = r_1(p_0) = \frac{7}{20}a + \frac{7}{20}p_0. \tag{11.2}$$

Figure 1 depicts the reaction curves of both firms on the price plane. From equations (11.1) and (11.2), we can confirm that if $\theta = 0$ (thus, $\mu = 0$), then both reaction curves become symmetric and the equilibrium is determined at point A on the 45 degree line. If $\theta > 0$ (thus, $\mu > 0$), then the slope of r_0 is steeper than that of r_1, and the reaction functions are no longer symmetric. If θ is relatively high, the equilibrium is determined at point B so that $p_0^N > p_1^N$. On the other hand, if θ is relatively low, the equilibrium is determined at point B' so that $p_0^N < p_1^N$. Here, the superscript 'N' denotes the market equilibrium.

Solving equations (11.1) and (11.2) yields the following equilibrium prices

$$p_0^N[\mu,\ \theta] = \frac{3a(21-8\theta+12\mu)}{117-36\theta+44\mu},\quad p_1^N[\mu,\ \theta] = \frac{7a(9-3\theta+4\mu)}{117-36\theta+44\mu}. \qquad (12)$$

By substituting equations (12) into (3), we can obtain the equilibrium outputs of both firms as follows

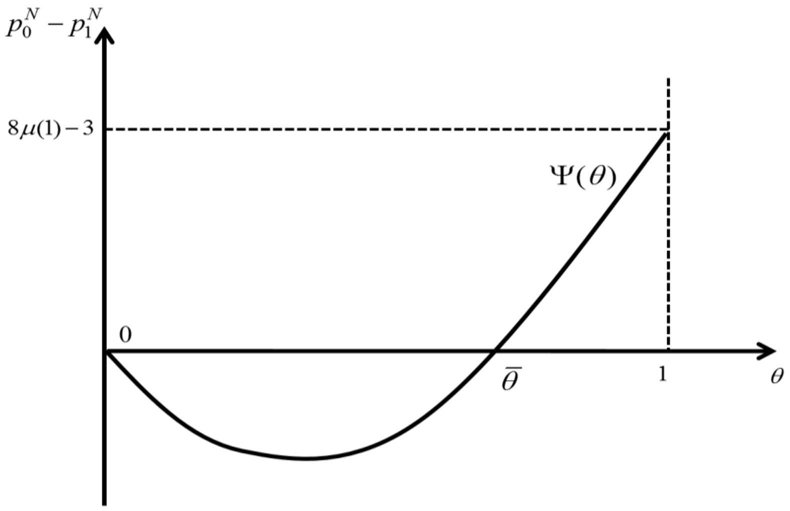
$$q_0^N[\mu,\ \theta] = \frac{6a(6-\theta)}{117-36\theta+44\mu},\quad q_1^N[\mu,\ \theta] = \frac{4a(9-3\theta+4\mu)}{117-36\theta+44\mu}. \qquad (13)$$

From equation (12) and (13), we can find that $p_0^N - p_1^N = -(1/2)(q_0^N - q_1^N) = a(-3\theta+8\mu)/117-36\theta+44\mu.$ That is, the sign of $p_0^N - p_1^N$ and $q_0^N - q_1^N$ depends on the magnitude of $8\mu - 3\theta,$ where $\mu = \mu(\theta).$ Letting $\Psi(\theta) \equiv 8\mu(\theta) - 3\theta,$ then we have $\Psi(0) = 0,$ $\Psi(1) = 8\mu(1) - 3,$ and $\Psi'(0) = 8\mu'(0) - 3$ from the assumption. If $\mu(1) > 3/8$ and $\mu'(0) < 3/8,$ then the function $\Psi(\theta)$ takes the shape shown in figure 2. Accordingly, there exists a unique value $\bar{\theta}$ in the domain of $\theta \in (0,\ 1]$ that satisfies $\Psi(\theta)\left[\equiv 8\mu(\theta) - 3\theta\right] = 0.$ If the degree of nationalization (θ) is relatively low (high), that is, $\theta \in [0,\ \bar{\theta}]$ ($\theta \in (\bar{\theta},\ 1]),$ $p_0^N \le p_1^N\ (p_0^N > p_1^N)$ and $q_1^N \le p_0^N\ (q_1^N > p_0^N)$ hold.

