

## **The Effects of Strategic Delegation on Price Competition with Product Compatibility and Network Externalities\***

Guangsug Hahn\*\* · Joon Yeop Kwon\*\*\* · Dong Chul Won\*\*\*\*

The paper analyzes the effects of strategic delegation on price competition in duopoly when product compatibility reinforces network externality. We extend Hoernig (2012) to the case where product compatibility is built into network effects. Product compatibility plays a role of enlarging the network size of a firm through spillover effects. Due to these effects, firm owners require managers to be more aggressive than profit maximizers when the degree of network externality lies in a lower range than that of Hoernig (2012).

JEL Classification: D43, L20

Keywords: price competition, strategic delegation, network externality, product compatibility

---

\* Received January 5, 2015. Revised March 3, 2015. Accepted March 26, 2015. This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2010-332-B00032).

\*\* Division of Humanities and Social Sciences, POSTECH, E-mail: econhahn@postech.ac.kr

\*\*\* Author for Correspondence, Graduate Program for Technology and Innovation Management, POSTECH, E-mail: jykwon@postech.ac.kr

\*\*\*\* School of Business Administration, Ajou Univeristy, E-mail: dcwon@ajou.ac.kr

## 1. INTRODUCTION

An important feature of modern corporations is the separation of ownership and management. While firm owners employ managers and want them to manage their firms in the owners' benefit, the managers' own best interests may differ from the owners' best interests. Owners can provide their managers incentive contracts to solve the agency problem due to the conflicts of interests between owners and managers.

Although standard economic theory presumes that the goal of firms is profit maximization, it is often observed that large corporations strive for maximizing their revenue or market share. The literature on strategic delegation such as Fershtman and Judd (1987) and Sklivas (1987) demonstrates that the owners of firms can make more profits by strategic delegation under imperfect competition, i.e., by offering their managers incentive contracts to achieve a specific goal other than profit maximization. Strategic delegation is typically modeled by the weights on profits and revenues. These weights are determined such that they bring more profits to owners than profit maximization. In particular, a higher weight on profit can induce the manager to act less aggressively than in the case of profit maximization by setting a higher price in equilibrium under price competition.

As information and communication technologies develop, the associated industries exhibit network externalities. Network externalities exist when the benefits of an agent from choosing a certain action increases as more peers choose the action. In the presence of network externalities in oligopoly markets, the utility that a customer obtains from purchasing a firm's product increases directly with the number of other customers purchasing the same product. In this case, product compatibility is a very important factor in making decisions in the standpoints of both customers (users) and producers (providers). Product compatibility can enhance the utilities of users of a product because they can interact with more other products' users for improving their performance. Network effects can be

more exploited by firms through product compatibility which enables firms to capture larger networks.

Recently, Hoernig (2012) studies strategic delegation in oligopoly with network externalities under price competition. He shows that firms' owners prefer their managers to be more aggressive if network externalities are strong enough. This implies that the results of Fershtman and Judd (1987) and Sklivas (1987) can be reversed in the presence of network externalities. Chirco and Scrimatore (2013) investigate the role of strategic delegation and network externalities in making the strategic choice between price and quantity competition in differentiated oligopoly. They show that, under strategic delegation, network externalities reverse the equilibrium outcome of Singh and Vives (1984) in that sufficiently strong network externalities fully support a unique equilibrium characterized by the symmetric choice of price strategy in equilibrium.<sup>1)</sup>

The goal of the paper is to analyze the effects of strategic delegation on price competition in duopoly when product compatibility reinforces network externalities and to investigate under what conditions firms' owners prefer their managers to be more aggressive than in the case of profit maximizer. To do this, we extend Hoernig (2012) to the case where product compatibility is built into network effects. For the convenience of analysis, we assume constant product compatibility instead of considering differential product compatibility or strategic choice of product compatibility. In our model, the owner of a firm may choose to make its manager more aggressive in a moderate range of network externalities, which may not invoke managerial aggressiveness in the framework of Hoernig (2012).

