Intergenerational Bequests and the Divergence of Income Distribution[*]

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This paper builds a model concerning intergenerational bequests and human capital investment. In the presence of credit market imperfection and liquidity constraint to human capital investment or education, income distribution can be divided into two groups: the poor and the rich. The author proposes a policy on the taxation of the rich's labor income and reducing the cost of education as a measure to prevent the income distribution from diverging. The policy measure induces 'double effects', and thus differs from a conventional redistributive policy which just transfers the income from the rich to the poor. One is the direct income increase effect by reducing the cost of education, and the other is the income increase effect by lessening the liquidity constraint which originates from credit market imperfection.

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1. INTRODUCTION

As Josan (2012) describes, human capital investment is made by individuals, families, enterprises, and states. It increases not only the individuals' earning capacity, but also the society's productivity. Cremer and Pestieau (2006) point out that the investment in education by families has a significance as a form of transfer between generations. The conditions for investments in human capital, or education,[1] however, depend upon the amount of bequest the individuals have in addition to their talents. The initial condition to attain education is more advantageous to the individuals with a large amount of bequest than those with little bequest and/or liquidity constraint. Another key factor for getting education is the credit market imperfection, which makes the borrowing interest rate higher than the lending interest rate due to the mortgage conditions of the individuals with little bequest and their monitoring cost.

Macroeconomic growth models across countries can be applied to models of income distribution within a country. Stiglitz (1969) constructed a model of income distribution among individuals by applying the neoclassical growth model, showing that income levels across countries converge into equilibrium. According to Stiglitz (1969), complete equalization of income distribution could occur in the absence of liquidity constraint and credit market imperfection. Benhabib and Spiegel (1994) emphasize 'catch-up effects' that the income levels across countries can be equalized by the effect of underdeveloped countries catching up with developed countries mainly due to human capital investment or education.[2] These models, however, do not take the effect of human capital investment and credit market imperfection into account.

Recent macroeconomic growth models including endogenous growth illustrate the impact of human capital on economic growth. Kim (2013)

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[1] The term 'education' has the similar meaning with human capital investment. We concern on whether it increases the productivity to be skilled labor or not.

[2] Benhabib and Spiegel (1994) provide the evidence using aggregate cross-country data concerning the role of human capital in economic development.

estimates the role of human capital on the cross-sectional income differences by introducing a measure that effectively quantifies the amounts of human capital. According to his empirical results, human capital enhances economic growth similarly to the case of endogenous growth model.[3] Galor and Zeira (1993) discuss that the income level difference could be kept across the countries in the presence of liquidity constraint to education and credit market imperfection. They use a dynamic model to analyze the divergence of income distribution into high and low income groups. Their result reveals the existence of multiple equilibria at the steady state. We also apply Galor and Zeira (1993)'s macroeconomic model across countries to the model of income distribution among individuals within a country.[4]

Torvik (1993) investigates the degree of human capital investment and education gap among individuals. He focuses on the heterogeneity of individuals' talent applying the model of Galor and Zeira (1993). As mentioned above, the difference of income levels across countries can be adopted to explain the difference of income distribution among individuals. Likewise, we can make use of the divergence of the income level across countries to describe the divergence of income distribution among individuals. However, at the policy level, these across- and within-countries compatibilities are a lot different. Whereas a relatively strong policy for equitable income distribution among individuals may be implemented within a country, at a global level it is difficult to take a redistributive fiscal policy to avoid the income distribution divergence, as there is no strong policy authority at world level.[5]

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[3] Endogenous growth models mainly deal with the existence of multiple equilibria and the importance of human capital in economic development. For example, see Romer (1986), Lucas (1988), and Barro (1991).

[4] Banerjee and Newman (1994) construct a model to investigate the possibility of poverty traps assuming moral hazard in capital markets. They do not discuss the relationship, however, between the polarization of income distribution and the policy tools to avoid it.

[5] Though there exist some international aid programs such as ODA, these are greatly restricted to implementing a powerful policy for income redistribution across countries. In the case of macroeconomic models that deal with economic growth across nations, it also has much difficulty to carry out the redistributive policy that either transfers income from rich countries to poor countries or reduces the liquidity constraint to human capital investment.

We argue that the income paths do not converge into the state of equitable income distribution but diverge into the rich and the poor income groups in the presence of credit market imperfection and liquidity constraint. Non-desirability of the income divergence requires implementing some policies to prevent it. We propose a policy that reduces the cost of education by using tax revenues from the rich who receive a large amount of bequests. The policy enables the individuals with liquidity constraint to get education.

The policy proposed here generates double effects. One is the 'direct income increase effect' through reducing the cost of education, which benefits the rich as well. The other is the income increase effect concerned with credit market imperfection. The high-ability individuals who want to be skilled labor will not be able to get education if there is liquidity constraint or little bequest. Reducing the cost of education enables those who want to get education to reduce their borrowing cost. As discussed in the models of Hendel *et al.* (2005) and Galor and Zeira (1993), credit market imperfection is reflected in the difference between the borrowing interest rate and the lending interest rate which is called "spread". The other income increase effect can be considered as 'spread effect', because it closely relates to a wide difference between the borrowing and the lending interest rates. The big difference between the two is a barrier to the credit market for high-ability individuals with liquidity constraint. This paper proves that the income transfer from the rich to the individuals is the same as 'spread effect'.

In what follows, section 2 introduces a basic model mainly explaining the types of skilled labor and unskilled labor. Section 3 deals with the supply and the demand of unskilled labor referring to Hendel *et al.* (2005). In section 4, we apply Galor and Zeira (1993)'s model across countries to the model of income distribution among individuals within a country. Section 5 shows that income distribution can be diverged in the presence of liquidity constraint and credit market imperfection. Section 6 discusses policy measures to prevent the income distribution from diverging. In section 7 the double effects of the redistributive policy are investigated. The final section is concluding remarks.