The intuitive explanation is as follows. Basically the price gap, $p_0^N - p_1^N$, depends on two factors: the efficiency gap (μ) and the degree of nationalization (θ).

Suppose that the efficiency gap between firms is exogenously given (i.e., μ is independent from θ) and is so small so as to be negligible, so that the gap is affected mainly by the degree of nationalization. In this case, the price of the private firm's good is higher than that of the public firm's good, because the public firm maximizes the weighted average of social welfare

Figure 2 The Price Gap between Goods



and its own profits, while the private maximizes only the profits. This price gap extends proportionally as θ increases. However, μ is not independent from θ. The efficiency gap, μ, is a positive function of θ and increases more than proportionally with θ. An increase in μ raises the price of good 0 via an increase in the marginal production cost of firm 0.

Thus, if θ is relatively low (i.e., $\theta \in [0, \bar{\theta}]$), then the former effects dominate the latter and the price of the private firm is higher than that of the public firm (i.e., $p_0^N \le p_1^N$). However, note that μ increases more than proportionally with θ. Consequently, if θ is sufficiently high (i.e., $\theta \in (\bar{\theta}, 1]$), then the latter effects dominate the former, and the price of the private firm is lower than that of the private firm (i.e., $q_1^N > p_0^N$).

Using equations (7), (8), (12), and (13), we can compare the market equilibriums with optimal allocations. Since the market is under duopolistic competition, a positive level of price-marginal cost mark-up exists, i.e., $p_i^N > MC$. As a result, both firms produce less with higher prices in the duopolistic competition than the quantities and prices in the optimal allocation ($p_0^N \ge p_0^*$, $p_1^N \ge p_1^*$, $q_0^N \le q_0^*$, $q_1^N \le q_1^*$).

Substituting equations (12) and (13) into (3), the equilibrium welfare is given as follows.

$$W^N[\mu(\theta),\ \theta] = \frac{2a^2\left\{9\theta^2(24-\mu)-9\theta(171+40\mu)+4(648+369\mu+56\mu^2)\right\}}{\left(117-36\theta+44\mu\right)^2}. \tag{14}$$

From equation (14), the derivative of W^N with respect to θ is obtained as follows:

$$\frac{dW^N}{d\theta} = \frac{\partial W^N}{\partial \mu}\frac{d\mu}{d\theta} + \frac{\partial W^N}{\partial \theta}, \tag{15.1}$$

where

$$\frac{\partial W^N}{\partial \theta} = \frac{18a^2(9-4\mu)\left\{81-60\theta+2\mu(11\theta-4)\right\}}{\left(117-36\theta+44\mu\right)^3}, \tag{15.2}$$

$$\frac{\partial W^N}{\partial \mu} = \frac{-18a^2(6-\theta)\left\{1026+36\theta^2+232\mu-\theta(573-44\mu)\right\}}{\left(117-36\theta+44\mu\right)^3} < 0. \tag{15.3}$$

In the right hand side of equation (15.1), the sign of $\partial W^N/\partial\theta$ is ambiguous while $\partial W^N/\partial\mu < 0$. As in equation (15.1), nationalization affects social welfare via two channels: the direct effect of nationalization on the social welfare and the indirect effect via the change in the technological efficiency of the public firm. In order to see whether or not there is an incentive for partial privatization, we estimate equation (15.1) at $\theta = 0$.

$$\left[\frac{dW^N}{d\theta}\right]_{\theta=0} = \frac{2a^2}{2197}\left\{9-76\left(\frac{d\mu}{d\theta}\right)_{\theta=0}\right\} \le (>) 0 \iff \left(\frac{d\mu}{d\theta}\right)_{\theta=0} \ge (<) \frac{9}{76}. \tag{16}$$