Recently many papers study the effects of network externalities and product compatibility on equilibrium outcomes of oligopoly, but they fail to address the problem of strategic delegation simultaneously. Product compatibility may belong to technological or physical attributes. However it can be also a firm's strategic instrument to attract consumers. Among

---

<sup>1)</sup> Without strategic delegation, network externalities do not change the equilibrium outcome of Singh and Vives (1984) since quantity is the symmetric equilibrium choice of firms without strategic delegation independently of the strength of the network effects.

others, Kim (2002) employs a Hotelling model to consider the compatibility decision as a signaling device of the quality of a newly introduced technology of which users are not informed. Chen and Chen (2011) consider Cournot competition under product compatibility and network externalities, where the degree of product compatibility is chosen by the rival firm, while Toshimitsu (2014) deals with the case where a firm can determine its own product compatibility.

The rest of the paper is organized as follows. Section 2 describes the model. In section 3, we characterize the optimal strategies of owners and managers at each stage game. Section 4 derives Bertrand duopoly equilibrium under strategic delegation in differentiated product market with network externalities and product compatibility. In this section, we provide conditions under which the managers are more aggressive. Section 5 concludes the paper.

## 2. THE MODEL

There are two firms  $i$  and  $j$  that produce differentiated goods (or systems) from each other with constant marginal cost  $c$ . However their products have network effects and are partially and symmetrically compatible with each other. The network effects for firm  $i$  is measured by  $f(S_i^e)$ , where  $S_i^e$  denotes the expected network size for firm  $i$ . We assume that firm  $i$  faces demand function

$$x_i = \alpha - p_i + \beta p_j + f(S_i^e), \quad (1)$$

where  $\alpha > 0$  is the potential market size,  $p_i$  and  $p_j$  the prices set by firm  $i$  and its rival firm  $j$ , and  $\beta \in (0, 1)$  indicates the degree of product substitutability. We assume that  $f(S_i^e) = nS_i^e$  where  $n \in (0, 1)$  is the degree of network externality for the network size of firm  $i$ . Furthermore, we assume that the expected network size is given by

$$S_i^e = y_i + \gamma y_j, \quad (2)$$

where  $\gamma \in [0, 1]$  denotes the degree of product compatibility between products, and  $y_i$  and  $y_j$  denote the expected equilibrium sales of firm  $i$  and its rival firm  $j$ . Thus, the expected network size of firm  $i$  consists of the own effect  $y_i$  and the spillover effect  $\gamma y_j$  of firm  $j$ . In particular, the spillover effect for network size of firm  $i$  depends on the degree of product compatibility which is exogenously and symmetrically given by the technology itself. If  $\gamma = 0$  ( $\gamma \in (0, 1)$ ,  $\gamma = 1$ ), then two products are perfectly incompatible (partially compatible, perfectly compatible, respectively). We focus on the case where two products are partially compatible. That is, components of system product  $i$  operate successfully on system  $j$  with the degree of product compatibility  $\gamma \in (0, 1)$ . The relations (1) and (2) lead to the demand function faced by firm  $i$ :

$$x_i = \alpha + n(y_i + \gamma y_j) - p_i + \beta p_j, \quad \forall i, j = 1, 2; j \neq i.$$

It is worth noting that the cases of  $\gamma = 0$  and  $n = 0$  correspond to Hoernig's (2012) model and Sklivas (1987) price competition model, respectively.

We assume that the owner of firm  $i$  makes a managerial incentive contract with the manager such that his manager  $i$  maximizes the objective function:

$$O_i = \lambda_i \pi_i + (1 - \lambda_i) R_i = (p_i - \lambda_i c) x_i, \quad (3)$$

where  $\pi_i = (p_i - c)x_i$  and  $R_i = p_i x_i$  are profits and revenue, respectively. Note that the weight  $\lambda_i$  on profits constitutes manager  $i$ 's perception of marginal cost. If  $\lambda_i = 1$ , manager  $i$  is simply a profit-maximizer; If  $\lambda_i < 1$ , manager  $i$  is said to be more aggressive (than profit maximizer); If  $\lambda_i > 1$ , manager  $i$  is said to be less aggressive (than profit maximizer). We will investigate the case where manager  $i$  is more aggressive, i.e.,

$\lambda_i < 1$ .