2. THE BASIC MODEL: TYPES OF SKILLED LABOR
AND UNSKILLED LABOR

We mainly refer to the models of Hendel *et al.* (2005), Galor and Zeira (1993), and Torvik (1993). These studies show that the size distribution of income can be divided into high and low income groups in the presence of liquidity constraint to human capital investment and credit market imperfection.

Human capital investment depends not only on the individual's talent but also on the bequest motive of the parents.[6] We assume an altruistic motive in which parents determine the amount of bequests. Their child can use the bequests for human capital investment or education. The high-ability individuals who face liquidity constraint, where the bequest is less than the cost of education, can finance the cost of education by borrowing money from credit market. Generally the poor are in a weak position on mortgage conditions or have barriers to credit market. In addition, they need monitoring cost is to perform financial intermediations and to present default. Hendel *et al.* (2005) and Galor and Zeira (1993) also assume that credit market is imperfect due to the possibility of default and monitoring cost. Credit market imperfection causes the borrowing interest rates to be higher than the lending interest rates. As a result, credit or capital market imperfection is reflected in the difference between the borrowing and the lending interest rates, or 'spread'.

There are skilled labor and unskilled labor in the market. Individuals who get education are classified as skilled labor, and others are considered as unskilled labor. Whether the individuals get education or not depends on conditions such as their talent or ability, the amount of bequest from the parents, and market conditions for borrowing. Skilled labor generates higher productivity than unskilled labor. Spence (1973) explores the 'signaling' role or the role of 'screening device' of education. Hendel *et al.* (2005) adopt

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[6] Torvik (1993) investigates the relationship between the economic growth and the income distribution in macroeconomic viewpoint focusing on agents' talent.

both the signaling role of education and the credit constraints. The signaling role of education implies that firms can only observe individuals' education level and set wages according to the expected value of their productivity. Denote w^s as the wage for skilled labor and w^u as the wage for unskilled labor. When firms employ individuals, w^s and w^u are conditional on whether they are educated or not.

$$w^s = \text{E(productivity} \mid \text{educated)}, \tag{1}$$

$$w^u = \text{E(productivity} \mid \text{not educated)}. \tag{2}$$

Firms pay higher wage to skilled labor than to unskilled labor: that is, $w^s > w^u$. For simplicity, normalize the number of individuals at a mass 1. Also assume that a proportion β of them is high-ability individuals and a proportion $(1 - \beta)$ is low-ability individuals. Let the productivity of the high-ability q^H, and that of the low-ability q^L, where $q^H > q^L$. In addition, time costs are different depending on the productivity of individuals. Generally, low-ability individuals require higher monetary cost or more effort than high-ability individuals. For example, low-ability individuals may need more expensive tutorials and materials relative to high-ability individuals. The monetary cost of low-ability individuals for getting education, or e_L, is greater than that of high-ability individuals, or e_H: that is, $e_L > e_H$.

All individuals who get education bear the cost of education, or c, to become skilled labor. Suppose that the cost of education for the low-ability individuals is so high that they do not participate in non-compulsory (e.g., postsecondary) education. Low-ability individuals instead invest a bequest (or wealth) from parents in security bonds (e.g., treasury bonds) with interest rate is r, which is the same as the lending interest rate. Then the condition that the highest possible skill premium, or $(q^H - q^L)$ of low-ability individual per dollar will be less than $1+r$.

$$\frac{q^H - q^L}{c + e_L} < 1 + r. \tag{3}$$

Those who invest in education to become skilled labor are high-ability individuals. Among them, at least the richest high-ability individuals are actually interested in getting education. When both low-ability and high-ability individuals are pooled together, the average productivity can be defined as $\bar{q} = (1-\beta)q^L + \beta q^H$. Then there will be a possible skill premium of high-ability individual with above the average productivity. In other words, the high-ability individuals who can get the skill premium above the cost of education are likely to invest in education when the profit per dollar investment is greater than 1+I, where i is the lending interest rate. The profitable condition can be written as follows.

$$\frac{q^H - \bar{q}}{c + e_H} > 1 + i. \tag{4}$$

By the assumption of credit market imperfection, the borrowing interest rate i is higher than the lending interest rate: that is, $i > r$. By merging equation (3) and (4), we obtain the inequality relation as in (5).

$$\frac{q^H - q^L}{c + e_L} < 1 + r < 1 + i < \frac{q^H - \bar{q}}{c + e_H}. \tag{5}$$

Equation (5) implies that there occurs the possibility of separation between unskilled labor and skilled labor in equilibrium. In addition, we can guarantee that there are no pooling equilibria in which both unskilled labor and skilled labor take their actions in the same manner. There exist equilibria where at least some high-ability individuals invest in education and low-ability individuals do not get non-compulsory (e.g., postsecondary) education.

3. SUPPLY AND DEMAND OF LABOR MARKET

Here we describe the income compositions of skilled labor and unskilled labor. The income of low-ability individuals, $Y_{u\,t+1}$, who do not invest in education, consists of labor (wage) income, w^u, the bequest or wealth from parent, B_t, and property or interest income earned by lending the bequest, rB_t.

$$Y_{u\,t+1} = w^u + (1+r)B_t. \tag{6}$$

On the other hand, the income of high-ability individuals who invest in education, $Y_{s\,t+1}$, can be classified into the two types according to the amount of bequest. One is the case with liquidity constraint in which the bequest is less than the cost of education: that is, $B_t < c$. The individual with liquidity constraint needs to borrow the rest of the education cost. The individuals with liquidity constraint have to pay higher interest amount for the borrowing due to credit market imperfection. Without loss of generality, when we assume that $e_H = 0$, the income of skilled labor with liquidity constraint, $Y_{s\,t+1}$, can be expressed as equation (7).

$$Y_{s\,t+1} = w^s + (1+i)(B_t - c). \tag{7}$$

While the individual with liquidity constraint earns the skilled wage income w^s, he/she owes the borrowed amount $(1+i)(B_t - c)\,(<0)$.

