

Education Competition and Redistribution^{*}

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The paper develops a general equilibrium model of education competition where entry into the higher education sector is a costly choice due to limited accommodation. We show that wealth distribution plays a crucial role in determining the extent of education competition. Redistribution via a simple tax-and-subsidy program may increase or decrease the extent of education competition depending on the initial configuration of the equilibrium. We also construct a measure of college premium and show that the redistribution program reduces the relative college premium for rich individuals.

JEL Classification: D3, I2

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1. INTRODUCTION

There is a significant gap in the average earnings between a four-year college graduate and a high school graduate, at about 30% and sometimes higher in South Korea. Depending on the universities and majors, this gap can be much higher, which makes competition to enter those universities extremely fierce. This provides individual households with pecuniary incentives to make efforts to send their kids to colleges and universities, especially good ones.¹⁾

It is well known that private education spending in Korea is quite high relative to other countries. Korean households spent the highest fraction of income on private education among OECD countries in 2004. This was more on private tutoring, at about 2% of GDP in 2009, than on tuitions and dormitories in private schools. Korean government offers various policies and programs to reduce private education expenditures every year to no avail.

One of the concerns is that a large part of time and money spent in the education competition is wasted. Due to the limited capacity of universities, especially the good ones, the graduates of those universities would enjoy rents.²⁾ It is then natural to think of households as engaging in rent seeking activities to enter the universities, which leads to the plausible possibility of social loss due to the education competition.

Although most of modern states provide either public education or public subsidies for education to their citizens, a substantial size of private education also coexists. In the literatures of human capital accumulation and endogenous growth, private education is generally accepted as a crucial component of human capital generation. For example, in Glomm and Ravikumar (1992) and Glomm and Kaganovich (2003), private education is an essential mechanism of human capital accumulation.³⁾

¹⁾ For a study of returns to elite college education in the US, see Brewer *et al.* (1999).

²⁾ Strayer (2002) finds that high school quality itself shows no direct significant association with earnings in the labor market. Instead, he finds a significant association of high school quality with earnings through college choices.

³⁾ They focus on the technology of human capital generation — proper combinations of public

We focus on private education as a competitive effort to realize the returns to human capital as well as an investment in human capital *per se*. There exists a literature on the market for higher education that focuses on competition among colleges to attract students (e.g., Epple *et al.*, 2006). There is also a literature on school choice by the households (e.g., De Fraja, 2002). Studies on the competition among students to enter colleges are relatively scarce. We add the element of rent seeking to education competition by adopting the “contest success function” to describe the probability of successful rent seeking through higher education.

In the context of rent-seeking literature, human capital accumulation may induce an entry competition for taking prize, and thus it requires private efforts that entail social costs. The opportunity for human capital accumulation may not be open to all. Rather, it seems to be fairly restrictive in the sense that the higher education sector has limited accommodation and thus competition for entry will prevail. As a result, the population will be partitioned into two groups: rent-seekers and producers as in Grossman and Kim (1996) and Murphy *et al.* (1993).

Given the cost to enter colleges and universities, acquiring higher education may not be an optimal choice for some people due to several reasons. The distribution of ability affects the choice of human capital investment in Mincer (1958) that is followed by a large empirical literature on returns to education. We will consider the distribution of the initial wealth endowment as the source of heterogeneity in the human capital investment.

An interesting finding in the empirical literature is that the return to schooling is much higher in the right tail of the income distribution, which suggests that the return to education or the college premium itself has a distribution. Our model below will associate the distribution of human capital investment and the resulting distribution of college premium with the distribution of initial wealth.

In what follows, we begin by setting up the analytical framework that

allows a continuum of agents with varying endowments to choose whether or not to invest in human capital. Then we analyze the configurations of the equilibrium and relationship between wealth distribution and composition of population which reflects simple aggregation of each individual's type. Furthermore, we will study the redistribution effects on this economy. We will also construct a measure of college premium and study the redistribution effects on inequality.