The price competition game consists of two stage games. In the first stage game, owners of firms  $i$  and  $j$  simultaneously and noncooperatively choose weights  $\lambda_i$  and  $\lambda_j$  in order to maximize firms' profits. In the second stage game, managers  $i$  and  $j$  compete with each other by setting prices  $p_i$  and  $p_j$  in order to maximize  $O_i$  and  $O_j$ , respectively. To analyze the outcomes of the whole game, we adopt the notion of subgame-perfect Nash equilibrium and impose the rational expectations condition that  $y_i = x_i$  for  $i = 1, 2$  at the second stage game.<sup>2)</sup>

### 3. PRICE COMPETITION AND STRATEGIC DELEGATION

This section discusses the equilibrium strategies of owners and managers. Since the whole game is composed of two stage games and the solution concept is subgame-perfect Nash equilibrium, equilibrium outcomes can be obtained via backward induction. We start with the second stage game between the managers. In this game, manager  $i$  maximizes his objective function by choosing price  $p_i$  against the rival manager  $j$ 's price choice  $p_j$  given expected equilibrium sales  $(y_1, y_2)$  of both firms and owner  $i$ 's choice  $\lambda_i$  of weight on profit in (3). First we derive a Nash equilibrium for the given expected equilibrium sales  $(y_1, y_2)$  and for the given weight on profits  $(\lambda_1, \lambda_2)$ . Assuming rational expectations for equilibrium sales, we then find a unique symmetric Nash equilibrium for the game of the managers for the given weight on profits  $(\lambda_1, \lambda_2)$ . By backward induction, we move to the first stage game between the owners who know the reaction functions of the managers. In this game, the equilibrium weights on the profits are determined as a Nash equilibrium.

---

<sup>2)</sup> Rational expectation is also assumed in a similar context in Katz and Shapiro (1985).

### 3.1. Price Competition

We will find a Nash equilibrium in the second stage game between the managers. For given the owners' choices of weights  $(\lambda_i, \lambda_j)$  and firm  $j$ 's price  $p_j$ , manager  $i$  chooses price  $p_i$  to maximize the objective function  $O_i$  with expecting the equilibrium sales of firms to be  $(y_i, y_j)$ . The first-order condition for maximization yields the best response function  $B_i$  of manager  $i$ :

$$B_i(p_j) = \frac{1}{2}[\alpha + \lambda_i c + \beta p_j + n(y_i + \gamma y_j)], \forall i = 1, 2, j \neq i.$$

The best response function  $B_i$  increases in  $p_j$ , implying that prices are strategic complements. A unique Nash equilibrium in the pricing game of managers is given by  $(p_1, p_2)$  such that

$$p_i = B_i(p_j), \forall i, j = 1, 2, j \neq i,$$

which implies that the Nash equilibrium is symmetric. As a consequence, we obtain equilibrium price  $p_i$  and corresponding quantity  $x_i$ : for each  $i = 1, 2$  with  $j \neq i$ ,

$$p_i = \frac{\alpha(2 + \beta) + n[2(y_i + \gamma y_j) + (y_j + \gamma y_i)\beta] + c(2\lambda_i + \beta\lambda_j)}{4 - \beta^2}, \quad (4)$$

$$x_i = \frac{\alpha(2 + \beta) + n[2(y_i + \gamma y_j) + (y_j + \gamma y_i)\beta] - c[(2 - \beta^2)\lambda_i - \beta\lambda_j]}{4 - \beta^2}.$$

Imposing the rational expectations condition  $y_i = x_i$ , we see that the equilibrium prices and quantities in (4) becomes:

$$\hat{p}_i = \frac{\alpha[2 + \beta - (1 - \gamma)n]}{(2 - n)^2 - \beta^2 - \gamma^2 n^2 - 2\beta\gamma n} + \frac{c[(2 + (1 - \gamma^2)n^2 - (3 + \beta\gamma)n)\lambda_i + (\beta - (\gamma + \beta)n)\lambda_j]}{(2 - n)^2 - \beta^2 - \gamma^2 n^2 - 2\beta\gamma n}, \quad (5)$$

$$\hat{x}_i = \frac{\alpha[2 + \beta - (1 - \gamma)n] - c[(2 - \beta^2 - (1 + \beta\gamma)n)\lambda_i - (\beta - (\gamma + \beta)n)\lambda_j]}{(2 - n)^2 - \beta^2 - \gamma^2 n^2 - 2\beta\gamma n},$$

for each  $i = 1, 2$  with  $j \neq i$ .