The other is the individual without liquidity constraint. This is the case where the bequest is larger than the cost of education: that is, $B_t > c$. The individual without liquidity constraint who gets education obtains not only the skilled labor (wage) income but also the property income by lending the rest of bequest that exceeds the cost of education. His/her income can be written as equation (8).

$$Y_{s\,t+1} = w^s + (1+r)(B_t - c). \tag{8}$$

Equation (8) shows that the skilled income without liquidity constraint $Y_{s\,t+1}$ is composed of the skilled labor income w^s, the wealth and the property income $(1+r)(B_t-c)\,(>0)$.

We look at the supply and the demand of labor market. Low-ability individuals do not invest in education. Therefore, with them it is enough to consider the labor market for unskilled labor, fixing the wage of skilled individuals at $w^s=q^H$. In the unskilled labor group, there are also high-ability individuals. Let $P(u|H)$ as the probability that a high-ability individual is unskilled. Also the cumulative density function $P(u|H)$ of the bequest distribution can be used for the supply curve of unskilled labor. High-ability individuals with small amount of bequest, i.e., less than c (that is, $B_t < c$) get education if the RHS (right hand side) of equation (7) is larger than the RHS of equation (6). That is,

$$w^s + (1+i)(B_t - c) > w^u + (1+r)B_t. \tag{9}$$

We can rewrite the equation (9) as follows.

$$B_t > \frac{(1+i)c + (w^u - w^s)}{i - r} \equiv \tilde{B}. \tag{9-1}$$

From equation (9-1), the supply curve of unskilled labor can be expressed as equation (10)

$$P(u|H) = P(B_t \leq \tilde{B}). \tag{10}$$

Equation (9-1) indicates that when w^u increases (decreases), the critical value of $\tilde{B}$ increases (decreases) and the value of $P(u|H)$ increases (decreases). As a result, the supply curve has a positive relationship between w^u and $P(u|H)$. If unskilled labor is normally distributed cdf $P(u|H)$ has an S-shape when the cumulative density $P(u|H)$ is on the y-axis and w^u is on the x-axis. We consider the normal form of supply curve (usually, price on

Figure 1   Supply and Demand of Unskilled Labor

cumulative density of unskilled labor $P(u|H)$
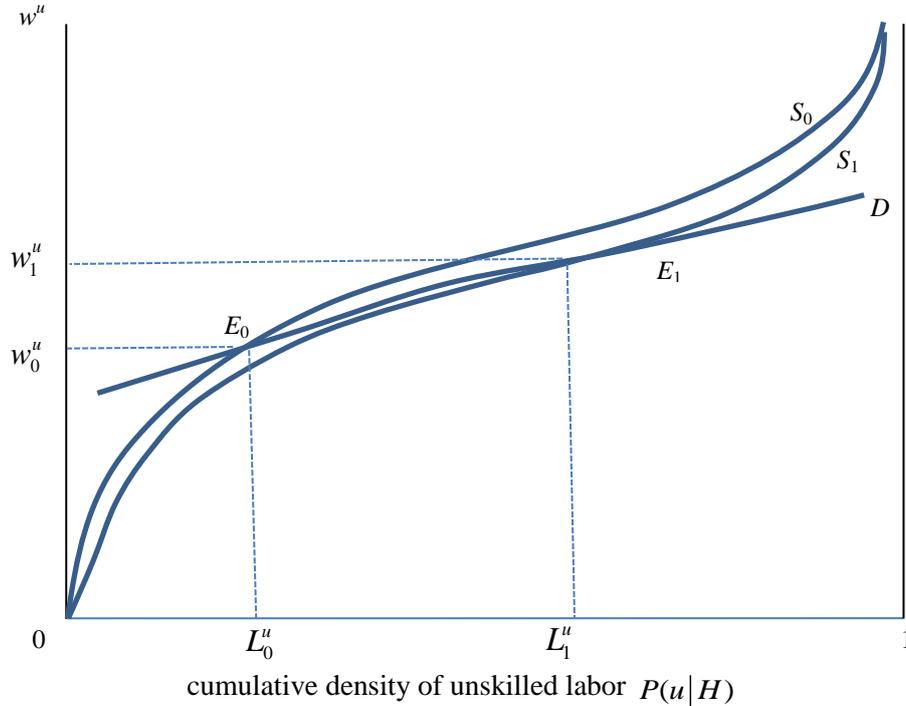
x-axis and quantity on *y*-axis) by changing their axis each other. Figure 1 depicts the supply curve corresponding to $P(u|H)$ on the *x*-axis and w^u on the *y*-axis. For example, the supply curve has the form of inverse *S*-shape like the S_0 in figure 1.

   An increase in the cost of education (c) raises $\tilde{B}$ in equation (9-1). Given other parameters, the increase of the cost of education (c) makes the return of education fall down and shifts the supply curve to the right (from S_0 to S_1 in figure 1), decreasing the value of cumulative density of $P(u|H)$. Increase in spread $(i - r)$, or the difference between the borrowing interest rate and the lending interest rate, drives the value of $\tilde{B}$ down, reduces the slope of supply curve, and shifts the curve to the right. The reduced slope of supply curve makes unskilled labor become more elastic or sensitive to wage level.

   When we adopt the 'signaling' role or the 'screening device' of education

explored by Spence (1973), firms employ unskilled labor paying a wage on the expected productivity. As unskilled labor is a mix of high-ability and low-ability individuals, the expected productivity would be: $(1-\beta)+$ $\beta P(u|H)$. In order to drive the demand curve of unskilled labor, we describe a relationship to determine the skill premium between skilled labor and unskilled labor by using their equilibrium wage levels of equation (1) and (2).

$$w^s - w^u = (q^H - q^L)\left(\frac{1-\beta}{1-\beta+\beta P(u|H)}\right). \qquad (11)$$

From equation (11), we can find a positive relationship between w^u and $P(u|H)$. The positive relationship proposed here can be interpreted as an upward sloping demand curve, which is shown as the curve D in figure 1. Hendel *et al.* (2005, p. 849) provides an intuitive explanation for the positive relationship between the two. "Intuitively, as fewer and fewer workers get an education firms realize that the average uneducated worker is more and more likely to be of high ability. Thus they are willing to pay more for unskilled workers".