If $(d\mu/d\theta)_{\theta=0} \geq 9/76$, then $\left[dW^N/d\theta\right]_{\theta=0} \leq 0$ holds, implying that full privatization improves social welfare in some neighborhood of $\theta = 0$. On the other hand, if $(d\mu/d\theta)_{\theta=0} < 9/76$, then $(dW^N/d\theta)_{\theta=0} > 0$ holds, implying that leaving from full privatization, i.e., partial privatization, is socially desirable.

Although the above argument holds locally in the neighborhood of $\theta = 0$. However, it suggests interesting insight because it explains the possibility of partial privatization through the improvement of inefficiency, $d\mu/d\theta$, while the previous literature tried to find the reasons for the partial privatization from the cost structure of the firms, e.g., Matsumura (1998), Fujiwara (2007).

When $(d\mu/d\theta)_{\theta=0} < 9/76$, how can we find the socially optimal level of privatization, θ^*, which maximizes the social welfare? Unfortunately, we cannot derive the optimal one in an explicit formula because of the complexity of the welfare function in our model. However, solving the inequality of $(dW^N/d\theta)_{\theta=1} < 0$ gives the sufficient condition for the partial privatization, i.e., $\theta^* \in (0,\ 1)$, since $(dW^N/d\theta)_{\theta=0} > 0$. To attain this end, we define the elasticity of X-inefficiency in the public firm as η; $\eta(\theta) = d\ln\mu/d\ln\theta$, $\eta(\theta) > 0$. Here, η implies the percentage increase in the inefficiency of production technology of the public firm over the percentage change in the magnitude of the nationalization. From equations (15.1), (15.2), and (15.3), we have

$$\left[\frac{dW^N}{d\theta}\right]_{\theta=1} \leq (>) 0 \quad \Leftrightarrow \quad \eta(1) \geq (<) \bar{\eta} \equiv \frac{7(9-4\mu(1))(3+2\mu(1))}{15\mu(1)(163+92\mu(1))}, \qquad (17)$$

where $\bar{\eta}$ is the critical value of η which satisfies $(dW^N/d\theta)_{\theta=1} = 0$. If η measured at $\theta = 1$ exceeds the critical level, leaving from full nationalization is socially desirable. By combining equation (17) with Proposition 1, the following proposition is obtained.

Proposition 1: *There exists a socially optimal level of privatization* θ^* *in*

(0, 1) *if* $0 \le \mu'(0) < \frac{9}{76}$ *and* $\eta(1) > \mathrm{Max}\left[\frac{7(9-4\mu(1))(3+2\mu(1))}{15\mu(1)(163+92\mu(1))},\ 0\right]$.

For example, let us look at the case of constant elasticity of X-inefficiency; i.e., $\mu = \alpha\theta^{\eta}$, where α is a parameter of the inefficiency gap and $\eta(>0)$ is a constant value. In this case, $(d\mu/d\theta)_{\theta=0} = 0 < 9/76$ and $\bar{\eta} = (7(9-4\alpha)(3+2\alpha))/(15\alpha(163+92\alpha))$. Therefore, if $\alpha < 9/4$ (resp. $\alpha \ge 9/4$), then partial privatization is socially desirable for $\eta > \bar{\eta}$ (resp. $\eta > 0$).

4. SUBSIDIZATION AND SOCIAL OPTIMUM

Equations (10) and (14) indicate that the effect of social welfare on the equilibrium is lower than that on the optimal allocation, i.e., $W^* > W^N$. This is quite natural because there is a market failure caused by imperfect competition. A lot of literature, such as Zikos (2007), points out that the optimal subsidization could restore the first best allocation.[6)] We investigate whether this proposition is still valid or not when we take into account the X-inefficiency.[7)]

As assumed earlier, a two-stage game is introduced. In the first stage, the government decides the level of subsidies, and in the second stage, the firms engage in price competition under the subsidies. Following the backward induction, given the subsidy level, we must obtain the market equilibrium in the second stage. That is, given the subsidy level by the government, firm 1 (firm 0) determines $p_1(p_0)$ to maximize $\pi_1(U)$. By solving the first order conditions, we now obtain the market equilibriums as follows:

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6) Lee and Lee (2013).