2. ANALYTICAL FRAMEWORK

Consider a simple general equilibrium model of a closed society with a continuum of agents. Each individual is indexed by $i \in \Omega = [0, 1]$. Assume that the initial wealth is distributed according to a distribution $F(\cdot)$ in $(0, \infty)$. Each person takes his initial wealth ω_i as given. The economy is composed of two distinctive sectors. Sector 1 is characterized by a conventional production technology with a diminishing marginal product. Sector 2 requires a college degree and uses a linear technology in production.

Given the initial wealth, each person chooses where to join, either sector 1 or sector 2.⁴⁾ Let individuals who choose to engage in sector 2 be classified as an aptitude type 2 and let individuals who choose to be in sector 1 be classified as an aptitude type 1 for simplicity. Individuals who want to choose aptitude type 2 face an essential trade off: allocation of the initial wealth between education competition and human capital investment. Each person decides these decisions individually and independently. Let θ be a nonnegative fraction of people who choose an aptitude type 2 and let $\Theta \equiv \theta / (1 - \theta)$ denote the ratio of people who choose an aptitude type 2 to aptitude type 1. The parameter Θ also measures the intensity of competition in sector 2.

⁴⁾ Murphy *et al.* (1993) and Grossman and Kim (1996) have constructed models where both individuals of two different aptitude types coexist in one economy. In this setting, the equilibrium allows coexistence of individuals with different aptitude types without identification of each individual.

A person who chooses aptitude type 1 can expect his or her consumables q_{1i} as

$$q_{1i} = \alpha \sqrt{\omega_i}. \quad (1)$$

In equation (1), α is the productivity parameter of sector 1. The production function exhibits diminishing marginal product technology.

A person who chooses to be of aptitude type 2 has to divide his or her initial resources into two parts, allocation for education competition and human capital investment. Thus their budget constraint can be expressed as $\omega_i = e_i + h_i$, where e_i is the expenditure for education competition and h_i is the resource allocation for human capital investment.⁵⁾ Let x_i be the ratio of expenditure for education competition to human capital investment, i.e., $x_i \equiv e_i / h_i$.

Due to limited accommodation in the colleges and universities, spending on education competition does not guarantee entry into the higher education sector. We adopt a standard contest success function to describe the probability of successful entry into the higher education sector:⁶⁾

$$p_i = \begin{cases} \frac{1}{1 + \Theta / x_i} & \text{for } x_i > 0 \\ 0 & \text{for } x_i = 0 \end{cases}. \quad (2)$$

According to equation (2), the probability of success is strictly increasing in one's allocation to education competition (x_i) and strictly decreasing in the intensity of education competition (Θ).⁷⁾ More severe competition

⁵⁾ One may argue that some activities can be good for both education competition and human capital competition. We do not consider such activities for simplicity.

⁶⁾ The probability p_i is not a contest success function itself in that it is not subject to logit condition (Dixit, 1987): that $\sum_i p_i = 1$, which is required for desirable contest success function (Hirshleifer, 2000). Rather, p_i roughly approximates uncertainty in future pay-off of sector 2 that originates from entry competition.

⁷⁾ Hirshleifer suggests that a standard contest success function of ratio form takes the form of

dilutes probability of success and thus it plays a critical role in determining aptitude type for each person by lowering expected return from choosing an aptitude type 2. More resource allocation toward education competition leads to higher probability of success.

With a college degree, one produces in sector 2 using a linear technology. Let γ be the productivity parameter of sector 2. Then the expected output for an aptitude type 2 is given as

$$q_{2i} = \gamma p_i h_i. \quad (3a)$$

In equation (3a), the production of consumable is linear in h_i , which contains uncertainty captured by the contest success function p_i . By using the relation such that $h_i = \omega_i / (1 + x_i)$, we can reformulate equation (3a) into

$$q_{2i} = \gamma p_i \frac{\omega_i}{1 + x_i}. \quad (3b)$$

Equation (3b) implies the trade-off relation in allocating resources of individuals of an aptitude type 2. Specifically, raising the relative size of resource allocation to education competition leads to higher probability of success p_i at the expense of lower allocation to human capital investment that leads to lower production of consumables.