Figure 1 takes an example of comparative statics when the cost of education increases. In that case, equilibrium point shifts from E_0 to E_1, resulting in the increase in the wage of unskilled labor (from w_0^u to w_1^u) as well as in the increase in the supply and the demand of unskilled labor (from L_0^u to L_1^u). There is no necessity to have a single crossing between the supply and the demand curve or a single equilibrium point. There may exist multiple equilibria when the supply and the demand intersect several times. Among such equilibria, some are stable and the others are unstable. Next section offers a simple dynamic model concerning the intergenerational income paths and discusses the possibility of divergence of income based on the basic model.

4. THE DYNAMIC MODEL

In section 2, we investigated the condition under which the individual chooses to invest in education. The condition depended on factors such as the amount of bequest, the individual's talent, the cost of education, and the credit market imperfection. Also we discussed the condition for individuals to get education considering its signaling role or the role of screening device. The condition for high-ability individuals depended on liquidity constraint. The high-ability individual gets education when the condition of equation (4) is satisfied. The wage level of skilled labor w^s is larger than that of unskilled labor w^u. This section shows the possibility of income divergence with the case that wage is exogenous, letting $w^s = qw$ and $w^u = w$ for simplicity. This is,

$$qw > w, \text{ where } q > 1. \tag{12}$$

In inequality (12), w is a wage rate of unskilled labor, and q is an index of the productivity of skilled labor. The inequality (12) describes that the wage rate of skilled labor is q times larger than that of unskilled labor. Then, we can rewrite equation (6), (7), and (8) in section 3 as the followings.

$$Y_{ut+1} = w + (1+r)B_t, \tag{6-1}$$

$$Y_{s\,t+1} = qw + (1+i)(B_t - c), \tag{7-1}$$

$$Y_{s\,t+1} = qw + (1+r)(B_t - c), \tag{8-1}$$

First, the individual with liquidity constraint chooses to invest in education when the income of skilled labor is larger than that of unskilled labor. The condition can be expressed by using equation (6-1) and (7-1): that is, the right hand side (RHS) of equation (7-1) is larger than the RHS of equation (6-1).

$$qw + (1+i)(B_t - c) > w + (1+r)B_t, \tag{13}$$

$$B_t > \frac{(1+i)c + w(1-q)}{i-r}. \tag{14}$$

The RHS of inequality (14) is a critical value of bequest that separates high-ability individuals with liquidity constraint getting or not getting education. As shown in the RHS of inequality (14), the critical value of the bequest depends on various parameters: a negative relationship with the productivity of skilled labor (because $q > 1$), a positive relationship with the cost of education, and a negative relationship with the spread or the difference between the borrowing and lending interest rate, $i–r$.

The individuals without liquidity constraint choose to invest in education when the income of skilled labor is larger than that of unskilled labor. The condition can be described by using equation (6-1) and (8-1): that is, the RHS of equation (8-1) is larger than the RHS of equation (6-1).

$$qw + (1+r)(B_t - c) > w + (1+r)B_t, \tag{15}$$

$$q > (1+r)c / w + 1 \equiv q_0. \tag{16}$$

Contrary to inequality (14), the inequality (16) indicates that the critical value of the productivity of skilled labor without liquidity constraint does not depend on the amount of bequest. In this case, the individual needs not to finance the cost of education because the amount of bequest exceeds it. As a result, the individual who reaches a certain level of productivity q_0 gets education to be skilled labor.[7] If the credit market is perfect, the borrowing interest rate and the lending interest rate are the same. As a result, the condition to get education is the same for both the individual with liquidity constraint and the individual without it. This can be shown by rewriting the inequality (13) as the following.

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[7] Note that the value q_0 is a constant in equation (16). The value depends on the lending rate r, the cost of education c, and the wage rate of unskilled labor w.

$$q > [(r-i)B_t + (1+i)c]/w + 1. \tag{14-1}$$

The value of the RHS of inequality (14-1) shows the same value of RHS of the inequality (16) when $i = r$. This means that the individual who reaches a certain level of productivity invests in education to be the skilled regardless of the amount of bequest. In other words, the credit market imperfection or the existence of 'spread' plays an important role in whether the individuals get education or not.

5. DIVERGENCE OF INCOME DISTRIBUTION

For simplicity, we assume an asexual reproduction in which each agent reproduces one child hence there is no population growth, as presented in the models of Banerjee and Newman (1994), Torvik (1993), Davis and Kuhn(1991), and others. The assumption that the individual survives only one period does not require distinguishing the flow variables such as income from the stock variables such as wealth or property. The individual maximizes his/her utility under altruistic bequest motive. Parent feels more satisfaction by giving a bequest to its child. The utility function of the parent, then, can be expressed as follows.

$$U = U(C_t, B_t). \tag{17}$$

The subscript t denotes parent generation, C_t is parent's consumption, and B_t is inheritance (or bequest for its child).[8] For simplicity, assume homothetic utility function that some proportion of the parent income is allocated to its own consumption and another proportion to bequest for child

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[8] With the equal division, the relation between inheritance (I) and bequest (B) is $I_{t+1}=B_t/(1+n)$ where n is the number of children. In our model, since $n=1$, $I_{t+1}=B_t$. See Atkinson and Stiglitz (2015, pp. 222-231) concerning the issues on intergenerational transmission of wealth.

as a result of utility maximization.[9]   We can rewrite the relationships between Y_t and C_t or B_t as equation (18) and (19).