### 3.2. Strategic Delegation

In the first game, owner  $i$  anticipates that his choice  $\lambda_i$  of weight on profit induces prices and quantities of (5) in the second stage game. That is, owner  $i$  knows the reaction function of his manager. Given owner  $j$ 's weight  $\lambda_j$ , owner  $i$  chooses weight  $\lambda_i$  to maximize firm  $i$ 's profit:

$$\max_{\lambda_i \in \mathbb{R}} \hat{\pi}_i \equiv (\hat{p}_i - c)\hat{x}_i, \quad (6)$$

where  $\hat{p}_i$  and  $\hat{x}_i$  are prices and quantities of (5). Solving the profit maximization problem (6), owner  $i$  obtains best response function  $R_i$  with respect to owner  $j$ 's strategy  $\lambda_j$ :

$$R_i(\lambda_j) = \frac{\delta + \theta\lambda_j}{\Delta},$$

where

$$\begin{aligned} \delta = & [2 - (1 - \gamma)n + \beta] [\alpha((1 - \gamma^2)n^2 - 2n + \beta^2) \\ & + c\{4 - 2\beta - 2\beta^2 + \beta^3 + (1 + \gamma)(1 + \beta\gamma)n^2 \\ & - (4 - \beta - \beta^2 + 2(1 + \beta - \beta^2)\gamma)n\}], \end{aligned}$$

$$\theta = c[\beta^3 - (1 - \gamma^2)(\gamma + \beta)n^3 + (2\gamma + 3\beta - \gamma^2\beta)n^2 - (2 + \gamma\beta + \beta^2)\beta n],$$

$$\Delta = 2c[4 - 2\beta^2 - (1 - \gamma^2)(1 + \beta\gamma)n^3 + (5 + 4\gamma\beta - \beta^2 - 2(1 - \beta^2)\gamma^2)n^2 - (8 - 3\beta^2 - (4 - \beta^2)\beta\gamma)n].$$

A unique Nash equilibrium  $(\lambda_1^*, \lambda_2^*)$  in the first stage game between owners is determined by the relation  $\lambda_i^* = R_i(\lambda_j^*)$  with  $i, j = 1, 2$  with  $j \neq i$ . That is, the unique symmetric Nash equilibrium is given by

$$\lambda_i^* = \lambda^* := 1 + \frac{[\alpha - (1 - \beta)c][(1 - \gamma^2)n^2 - 2n + \beta^2]}{cK(\beta, \gamma, n)}, \quad \forall i = 1, 2, \quad (7)$$

where

$$K(\beta, \gamma, n) = (1 + \gamma)[2 - \beta - (1 - 2\beta)\gamma]n^2 - [6 - 3\beta - \beta^2 + 2(1 + \beta - \beta^2)\gamma]n + (4 - 2\beta - \beta^2).$$

#### 4. MAIN RESULTS

In this section, we will determine the final equilibrium of the whole game in the market. To find the equilibrium, we compute again the equilibrium prices and quantities in (5) with the equilibrium weights  $(\lambda_1, \lambda_2) = (\lambda^*, \lambda^*)$  in (7).

Putting (7) into (5), we obtain equilibrium prices and quantities: for each  $i = 1, 2$ ,

$$p_i^* = \frac{c[1 - (1 + \gamma)n][2 - \beta^2 - (1 + \beta\gamma)n] + \alpha[(1 - \gamma^2)n^2 - (3 + \beta\gamma)n + 2]}{K(\beta, \gamma, n)},$$

$$x_i^* = \frac{[\alpha - (1 - \beta)c][2 - \beta^2 - (1 + \beta\gamma)n]}{K(\beta, \gamma, n)}.$$

Thus the final equilibrium of the whole game is summarized in the following proposition.