3. INDIVIDUAL CHOICE

Individuals choose their own aptitude types, that is, whether to engage in sector 1 or sector 2. Those who choose to engage in sector 2 make an

$p_i = (b_1 F_1)^m / [(b_1 F_1)^m + (b_2 F_2)^m]$ where b_i is effectiveness parameter for agent i and m is decisiveness parameter in two agent model. The probability of success defined in present model resembles special case of contest success function of ratio form with decisiveness parameter, $m = 1$ and with relative effectiveness, $b_i / b_j = 1$ for $i \neq j$.

additional choice of resource allocation between education competition to receive higher education and human capital investment. We first study the latter then move on to the former.

If a person chooses to engage in sector 2, he or she has to decide how to allocate the initial wealth in order to maximize q_{2i} as given in equation (3b). This problem can be rewritten as follows:

$$\max_{\{x_i\}} q_{2i} = \gamma \frac{x_i}{x_i + \Theta} \frac{\omega_i}{1 + x_i}. \quad (4)$$

An individual chooses the ratio x_i taking Θ as given. The first order necessary condition, $dq_{2i} / dx_i = 0$, implies the following resource allocation rule:

$$x_i = \sqrt{\Theta}. \quad (5)$$

Given the optimal allocation rule in equation (5), we can characterize the probability of successful human capital investment as inversely related to the intensity of education competition (Θ):

$$p_i = \frac{1}{1 + \sqrt{\Theta}}. \quad (6)$$

Using equation (6), we can express the maximized value of engaging in sector 2 as follows:

$$q_{2i} = \frac{\gamma \omega_i}{(1 + \sqrt{\Theta})^2}. \quad (7)$$

Now we turn to the individual choice of an aptitude type. To decide whether or not to invest in human capital, and thus join in education competition, each person compares the expected returns to engaging in the

two sectors, as given by equations (1) and (7). To analyze this decision problem, we construct a criterion function ρ for the choice of an aptitude type as follows:

$$\frac{q_{2i}}{q_{1i}} = \frac{\sqrt{\omega_i}}{\lambda(1+\sqrt{\Theta})^2} \equiv \rho(\omega_i; \Theta, \lambda), \quad (8)$$

where $\lambda \equiv \alpha/\gamma$ is the relative productivity parameter. Equation (8) suggests that the individual choice of an aptitude type hinges on the relation $\rho(\omega_i; \Theta, \lambda) \gtrless 1$: If $\rho > 1$, then an individual chooses to engage in sector 2. If $\rho < 1$, then an individual chooses to engage in sector 1. If $\rho = 1$, an individual has no strict preference between the two aptitude types.

The criterion function $\rho(\omega_i; \Theta, \lambda)$ gives the relative return to participating in education competition and engaging in sector 2. Thus, we interpret $\rho(\omega_i; \Theta, \lambda)$ as the *ex ante* college premium. Note that the *ex ante* college premium depends on the individual endowment as well as the intensity of education competition and the relative productivity of the two sectors. Because ρ is a monotone increasing function of ω_i , individuals with larger endowments choose to engage in sector 2 whereas individuals with smaller endowments choose to engage in sector 1, given the intensity of education competition and relative productivity. Thus, given Θ and λ , equation (8) identifies the threshold level of wealth, denoted by $\tilde{\omega}$, with which an individual is indifferent between choosing sector 1 and choosing sector 2:

$$\tilde{\omega} = \lambda^2(1+\sqrt{\Theta})^4 \equiv \tilde{\omega}(\Theta, \lambda), \quad (9)$$

where $\partial\tilde{\omega}(\Theta, \lambda)/\partial\Theta > 0$, $\partial\tilde{\omega}(\Theta, \lambda)/\partial\lambda > 0$ and $\tilde{\omega}(0, \lambda) = \lambda^2$. Given Θ and λ , all individuals with $\omega_i > \tilde{\omega}(\Theta, \lambda)$ choose to participate in education competition and engage in sector 2, and all individuals with $\omega_i < \tilde{\omega}(\Theta, \lambda)$ choose not to participate in education competition and engage in sector 1.

4. EQUILIBRIUM

So far, we have studied the problem of individual choices given the intensity of education competition. The intensity of education competition, however, is determined in equilibrium as a result of individual choices in the aggregate. We use the fixed point argument to find equilibrium.