$$C_t = (1-\theta)Y_t, \tag{18}$$

$$B_t = \theta Y_t. \tag{19}$$

Y_t is the parent's income and θ is its bequest ratio. We assume that the wage rate w is given or exogenous. The equations (6) to (8), however, just show the income compositions of unskilled labor and skilled labor with and without liquidity constraint.   They are not the consequences of utility maximization. Under the homothetic utility function, dynamic income paths between generations can be derived by inserting the equation (19) into the equations (6-1) to (8-1), respectively. The results are

$$Y_{ut+1} = w + (1+r)\theta Y_t, \tag{20}$$

$$Y_{s\,t+1} = qw + (1+i)(\theta Y_t - c) \ \ \text{if} \ B_t < c, \tag{21}$$

$$Y_{s\,t+1} = qw + (1+r)(\theta Y_t - c) \ \ \text{if} \ B_t \geq c. \tag{22}$$

We investigate the dynamic income path and discuss the divergence of income distribution using equation (20) to (22). Suppose that $(1+r)\theta < 1$ and $(1+i)\theta > 1$,[10] which means that there is a wide difference between the lending rate and the borrowing rate. This assumption has an important role resulting in multiple equilibria. In other words, the degree of the spread

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[9] Davis and Kuhn (1991) and Becker and Tomes (1979) also analyze an intergenerational income distribution with a homothetic utility function. Davis and Kuhn (1991) assume that the parents' utility is a function of their own consumption and children's income. Becker and Tomes (1979) suppose that the parents' utility is a function of their own consumption and the bequest to children. The homothetic utility function leads to the result that the bequest has a linear relationship with the parent's income.

[10] The same assumption is also shown in the models of Galor and Zeira (1993, p. 41) and Tovik (1993, p. 589).

Figure 2   Divergence of Income Distribution
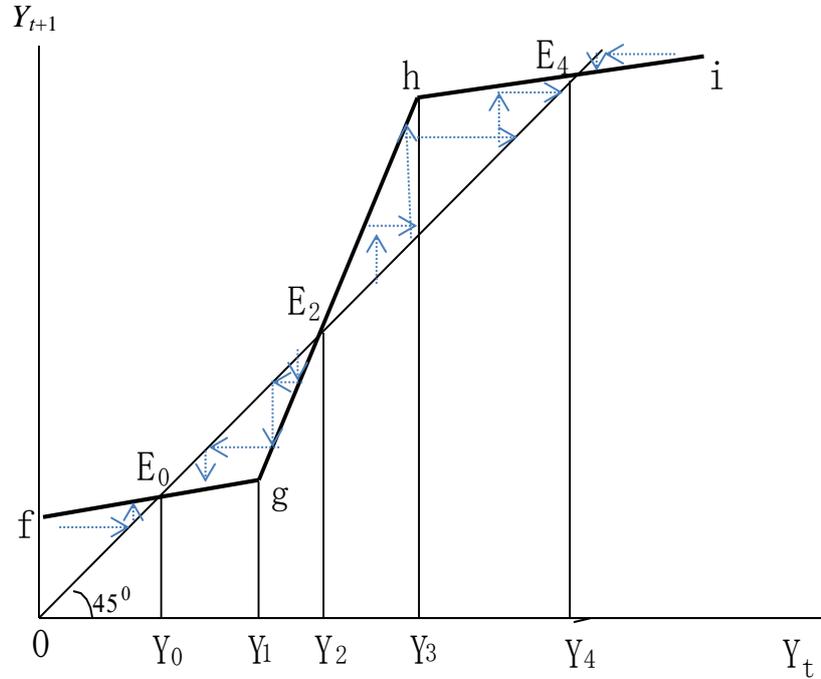
operates as an important factor in our dynamic income paths. Referring to figure 2, we try to explain the relationship between income distribution and the income paths (20) to (22).

The equilibrium is a point where income paths cross the 45 degree line from the origin. The lines *fg*, *gh*, and *hi* correspond to equation (20), (21), and (22), respectively. The line *fg* depicts the range where the individual works as unskilled labor and earns property income by lending the bequest. The line *gh* shows the interval in which the individual has to finance a part of education cost that exceeds the amount of bequest. And the *hi* line illustrates the range that the agent without liquidity constraint works as skilled labor. The high-ability individuals can also earn property income by lending the amount of bequest that exceeds education cost. The slope of *gh* is steeper than that of the lines *fg* and *hi*, because the individuals of the range of *gh*

with liquidity constraint have to borrow the money that falls short of the cost of education, paying high borrowing interest rate.

Income divergence in the presence of liquidity constraint and credit market imperfection can be shown by tracing the income paths in figure 2. There are three equilibrium points, E_0, E_2, and E_4. In figure 2, where Y_t is on the x-axis and Y_{t+1} is on the y-axis. Also, 45 degree line is drawn to show the equilibria and their stability. If we trace the income paths along the dotted arrow lines in figure 2, we can judge whether the equilibria are stable or not.[11] Among them, E_2 is an unstable equilibrium, E_0 and E_4 are stable equilibria. In the left side of E_2 (or Y_2), the income level converges towards the equilibrium E_0, whose income level is Y_0. In the right side of E_2, the income level converges towards the equilibrium E_4, whose income level is Y_4. The result shows that the income distribution gets worse or polarized.[12] The income groups are divided into two groups: the poor income group that converges into Y_0 and the rich income group that converges into Y_4.[13] We may call this phenomenon as 'divergence of income distribution'. When we face the divergence of income distribution, some policy measures are required, taking equity and efficiency criteria into account.

6. DISCUSSION ON POLICY TOOLS

Stiglitz (1969) shows that income distribution can be equalized completely as time passes when the macroeconomic model of neoclassical framework is applied to the model of income distribution among individuals. As explained in the previous section, however, income distribution gets worse when we consider liquidity constraint and credit market imperfection. Lee and Lee

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[11] In short, if the slope of each line at equilibrium is smaller (larger) than one (the slope of 45 degree line), then the equilibrium point is stable (unstable).

[12] Keum and Yi (2013) analyze a large part of the wage differential between regular and non-regular workers in Korea. They mention out that the large wage gap can be attributed to differences in human capital and productivity.

[13] The point E_0 is the point that corresponds to the poverty trap in macroeconomic growth model. See Banerjee and Newman (1994).