7) Even though we consider the price competition, a specific subsidization is introduced in our model, following the previous literature.

$$p_0^S[\mu(\theta),\ \theta;\ s_0,\ s_1] = \frac{3a(21-8\theta+12\mu)-40(1-\theta)s_0-2(7+4\theta+4\mu)s_1}{117-36\theta+44\mu}, \quad (18)$$

$$p_1^S[\mu(\theta),\ \theta;\ s_0,\ s_1] = \frac{7a(9-3\theta+4\mu)-14(1-\theta)s_0-4(5-\theta+2\mu)s_1}{117-36\theta+44\mu}, \quad (19)$$

$$q_0^S[\mu(\theta),\ \theta;\ s_0,\ s_1] = \frac{6a(6-\theta)+44(1-\theta)s_0-8(1-2\theta)s_1}{117-36\theta+44\mu}, \quad (20)$$

$$q_1^S[\mu(\theta),\ \theta;\ s_0,\ s_1] = \frac{4a(9-3\theta+4\mu)-8(1-\theta)s_0+4(11-4\theta+4\mu)s_1}{117-36\theta+44\mu}, \quad (21)$$

where $s_0(s_1)$ denotes the public (private) firm's output subsidy, and superscript '*s*' stands for the equilibrium with asymmetric subsidization. If $s_0 = s_1 = 0,$ we can easily confirm that $p_i^S = p_i^N$ and $q_i^S = q_i^N$ $(i = 0,\ 1).$

We now move to the first stage of the game. Equilibrium welfare is determined by inserting equations (18) to (21) into (6): $W^S[\mu(\theta),\ \theta;\ s_0,\ s_1]$ $= W[\mu;\ p_0^S(\mu(\theta),\ \theta;\ s_0,\ s_1),\ p_1^S(\mu(\theta),\ \theta;\ s_0,\ s_1)].$ Solving the welfare maximization problem for s_0 and s_1, i.e., $Max_{s_0,\ s_1} W^S[\mu(\theta),\ \theta;\ s_0,\ s_1],$ yields the following optimal production subsidies:

$$s_0[\mu] = \frac{9a}{30+16\mu},\ s_1[\mu] = \frac{3a(3+2\mu)}{30+16\mu}. \quad (22)$$

To look into whether the optimal subsidization could restore the first best allocation, we substitute equation (22) into the market equilibriums in equations (18) to (21), then we can obtain that

$$p_0^S[\mu,\ \theta;\ s_0(\mu),\ s_1(\mu)] = \frac{6a(1+\mu)}{15+8\mu} = p_0^*[\mu], \quad (23)$$

$$p_1^S[\mu,\ \theta;\ s_0(\mu),\ s_1(\mu)] = \frac{2a(3+2\mu)}{15+8\mu} = p_1^*[\mu], \quad (24)$$

$$q_0^S[\mu,\ \theta;\ s_0(\mu),\ s_1(\mu)] = \frac{6a}{15+8\mu} = q_0^*[\mu], \tag{25}$$

$$q_1^S[\mu,\ \theta;\ s_0(\mu),\ s_1(\mu)] = \frac{2a(3+2\mu)}{15+8\mu} = q_1^*[\mu]. \tag{26}$$

We can confirm from the above equations that equilibrium prices and outputs with optimized subsidy coincide with those in the social optimum. This means that asymmetric subsidization can restore the social optimum, and the proposition of Zikos (2007) holds, irrespective of the degree of privatization and X-inefficiency. One more interesting finding is that the optimal subsidies are not directly related to the degree of nationalization, θ, but are affected by the change in efficiency gap, μ. Equation (22) indicate this fact.