From equations (8) and (9), it is straightforward to see that $\rho \geq 1$ if and only if $\omega_i \geq \tilde{\omega}$. Thus, the relative size of the two aptitude types in equilibrium must be such that

$$\Theta = \frac{1 - F(\tilde{\omega})}{F(\tilde{\omega})}. \quad (10)$$

By substituting $\tilde{\omega}$ with $\tilde{\omega}(\Theta, \lambda)$, we have

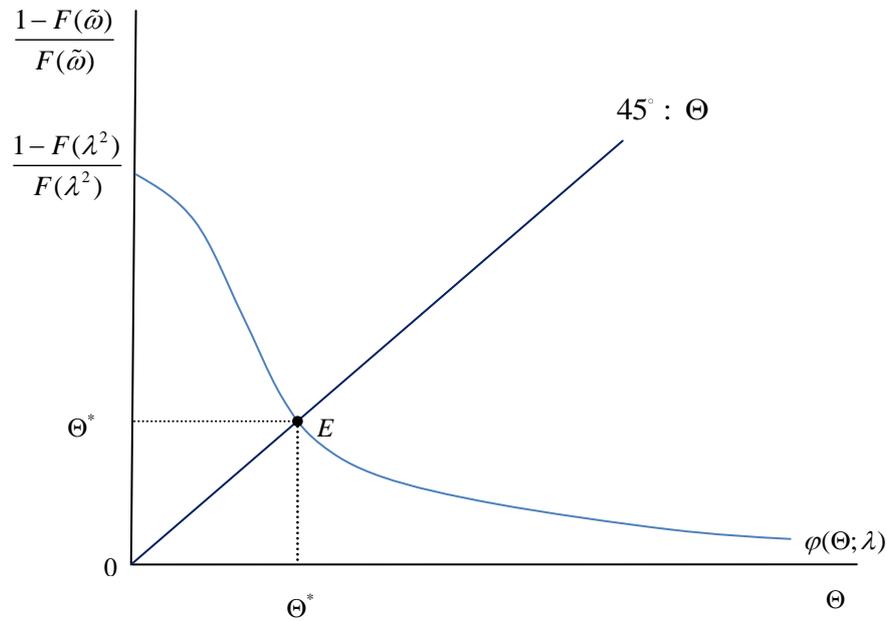
$$\Theta = \frac{1 - F(\tilde{\omega}(\Theta, \lambda))}{F(\tilde{\omega}(\Theta, \lambda))} \equiv \varphi(\Theta; \lambda), \quad (11)$$

where $\partial\varphi/\partial\lambda \leq 0$, $\partial\varphi/\partial\Theta \leq 0$, $\varphi(0; \lambda) = (1 - F(\lambda^2))/F(\lambda^2) > 0$, and $\lim_{\Theta \rightarrow \infty} \varphi(\Theta; \lambda) = 0$.

Equilibrium is defined as the relative size of the two aptitude types, Θ , that satisfy equation (11). Because $\varphi(\Theta; \lambda)$ is continuous, $\varphi(0; \lambda) > 0$, and $\lim_{\Theta \rightarrow \infty} \varphi(\Theta; \lambda) = 0$, an equilibrium Θ^* that solves equation (11) exists in $[0, \infty)$ as illustrated in figure 1. The uniqueness of equilibrium follows from $\partial\varphi(\Theta; \lambda)/\partial\Theta \leq 0$.

Using equations (5) through (9), we obtain a general equilibrium which is completely characterized by Θ^* . From equation (5), the equilibrium resource allocation ratio for each person of aptitude type 2 is given by

$$x_i^* = \sqrt{\Theta^*}. \quad (12)$$

Figure 1 Existence and Uniqueness of Θ^* 

From equation (6), the probability of successful human capital investment in equilibrium is given by

$$p_i^* = \frac{1}{1 + \sqrt{\Theta^*}}. \quad (13)$$

From equation (7), the expected return to engaging in sector 2 at equilibrium is given by

$$q_{2i}^* = \gamma \frac{\omega_i}{(1 + \sqrt{\Theta^*})^2}. \quad (14)$$

Combining equations (8) and (9), we obtain the *ex ante* college premium in equilibrium as follows:

$$\rho(\omega_i; \Theta^*, \lambda) = \sqrt{\frac{\omega_i}{\tilde{\omega}(\Theta^*, \lambda)}} \text{ for all } i \in \Omega. \quad (15)$$