(2013, p. 647) point out that Korea faces bipolarization in financial access, and around two million people are estimated to face difficulties in financial access. According to their results, income inequality and polarization have proceeded while the potential growth rate has decreased in Korea since the economic crisis in 1997. They suggest some policies including the one to reduce the income inequality and economic bipolarization, but they do not discuss concrete policy measures.[14]

There are some policy measure possibilities. Government can influence the borrowing and the lending interest rates, wage levels, and the cost of education, and other factors. Policy measures are related to these parameters shown in income paths (20) to (22).[15] The policy measures to equalize income distribution among individuals closely relate to how we deal with liquidity constraint and credit market imperfection. As mentioned in section 4, the critical value of productivity of skilled labor depends on the amount of bequest, the education cost, the difference between the borrowing and the lending interest rates. If the government policies can elaborate on these parameters, it may also be able to prevent the income diversion from getting worse. For example, government can intervene in easing (reducing) the credit market imperfection, resulting in smaller gap between the borrowing and the lending interest rates: i.e., in the decrease of the spread. The policy results in reducing the slope of line hg to hb or hb' as shown in figure 3.

We cannot solve the divergence of income distribution in the case of hb, because there still remain income divergences towards E_0 and E_4. The divergence disappears in the case of line hb', because there is only one stable equilibrium at E_4. On the other hand, government can spend money for the poor using a welfare program, which can shift the line fg up to $f'g'$ in figure 3. As a result, the equilibrium E_0 (or income level Y_0) rises to E_0' (or Y_0'). Note that the government is confronted with budget constraint, however,

[14] Lee and Lee (2013, p. 646) suggest Korea's new economic growth strategies which cover (i) the creation of efficient productive economic opportunities, (ii) the reduction of income inequality and economic bipolarization, (iii) the improvement of human capacities especially in health and education, and (iv) the strengthening of social protection.
[15] In figure 2 each point Y_0 to Y_4 can be calculated related with policy measures or measures.

Figure 3   Some Policy Measures
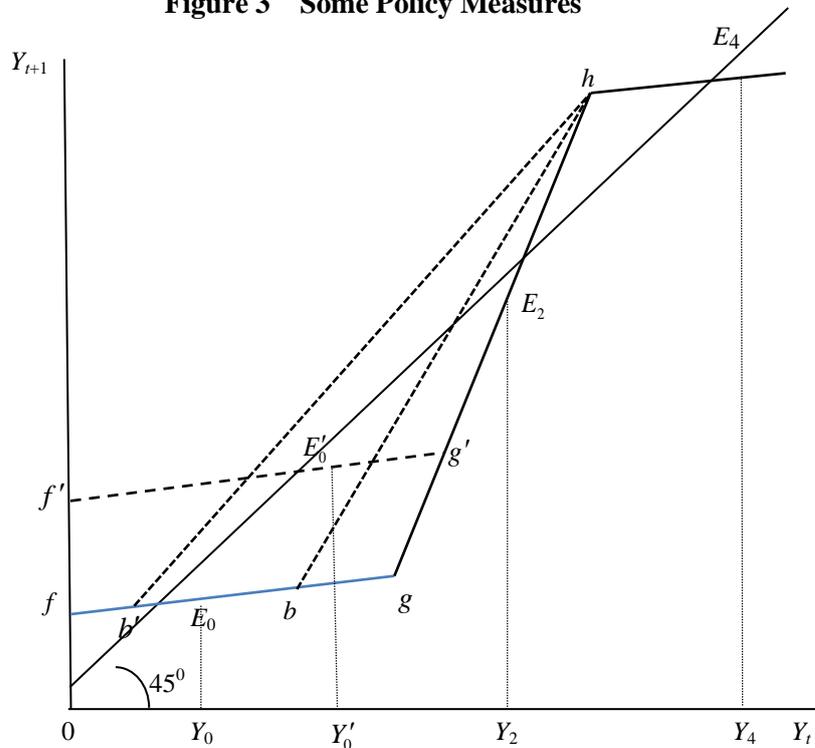
to implement any welfare programs. For example, policy authority may achieve income equalization through welfare programs for the poor by imposing a bequest tax to the rich. Equity as well as efficiency are important criteria when implementing the policies. A redistributive policy induces the income of the rich to decrease by way of lowering their work incentives, resulting in the decrease of total income in the society. This is a typical trade-off between equity and efficiency.   Hendel *et al.* (2005) state that imposing a progressive tax lowers the skill premium.

If government makes use of a progressive bequest tax only, low-ability individuals would take a relatively heavy burden by the policy under some circumstances. The reason is that government generally imposes relatively high taxes on the bequest with a heavy progressive tax schedule. Then, the individuals who obtain a large amount of bequest but do not invest in

education should pay large amount of bequest tax. Compared to them, relatively lower bequest tax would be imposed on the bequest of skilled labor who invests in education, because the skilled labor can use some portion of bequest for getting education. As a result, their bequest tax gets smaller. In other words, tax burden of skilled labor will be relatively lower than that of unskilled labor if labor income tax is not so progressive. Generally, bequest tax is more progressive than labor income tax. In addition, the labor income of skilled labor is higher than that of unskilled labor. Under these circumstances, a progressive bequest tax may lead to a lower burden on skilled labor relative to unskilled labor.[16] In short, making use of a heavy progressive bequest tax only can be unequitable if we do not take the effect of bequest on education or earning capacity into consideration.

7. DOUBLE EFFECTS OF REDUCING THE COST OF EDUCATION

7.1. Policy Measure

As discussed in section 2 and 3, Hendel *et al*. (2005) show that the reduction of the education cost decreases the demand of unskilled labor under the assumption of the signaling role of education explored by Spence (1973). However, they do not take the government budget constraint into account. Reducing the cost of education enables those who were confronted in liquidity constraint to ease for getting education. As a result, their income increases because reducing the cost of education decreases the critical value

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[16] There are two effects of taxation when individuals use the bequest to get education. Those are 'conversion effect' and 'deferral effect'. The conversion effect of taxation indicates that the bequest tax is converted into labor tax because the investment in education carried out by the bequest enhances the earning capacity. The bequest tax is not levied on the some portion when it is used for education. For example, in Japan, a lot of amount of gift or bequest is not taxed when parents (or grandparents) transfer the bequest to children for their education. The deferral effect of taxation occurs when the tax is not imposed at the period the individual gets education whereas the tax is imposed when the individual earns labor income in the future. See Kook (2014) in more detail.