**Proposition 4.1:** *A unique symmetric subgame-perfect Nash equilibrium is given by*

$$p_i^* = \frac{c[1 - (1 + \gamma)n][2 - \beta^2 - (1 + \beta\gamma)n] + \alpha[(1 - \gamma^2)n^2 - (3 + \beta\gamma)n + 2]}{K(n; \beta, \gamma)},$$

$$x_i^* = \frac{[\alpha - (1 - \beta)c][2 - \beta^2 - (1 + \beta\gamma)n]}{K(n; \beta, \gamma)},$$

$$\lambda_i^* = 1 + \frac{[\alpha - (1 - \beta)c][(1 - \gamma^2)n^2 - 2n + \beta^2]}{cK(n; \beta, \gamma)},$$

where  $i = 1, 2$  and

$$K(\beta, \gamma, n) = (1 + \gamma)[2 - \beta - (1 - 2\beta)\gamma]n^2 \\ - [6 - 3\beta - \beta^2 + 2(1 + \beta - \beta^2)\gamma]n + (4 - 2\beta - \beta^2).$$

To examine the behavior of managers in equilibrium, we will assume the following condition:<sup>3)</sup>

$$(A1) \quad 0 < c < (1 - \beta)^{-1}\alpha.$$

In addition, we should consider  $n \in (0, 1)$  such that

---

<sup>3)</sup> This condition is introduced in Hoernig (2012) to characterize price competition in the presence of network effects without product compatibility.

$$p^* - c = \frac{[\alpha - (1 - \beta)c][(1 - \gamma^2)n^2 - (3 + \beta\gamma)n + 2]}{K(n; \beta, \gamma)} > 0, \quad (8)$$

and

$$x_i^* = \frac{[\alpha - (1 - \beta)c][2 - \beta^2 - (1 + \beta\gamma)n]}{K(n; \beta, \gamma)} > 0. \quad (9)$$

Under Assumption (A1), to ensure (8) and (9), we need to check the signs of  $K(n; \beta, \gamma)$ ,  $(1 - \gamma^2)n^2 - 2n + \beta^2$ , and  $2 - \beta^2 - (1 + \beta\gamma)n$ . Now we observe that

$$K(n; \beta, \gamma) > 0 \text{ if and only if } n < n^*, \quad (10)$$

where  $n^*$  satisfies  $K(n^*; \beta, \gamma) = 0$  with

$$n^* = \frac{[6 - 3\beta - \beta^2 + 2(1 + \beta - \beta^2)\gamma]}{2(1 + \gamma)[2 - \beta + (2\beta - 1)\gamma]} - \frac{(1 - \beta)\sqrt{4 + 8\gamma + 20\gamma^2 + (\beta + 2\beta\gamma)^2 + 4\beta(1 + 5\gamma + 2\gamma^2)}}{2(1 + \gamma)[2 - \beta + (2\beta - 1)\gamma]}.$$

Next we see that

$$(1 - \gamma^2)n^2 - (3 + \beta\gamma)n + 2 > 0 \text{ if and only if } n < n_1,$$

where  $n_1$  satisfies  $(1 - \gamma^2)n_1^2 - (3 + \beta\gamma)n_1 + 2 = 0$  with

$$n_1 = \frac{3 + \beta\gamma - \sqrt{(3 + \beta\gamma)^2 - 8(1 - \gamma^2)}}{2(1 - \gamma^2)}.$$

Similarly, it holds that

$$2 - \beta^2 - (1 + \beta\gamma)n > 0 \text{ if and only if } n < n_2,$$

where  $n_2$  satisfies  $2 - \beta^2 - (1 + \beta\gamma)n_2 = 0$  with

$$n_2 = \frac{2 - \beta^2}{1 + \beta\gamma}.$$

In Lemma A.1 of Appendix, it is shown that  $0 < n^* < n_1 < n_2$  with  $n_1 < 1$ . As a consequence, for (8) and (9) to hold, it is necessary that  $n < n^*$  or  $n > n_2$ .