Note that $\rho(\omega_i; \Theta, \lambda) / \partial \omega_i > 0$. Equation (15) implies that the *ex ante* college premium for each individual is a monotone increasing function of his or her initial wealth relative to the threshold level $\tilde{\omega}(\Theta, \lambda)$ in a given equilibrium.

5. COMPARATIVE ANALYSIS

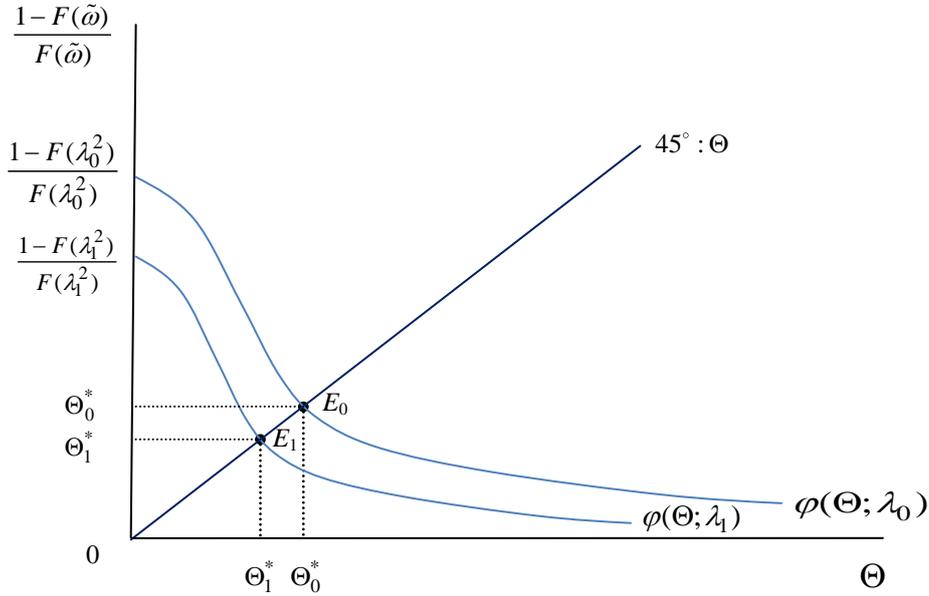
5.1. Productivity Shocks

We now analyze the effects of a change in relative productivity between two sectors on individual choices and the equilibrium. Suppose that the relative productivity of sector 1, denoted by λ , increases by exogenous forces such as technological progress bias toward sector 1 or by some regulations on sector 2 that reduces the expected returns to higher education. According to equation (11), an increase in λ lowers Θ^* as illustrated in figure 2. An increase in λ makes engaging in sector 2 relatively less attractive, and thus lowers the intensity of education competition, Θ^* . According to equations (12) and (13) this leads to a lower x_i^* and a higher p_i^* .

According to equation (10), lower intensity of education competition implies a higher threshold level of wealth, $\tilde{\omega}(\Theta^*, \lambda)$, given the distribution, $F(\cdot)$. Then, equation (15) says that the *ex ante* college premium, $\rho(\omega_i; \Theta^*, \lambda)$, decreases for all individuals accordingly. In sum, we have the following proposition:

Proposition 1: *Any relative productivity shock biased favorably toward sector 1 leads to lower level of education competition. That is, $\partial \Theta^* / \partial \lambda \leq 0$. Furthermore, the general equilibrium effects are such that $\partial \tilde{\omega} / \partial \lambda > 0$, $\partial x_i^* / \partial \lambda < 0$, and, $\partial p_i^* / \partial \lambda > 0$. The *ex ante* college premium decreases accordingly.*