Figure 4   A Policy to Prevent Income Distribution from Diverging
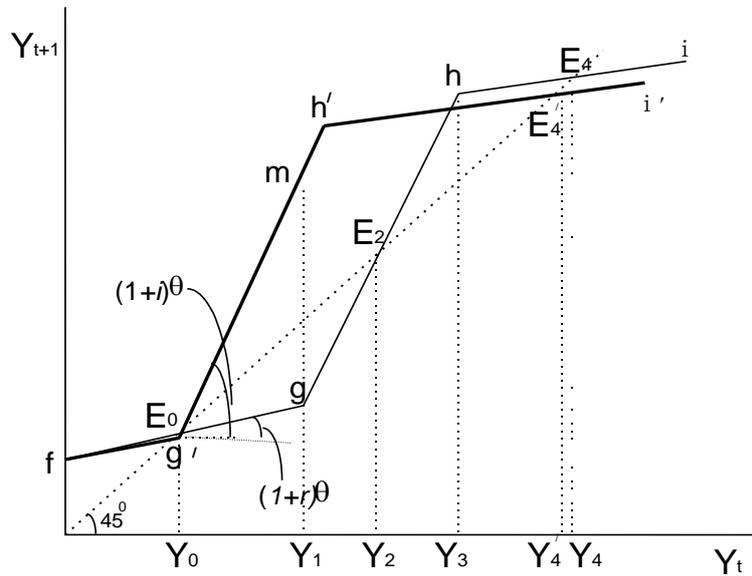
for high-ability individuals to get education as shown in equation (14).

Though the policy increases the income of the rich by the reduction of the cost of education, the rich who invest in education have to pay labor income (wage) tax at the same time. We try to calculate the net effect of the policy on the income of the rich. We need to consider both the income decreasing effect of income tax and the income increasing effect of the reduction in the education cost. We propose a redistributive policy in which the government decreases the cost of education by tax imposed on the income of skilled labor, considering the government budget constraint. What amount of the reduction in the cost of education will prevent the divergence of income distribution? Figure 4 helps an intuitive understanding regarding the policy measure.

In figure 4, the income path of *fghi* is the same as in figure 2. The method to avoid the divergence of income distribution is to shift the line *gh* up to

$g'h'$ or more by reducing the cost of education.[17] The line gh corresponds to the equation (21), from which we know that the line shifts upwards if the cost of education goes down. As shown in figure 4, it needs to finance at least the amount of $mg(=c^*)$ in order to move the line gh up to $g'h'$ or more to avoid the divergence of income distribution. How can we calculate the amount of reduction that shifts the line gh up to $g'h'$? We can derive the amount of the reduction from equation (21). In figure 4, the line $g'h'$, or the income path after reducing the cost of education, can be written as equation (23).

$$
\begin{aligned}
Y_{s\,t+1} &= qw + (1+i)(\theta Y_t - c_1) \\
&= qw + (1+i)(\theta Y_t - c) + (1+i)(c - c_1).
\end{aligned}
\tag{23}
$$

The c_1 is the cost of education after its reduction, and $c > c_1$. Therefore, c^* is expressed as follows.

$$
c^* = (1+i)(c - c_1).
\tag{24}
$$

Government should finance c^* for the reduction of the education cost. We propose a redistributive policy that imposes tax on the income of skilled labor. While the rich who invest in education benefit from the reduction of the education cost, they should take a burden of income tax. If tax authority imposes wage tax with tax rate τ, the tax revenue from the rich is τqw. We use equation (22) to evaluate the net effect of the policy on the rich. The policy shifts the income path (22) of the rich downwards as shown in equation (25).

$$
Y_{s\,t+1} = (1-\tau)qw + (1+r)(\theta Y_t - c_1).
\tag{25}
$$

From equation (25), we know that the tax payment moves the income path

---

[17] In order to prevent income distribution from diverging, it is necessary to implement a policy in which the income path gh is shifted up to crossing the 45 degree line only once in figure 4.

of the rich who invest in education downwards, whereas the reduction of the education cost shifts the path upwards. Whether the rich who invest in education will benefit or not from the policy depends on whether the income path of the rich moves upwards or downwards. Equation (25) can be rewritten as equation (26).

$$Y_{s\,t+1} = qw + (1+r)(\theta Y_t - c) - \tau qw + (1+r)(c - c_1). \tag{26}$$

Look at the relationship between the equation (22) and (25). From equation (26), the size of the movement of the income path, or net benefit of the rich, by the policy, or NB^r, can be expressed as,

$$NB^r = -\tau qw + (1+r)(c - c_1). \tag{27}$$

Equation (27) means that while the rich who invest in education are burdened by the tax τqw, their income increases to some extents from the effect of reducing the cost of education. The benefit size is the second term of RHS of equation (27), or $(1+r)(c - c_1)$. As mentioned above, the increase of tax burden shifts the income path, hi, downwards whereas the reduction of the cost of education shifts the path upwards. Under the balanced budget, the tax revenue is equal to the amount of reducing the cost of education. The balanced budget or government budget constraint can be written as equation (28).

$$c^* = \tau qw \ \text{ or } \ (1+i)(c - c_1) = \tau qw. \tag{28}$$

The second equation of (28) is obtained from equation (24). Inserting equation (28) into equation (27), we can calculate the net benefit of the policy. It becomes

$$NB^r = -(1+i)(c - c_1) + (1+r)(c - c_1) = -(i - r)(c - c_1) < 0. \tag{29}$$

Equation (29) shows that the net benefit of the rich is negative because the borrowing interest rate is higher than the lending interest rate by the assumption of credit market imperfection. As a result, the policy proposed here shifts the income path downwards with the size of $(i-r)(c-c_1)$. Figure 4 depicts it. The policy shifts the income path of hi downwards to $h'i'$ in figure 4.   The equilibrium E_4 changes to E_4', or the income level corresponding to the equilibrium decreases from Y_4 to Y_4'.