On the other hand, to see if managers are more aggressive under Assumption (A1), i.e.,

$$\lambda_i^* = 1 + \frac{[\alpha - (1 - \beta)c][(1 - \gamma^2)n^2 - 2n + \beta^2]}{cK(n; \beta, \gamma)} < 1. \quad (11)$$

We need to check the signs of  $K(n; \beta, \gamma)$  and  $(1 - \gamma^2)n^2 - 2n + \beta^2$ . It is noted that

$$(1 - \gamma^2)n^2 - 2n + \beta^2 < 0 \text{ if and only if } n > n_0, \quad (12)$$

where  $n_0$  satisfies  $(1 - \gamma^2)n_0^2 - 2n_0 + \beta^2 = 0$  with

$$n_0 = \frac{1 - \sqrt{1 - (1 - \gamma^2)\beta^2}}{1 - \gamma^2}.$$

One can verify that  $n_0$  belongs to  $(0, 1)$ . To determine the size of  $\lambda^*$ , we need to compare  $n_0$  with  $n^*$ . Lemma A.2 in Appendix shows that  $n_0 < n^*$  for all  $\beta \in (0, 1)$  and  $\gamma \in (0, 1)$ . It is observed that there is no equilibrium if  $n^* \leq n \leq n_2$ , since  $K(n; \beta, \gamma) = 0$  or one of (8) and (9) is violated.

**Theorem 4.1:** *Under Assumptions (A1), if the degree of network externality satisfies  $n \in (n_0, n^*)$ , then the owners want the managers to be more aggressive, i.e.,  $\lambda^* < 1$ .*

**Proof:** Consider the case where  $n > n^*$  for which  $K(n; \beta, \gamma) < 0$ . In view of Lemma A.1, for (8) and (9) to hold, it is necessary that  $n > n_2$ . Since Lemma 4.2 and (12) imply  $(1 - \gamma^2)n^2 - 2n + \beta^2 < 0$ , we have  $\lambda^* > 1$ .

On the other hand, in the case of  $n < n^*$ , we see that  $K(n; \beta, \gamma) > 0$ . Note that (8) and (9) are satisfied due to Lemma A.1. From Lemma A.2 and (12), the relation (11) holds, i.e.,  $\lambda^* < 1$  if  $n \in (n_0, n^*)$ .

When  $\gamma = 0$ , we have  $n_0 = 1 - \sqrt{1 - \beta^2}$  and  $n^* = 1$ , and therefore confirm the result of Hoernig (2012), which is restated in the following corollary.

**Corollary 4.1:** *Suppose that Assumptions (A1) holds and  $\gamma = 0$ . If network effects are sufficiently strong, such that  $n \in (1 - \sqrt{1 - \beta^2}, 1)$ , then in equilibrium the owners want the managers to be more aggressive, i.e.,  $\lambda^* < 1$ .*

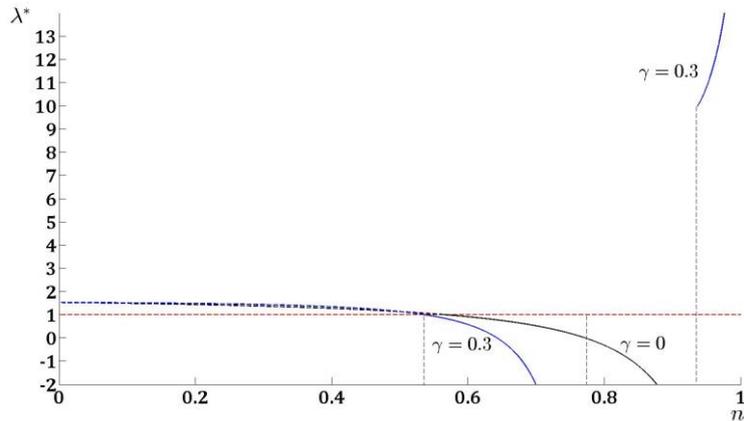
Theorem 4.1 states that the owners want the managers to be more aggressive if the degree of network externality takes intermediate values in  $(0, 1)$ . If the degrees of network externality is sufficiently large but not too large such that  $n \in (n_0, n^*)$ , then owners' weights on profits are strategic substitutes.<sup>4)</sup> Thus competition between owners leads them to choose delegation parameters less than unity, for which both managers strive for larger market share.