Figure 2 The Effect of an Increase of λ on Θ^* for $\lambda_0 < \lambda_1$



5.2. Distributional Shocks

This section analyzes the effects of distributional changes. Consider an arbitrary distributional shock that changes the distribution of endowments from $F(\cdot)$ to $G(\cdot)$. The distributional shock may affect individual optimization for two reasons: First, for some individuals, the distributional shock may change their own endowments from ω_i^F to ω_i^G and affect their *ex ante* college premium in the given equilibrium from $\rho(\omega_i^F; \Theta^F, \lambda)$ to $\rho(\omega_i^G; \Theta^F, \lambda)$ such that they change their choices of aptitude type. Second, these changes then may affect the relative size of aptitude types, Θ , which feeds back to the individual *ex ante* college premium. In the end, we will have a new equilibrium, Θ^G .

As a result of a distributional shock, the relative size of aptitude types in equilibrium, Θ^* , and the resulting threshold level of wealth, $\tilde{\omega}(\Theta^*, \lambda)$, may increase or decrease depending on the nature of the shock. For convenience, let us denote the threshold level of wealth in the initial

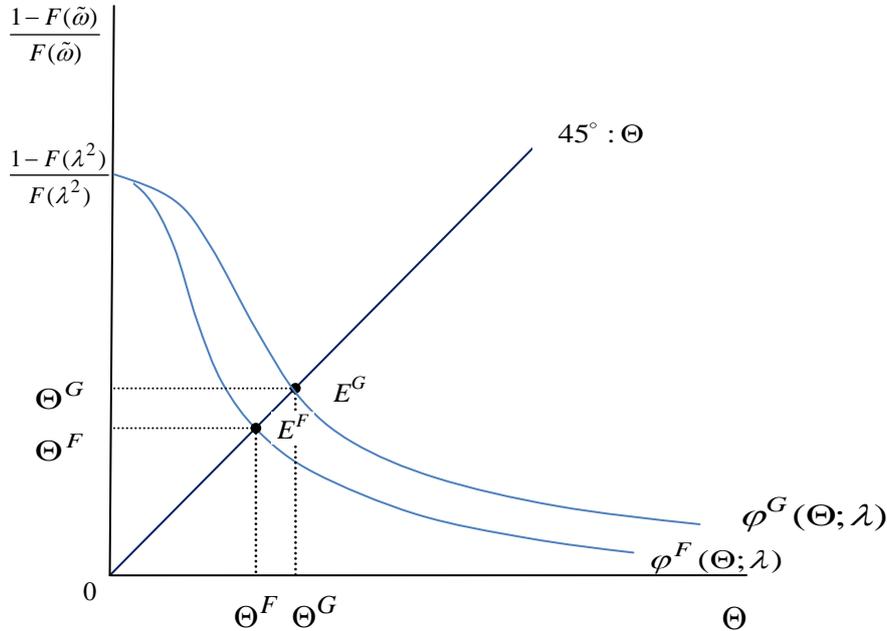
equilibrium by $\tilde{\omega}^F \equiv \tilde{\omega}(\Theta^F, \lambda)$ and the threshold level of wealth in the equilibrium after the distributional shock by $\tilde{\omega}^G \equiv \tilde{\omega}(\Theta^G, \lambda)$. Consider first a distributional shock that does not affect the endowment ranking of an individual with $\tilde{\omega}^F$, that is, $F(\tilde{\omega}^F) = G(\tilde{\omega}^F)$. In this case, $\Theta^F = (1 - F(\tilde{\omega}^F)) / F(\tilde{\omega}^F) = (1 - G(\tilde{\omega}^F)) / G(\tilde{\omega}^F) = \Theta^G$, that is, the equilibrium does not change. Because $F(\tilde{\omega}^F) = G(\tilde{\omega}^F)$, the distributional shock does not change the number of people whose *ex ante* college premium is larger than one.