In addition, we can confirm that $\partial s_0/\partial\mu < 0$ and $\partial s_1/\partial\mu > 0$. The larger the efficiency gap between the private and public firm, the lower the optimal production subsidy for the public firm, and the higher for the private firm that has more efficient technology in production. This result suggests the reason why symmetric subsidization could not restore the social optimum.

We can also confirm the following relation.

$$s_1 - s_0 = \frac{3a\mu}{15+8\mu} > 0,\ \ \frac{d(s_1-s_0)}{d\mu} > 0.$$

This shows that the government should subsidize the private firm more, rather than the public firm for welfare maximization. Furthermore the subsidization gap increases with μ.

Proposition 2: *Suppose that the public firm is less efficient than the private firm and that the government provides output subsidies. (1) Asymmetric subsidization could restore the social optimum. (2) The optimal subsidization level is not directly related to the degree of nationalization but is affected via the efficiency gap. (3) The optimal output* *subsidy for the private firm is larger than that for the public firm. The larger the efficiency gap between the private and public firm, the lower the optimal production subsidy for the public firm, and the higher for the private firm that has more efficient technology in production.*

5. CONCLUDING REMARKS

A large literature points out the production inefficiency of public firm, and also raises this issue as a main reason when considering privatization. The motivation of this paper is to study the privatization issue with explicitly introducing X-inefficiency in the model explicitly.

The main results are as follows. First, without output subsidy by the government, the price gap between the goods depends on two factors: efficiency gap, μ, and the degree of nationalization of the public firm, θ. If θ is relatively low, then the price of the public firm's good tends to be lower than that of the private firm's good. However, if θ is high enough, then the price of the public firm's good tends to be higher than that of the private firm's good. This is because the marginal production costs of the public firm increases more than proportionally, as θ increases. Second, in determining whether the partial privatization is socially desirable, the following two kinds of information such as the marginal effect of nationalization on X-inefficiency measured at $\theta = 0$ and the elasticity of X-inefficiency in the public firm measured at $\theta = 1$ matter. If the marginal effect is relatively low (i.e., $(d\mu/d\theta)_{\theta=0} < 9/76$) and the elasticity of X-inefficiency measured at $\theta = 1$ is greater than the critical level (i.e., $\eta > \overline{\eta}$), then the partial privatization is the optimal strategy from the social welfare viewpoint. Third, when an efficiency gap exists between firms, asymmetric subsidization could restore the social optimum. And the optimal subsidization level is not directly related to the degree of nationalization but is affected by the efficiency gap. In addition, the subsidy for the private firm is larger than that for the public firm, which is relatively inefficient, compared to the private firm.

Even though this result is quite natural so that many governments raise this point as a main reason when they take into account privatization of their public firm, there are few, however, who have handled this issue theoretically, until now. In this respect, our results suggest some important insights into the privatization policy by governments. They imply that when considering privatization, government should consider the improvement of inefficiency by privatization (Proposition 1). While Lee and Lee (2013) introduce this factor into the model, it focuses on the optimal subsidization so it is not sufficient to explain the relationship between efficiency gap and privatization.

In this paper, we handled the issue of privatization in the context of industrial organization. However, when the policy makers take into account this issue, they should consider this from not only the viewpoint of industrial organization but also that of public finance. Actually IMF recommended small government as a measure reform in the economic crisis of Korea in 1997. Following this recommendation, reduction in the number and size of the government organizations and personnel, and privatization of SOE were taken in Korea. Yoo (2008) summarized this situation in detail. In addition, the assumption about efficiency gap, μ, is very important in our paper because the main results are affected by this factor. This means that when we consider the privatization problem, more plausible definition of this factor is crucial. It will be a meaningful work to estimate the marginal efficiency gap empirically. This remains for future research.

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