Since  $n_0 < 1 - \sqrt{1 - \beta^2} < n^* < 1$ , the interval for managers to be more aggressive of our model partially overlaps that of Hoernig (2012). In other words, the interval of our model is located leftward relative to the interval of Hoernig (2012) with a nonempty intersection. Since the spillover effect due to product compatibility builds a larger network size, smaller degrees of network

---

<sup>4)</sup> One can verify that  $\theta / \Delta < 0$  if  $n \in (n_0, n^*)$ .

**Figure 1** For  $\alpha = c = 1$ ,  $\beta = 0.9$ , and  $\gamma = 0.3$ , it holds that  $\lambda^* < 1$  for  $n \in (n_0, n^*)$  where  $n_0 = 0.5355$ ,  $n^* = 0.7743$ , and  $n_2 = 0.937$



externality are sufficient to induce the managers to be more aggressive.

More interestingly, in contrast to Hoernig (2012), our model also shows that the owners do not prefer the managers to be more aggressive when network externality is too strong. More precisely, the managers are not more aggressive in our model if network externality is sufficiently strong such that  $n \in [n^*, 1)$  (as long as the equilibrium is well defined),<sup>5)</sup> in which the managers are more aggressive in the model of Hoernig (2012).

## 5. CONCLUSION

By incorporating exogenous product compatibility into the model of Hoernig (2012), we investigate under what conditions the firms' owners prefer their managers to be more aggressive under price competition with

<sup>5)</sup> Note that  $\gamma > \beta^{-1} - \beta$  if and only if  $n_2 \in (0, 1)$ . In this case, if  $n \in (n_2, 1)$ , the owners want their managers to be less aggressive, i.e.,  $\gamma^* > 1$ .

product compatibility and network externalities. It is shown that the managers are more aggressive in equilibrium if the degree of network externality takes intermediate values. This result coincides with that of Hoernig (2012) when  $\gamma = 0$ , but is different from it in that the degree of network externality should not be too large to ensure the more aggressiveness of managers.

It is perceived that the assumption of common exogenous product compatibility may be strong. One of future research is to extend this model by endogenizing product compatibility as an equilibrium outcome of the game between firms.<sup>6)</sup> One may consider a case where a firm's product compatibility is a signaling device to its rival firm and users about its product quality.<sup>7)</sup> It is also important to investigate, under what conditions, which combination between price and quantity competition arises as an equilibrium outcome.<sup>8)</sup>

## APPENDIX

**Lemma A.1:** *For every  $\beta \in (0, 1)$  and  $\gamma \in (0, 1)$ , it holds that*

$$n^* < n_1 < n_2.$$

**Proof:** We now take any  $\beta \in (0, 1)$  and  $\gamma \in (0, 1)$ . Let  $f(n; \beta, \gamma) = 2 - \beta^2 - (1 + \beta\gamma)n$ . Then one can check

$$K(n_1; \beta, \gamma) < 0 < f(n_1; \beta, \gamma).$$

Since  $K(n^*; \beta, \gamma) = 0$  and  $f(n_2; \beta, \gamma) = 0$ , this implies that  $n^* < n_1 < n_2$ .

---

<sup>6)</sup> See Chen and Chen (2011) and Toshimitsu (2014) for quantity competition without strategic delegation.

<sup>7)</sup> See Kim (2002) for Hotelling competition without strategic delegation.

<sup>8)</sup> See Chirco and Scrimatore (2013) for the case without product compatibility.

**Lemma A.2:** For every  $\beta \in (0, 1)$  and  $\gamma \in (0, 1)$ , it holds that

$$n_0 < n^*.$$

**Proof:** We observe that

$$(2 + \beta)(1 + 2\gamma) > \sqrt{4 + 8\gamma + 20\gamma^2 + (\beta + 2\beta\gamma)^2 + 4\beta(1 + 5\gamma + 2\gamma^2)},$$

where the term on the right-hand side appears in  $n^*$ . This implies that

$$n^* > \frac{[6 - 3\beta - \beta^2 + 2(\gamma + \beta - \beta^2)\gamma] - (1 - \beta)(2 + \beta)(1 + 2\gamma)}{2(1 + \gamma)[2 - \beta + (2\beta - 1)\gamma]} = \frac{1}{1 + \gamma}.$$

Then we have

$$n^* - n_0 > \frac{1}{1 + \gamma} - \frac{1 - \sqrt{1 - (1 - \gamma^2)\beta^2}}{1 - \gamma^2} = \frac{-\gamma + \sqrt{1 - (1 - \gamma^2)\beta^2}}{1 - \gamma^2} > 0,$$

which implies that  $n_0 < n^*$ .