Consider now a distributional shock that lowers the endowment ranking of an individual with $\tilde{\omega}^F$ that is, $F(\tilde{\omega}^F) > G(\tilde{\omega}^F)$.⁸⁾ In this case, it must be that $\Theta^F < \Theta^G$. Otherwise, there is a contradiction. Suppose that $\Theta^F = \Theta^G$, which implies $\tilde{\omega}^F = \tilde{\omega}(\Theta^G, \lambda)$ and $F(\tilde{\omega}^F) > G(\tilde{\omega}^G)$. Then, $\Theta^F = (1 - F(\tilde{\omega}^F)) / F(\tilde{\omega}^F) < (1 - G(\tilde{\omega}^G)) / G(\tilde{\omega}^G) = \Theta^G$, which is a contradiction. Suppose that $\Theta^F > \Theta^G$, which implies, due to the monotonicity of $\tilde{\omega}(\Theta, \lambda)$, $\tilde{\omega}^F = \tilde{\omega}(\Theta^F, \lambda) > \tilde{\omega}(\Theta^G, \lambda) = \tilde{\omega}^G$ and $F(\tilde{\omega}^F) > G(\tilde{\omega}^F) > G(\tilde{\omega}^G)$. Then, $\Theta^F = (1 - F(\tilde{\omega}^F)) / F(\tilde{\omega}^F) < (1 - G(\tilde{\omega}^G)) / G(\tilde{\omega}^G) = \Theta^G$, which is a contradiction. Figure 3 shows the effect of this distributional shock on Θ^* . Similarly, we can show that a distributional shock that raises the endowment ranking of an individual with $\tilde{\omega}^F$ reduces the intensity of education competition.

By carrying this qualitative result over to equation (12) and (13), we can analyze the general equilibrium effect of these distributional shocks. If the distributional shock raises the equilibrium intensity of education competition ($\Theta^F < \Theta^G$), then the relative allocation to education competition increases ($x_i^F < x_i^G$) and the probability of successful human capital investment decreases ($p_i^F > p_i^G$). By analogy, we can deduce that an exogenous distributional shock that reduces the fraction of people whose wealth is higher than current threshold level of wealth — that is, if $F(\tilde{\omega}^F) < G(\tilde{\omega}^F)$ holds — have opposite effects on Θ^* , $\tilde{\omega}$, x_i^* , and p_i^* . Proposition 2 summarizes these qualitative results.

⁸⁾ These kinds of distributional shocks are clearly competition-accelerating in a sense that they monotonically lead to more severe entry competition in sector 2. For example, proportional growth and lump-sum fashion growth induce such distributional changes.

Figure 3 The Effect of Positive Distributional Shocks on Θ^*



Proposition 2: An exogenous distributional shock that changes wealth distribution from $F(\cdot)$ to $G(\cdot)$, has general equilibrium effects such that $\Theta^G \geq \Theta^F$, $\tilde{\omega}^G \geq \tilde{\omega}^F$, $x_i^G \geq x_i^F$, $p_i^G \leq p_i^F$ if and only if $G(\tilde{\omega}^F) \leq F(\tilde{\omega}^F)$.

Note that, however, the effect on expected pay-off is not clarified without imposing more restrictions on specific forms of distributional changes since expected pay-off depends on the identity and status of each individual. The prediction is confined to an analysis on the effect of distributional changes on the configuration of aggregate economy, and thus the model does not give any information on individual-specific behavior without further specification of exogenous distributional shocks.

An immediate result that follows Proposition 2 is that, if the wealth endowment of each and every individual grows at the same positive rate, then the intensity of education competition rises because more people can afford such competition.

5.3. Redistribution by Tax-and-Subsidy

Redistribution by the government is an example of exogenous distributional shocks to initial endowments. We now consider a specific redistribution program of tax-and-subsidy: $\omega_i^G = (1 - \tau)\omega_i^F + \tau\omega$, where $\tau \in (0, 1)$ is common flat tax rate and ω is the average wealth. We step aside from the issues of political feasibility and enforceability of this program and focus on its effects. Note that this distributional shock affects neither the average wealth nor the endowment raking of individuals.