Though the policy may prevent the income distribution from diverging in the presence of liquidity constraint and credit market imperfection, it decreases of the equilibrium income level from Y_4 to Y_4'. This means that a trade-off occurs between the income equalization and the increase of income level at the equilibrium.[18]   The decrease of income caused by the redistributive policy suggested here, however, is smaller than the tax payment of the rich. This is because the redistributive policy not only results in income increase effect of the individuals with liquidity constraint but also raises some amounts of the income of the rich through reducing the cost of education.   The policy has a distinguishing feature compared to a conventional redistributive policy as it only transfers the tax revenue from the rich directly to the poor.

7.2. Double Effects of the Redistributive Policy

The labor income tax imposed on the rich who invest in education is used for financing the reduction of the education cost. Reducing the cost of education generates the effect of increasing the income of not only the individual with liquidity constraint but also, to some extent, of the rich without it. As shown in inequality (14), the reduction of the education cost decreases the critical value of the bequest of high-ability individuals with

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[18] If a technological innovation that increases overall income level in the economy occurs, there may be no trade-off between income inequality and income growth. The case would be the situation that the line of *gh* in figure 4 moves upwards sufficiently to the extent that the line of *ghi* intersects the 45 degree line only once and the equilibrium converges into the high income level. Refer to Kuznets (1955) about the classical relations between economic growth and income inequality in macro economy.

liquidity constraint,[19] which increases their income. In figure 4 the reduction of the education cost from c to c_1 shifts the income path (i.e., gh) of the individual with liquidity constraint to $g'h'$.[20]

Note that the individual with liquidity constraint has more income increase effect than the rich without it. It is because the policy results in double effects of income increase for high-ability individuals with liquidity constraint: direct income increase effect by reducing the cost of education and the income increase effect by lessening the liquidity constraint. While the former effect also benefits the rich, the latter effect benefits only the high-ability individuals with liquidity constraint. The latter effect originates from the existence of credit market imperfection.

The double effects can be described more precisely using an equation. According to equation (23), the income increase effect by the policy which benefits high-ability individuals with liquidity constraint is $(1+i)(c-c_1)$. The high-ability individuals who invest in education obtain double effects. We can decompose $(1+i)(c-c_1)$ into two terms as the following.

$$(1+i)(c - c_1) = (1+r)(c - c_1) + (i - r)(c - c_1).   (30)$$

The first term of the RHS in equation (30) is a direct income increase effect by the reduction of the education cost. Equation (29) shows that the rich have the same effect with the exact size of the first term of $(1+r)(c-c_1)$ in equation (30). The second term of the RHS in equation (30) is the income increase effect caused by easing the liquidity constraint. The effect of the second term originates from credit market imperfection, which is reflected in the difference between the borrowing interest rate and the lending interest rate, which is called 'spread'. Therefore, we may rename the income increase effect caused by reducing the liquidity constraint as 'spread effect'. From the equation (29) we can observe that the decrease of income of the

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[19] On the other hand, the bequest tax will change the slope of the income path.

[20] If the income paths are non-linear, they will become curve–shapes. In those cases many multiple equilibria may appear. The qualitative feature between the income level and the income distribution, however, will not be changed.

rich who burden taxes is as large as the 'spread effect'. In other words, the amount of income transfer from the rich to the individuals with liquidity constraint is the same as the 'spread effect'.

8. CONCLUDING REMARKS

Kim and Choi (2010, p. 413) construct a measure of *ex ante* college premium and show that the premium is positively related to individual wealth endowment. They conclude that redistribution raises the *ex ante* college premium for relatively poor people and decreases that for relatively rich people. They offer the estimation results, however, lacking in theoretical discussion. The important role of human capital relates to the earning capacity through which individuals are able to raise their income level. We showed the possibility of the divergence of income distribution in the presence of liquidity constraint and credit market imperfection.

After the discussion on the static basic model, we derived the dynamic income paths from the model of intergenerational transfers. We also suggested a redistributive policy to avoid the divergence of income distribution.[21] The policy has some characteristics distinguished from a conventional redistribution that only transfers the income from the rich to the poor. Though the rich skilled labor should pay the income (wage) tax, the policy proposed here has an income increase effect even for the rich who invest in education through the reduction of the cost of education.

The redistributive policy generates double effects. One is the direct income increase effect by reducing the cost of education and the other is the income increase effect by lessening the liquidity constraint. While the rich who invest in education obtain only the former effect, the individuals with liquidity constraint benefit from both effects. The latter effect originates

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[21] Kopczuk (2013) provides excellent surveys concerning the issues on the taxation of intergenerational transfers and wealth, including the topics of behavioral response to transfer taxation and redistribution. Also Atkinson and Stiglitz (2015, Ch. 9) explain the distributional effect of taxation.

from the credit market imperfection, reflected in the difference between the borrowing interest rate and the lending interest rate, or spread. We renamed the latter effect as 'spread effect'.

Our discussion is basically related to private human capital investment because the individuals choose to invest in human capital with the assumptions of the liquidity constraint and of the credit market imperfection. In addition to the private sector, the government can strongly affect the income distribution among individuals by measures such as providing public education and regulating private education. Further studies are required to analyze the role of government relating to public education and income inequality.[22] On the other hand, Lucas (1988) focuses on the sources of economic development emphasizing the externality of human capital. He points out that human capital is a critically important factor for economic development.   Also Romer (1986) specifies the increasing marginal productivity of human capital. We did not consider an explicit model, however, concerning the external effect of human capital.

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[22] Glomm and Ravikumar (1992) analyze the effect of private vs. public human capital investment on income inequality.

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