## REFERENCES

- Baake, P. and A. Boomb, "Vertical Product Differentiation, Network Externalities, and Compatibility Decisions," *International Journal of Industrial Organization*, 19(1-2), 2001, pp. 267-284.
- Cabral, L., "Dynamic Price Competition with Network Effects," *Review of Economic Studies*, 78(2), 2011, pp. 83-111.
- Chen, H-C. and C-C. Chen, "Compatibility under Differentiated Duopoly with Network Externalities," *Journal of Industry, Competition, and Trade*, 11(1), 2011, pp. 43-55.

- Chirco, A. and M. Scrimatore, "Choosing Price or Quantity? The Role of Delegation and Network Externalities," *Economics Letters*, 121, 2013, pp. 482-486.
- Choi, J., "Network Externality, Compatibility Choice, and Planned Obsolescence," *Journal of Industrial Economics*, 42(2), 1994, pp. 167-181.
- Cremer, J., P. Rey, and J. Tirole, "Connectivity in the Commercial Internet," *Journal of Industrial Economics*, 48(4), 2000, pp. 433-472.
- De Palma, A., L. Leruth, and P. Regibeau, "Partial Compatibility with Network Externalities and Double Purchase," *Information Economics and Policy*, 11(2), 1999, pp. 209-227.
- Economides, N., "Network Externalities, Complementarities, and Invitations to Enter," *European Journal of Political Economy*, 12(2), 1996, pp. 211-233.
- Farrell, J. and G. Saloner, "Standardization, Compatibility, and Innovation," *RAND Journal of Economics*, 16(1), 1985, pp. 70-83.
- \_\_\_\_\_, "Installed Base and Compatibility," *American Economic Review*, 76(5), 1986, pp. 940-955.
- \_\_\_\_\_, "Converters, Compatibility, and the Control of Interfaces," *Journal of Industrial Economics*, 40(1), 1992, pp. 9-35.
- Fershtman, C. and K. Judd, "Equilibrium Incentives in Oligopoly," *American Economic Review*, 77(5), 1987, pp. 927-940.
- Foros, O. and B. Hansen, "Competition and Compatibility among Internet Service Providers," *Information Economics and Policy*, 13(4), 2001, pp. 411-425.
- Hoernig, S., "Strategic Delegation under Price Competition and Network Effects," *Economics Letters*, 117, 2012, pp. 487-489.
- Katz, M. and C. Shapiro, "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75(3), 1985, pp. 424-440.
- \_\_\_\_\_, "Technology Adoption in the Presence of Network Externalities," *Journal of Political Economy*, 94(4), 1986, pp.

822-841.

\_\_\_\_\_, “Systems Competition and Network Effects,” *Journal of Political Economy*, 8(2), 1994, pp. 93-115.

Kim, J., “Product Compatibility as a Signal of Quality in a Market with Network Externalities,” *International Journal of Industrial Organization*, 20(7), 2002, pp. 949-964.

Shy, O., *Industrial Organization: Theory and Application*, Cambridge: MIT University Press, 1995.

\_\_\_\_\_, “Technology Revolutions in the Presence of Network Externalities,” *International Journal of Industrial Organization*, 14(6), 1996, pp. 785-800.

\_\_\_\_\_, *The Economics of Network Industries*, Cambridge: Cambridge University Press, 2001.

Singh, N. and X. Vives, “Price and Quantity Competition in a Differentiated Duopoly,” *RAND Journal of Economics*, 15(4), 1984, pp. 546-554.

Sklivas, S., “The Strategic Choice of Managerial Incentives,” *RAND Journal of Economics*, 18(3), 1987, pp. 452-458.

Toshimitsu, T., “Endogenous Product Compatibility Choice under Cournot Competition with a Network Externality,” mimeo, 2014.