The general equilibrium effects of the tax-and-subsidy program hinges crucially on the configuration of the equilibrium before the policy shock. Specifically, if the threshold level of wealth is initially lower than the average, i.e., $\tilde{\omega}^F < \omega$, then $F(\tilde{\omega}^F) > G(\tilde{\omega}^F)$ holds. Thus, it follows from Proposition 2 that the tax-and-subsidy program increases the intensity of education competition, induces more resources to education competition, and lowers the probability of successful human capital investment. The new threshold level of wealth will be higher than the old threshold level of wealth: $\tilde{\omega}^F < \tilde{\omega}^G$. This result obtains because redistribution increases the wealth of some of the initially poor individuals and, hence, push their *ex ante* college premium above one.

If the threshold level of wealth is initially higher than the average, i.e., $\tilde{\omega}^F > \omega$, then $F(\tilde{\omega}^F) < G(\tilde{\omega}^F)$ holds. Thus, it follows from Proposition 2 that the tax-and-subsidy program decreases the intensity of education competition, induces less resources to education competition, and raises the probability of successful human capital investment. The new threshold level of wealth will be lower than the old threshold level of wealth: $\tilde{\omega}^F > \tilde{\omega}^G$. This result obtains because redistribution decreases the wealth of some of the initially rich individuals and, hence, push their *ex ante* college premium below one. We summarize the general equilibrium effects of redistribution by tax-and-subsidy as follows:

$$\frac{\partial \Theta^*}{\partial \tau} \geq 0, \quad \frac{\partial \tilde{\omega}}{\partial \tau} \geq 0, \quad \frac{\partial x_i^*}{\partial \tau} \geq 0, \quad \frac{\partial \rho_i^*}{\partial \tau} \leq 0 \quad \text{if and only if} \quad \tilde{\omega}^F \leq \omega.$$

We now turn to the effect of redistribution on the *ex ante* college premium. We proceed by examining the ratio of the *ex ante* college premium after redistribution (ρ_i^G) to the *ex ante* college premium before redistribution (ρ_i^F).

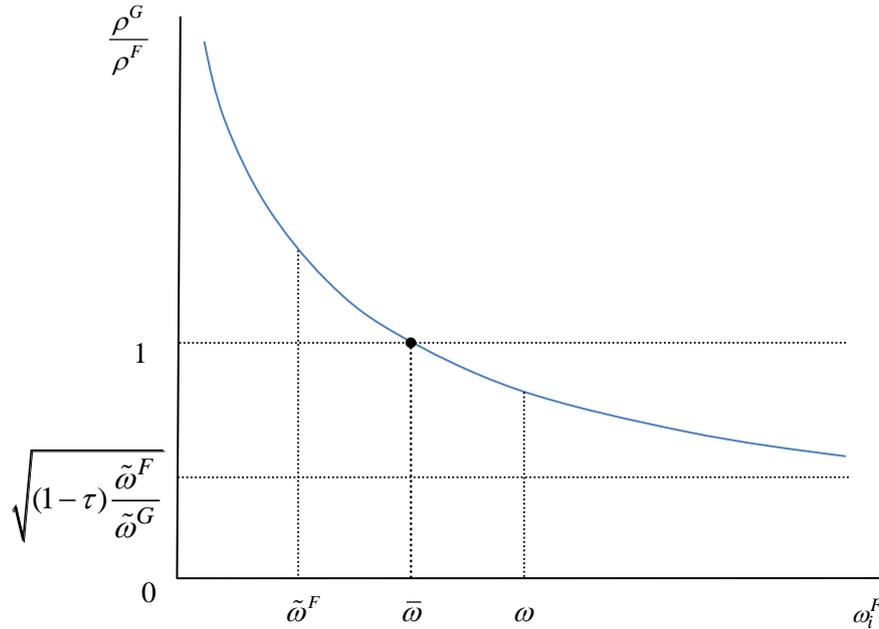
$$\frac{\rho_i^G}{\rho_i^F} = \sqrt{\frac{\omega_i^G / \tilde{\omega}^G}{\omega_i^F / \tilde{\omega}^F}} = \sqrt{\frac{\tilde{\omega}^F}{\tilde{\omega}^G}} \sqrt{\frac{(1-\tau)\omega_i^F + \tau\omega}{\omega_i^F}} \quad \text{for all } i \in \Omega. \quad (16)$$

Note that $\tilde{\omega}^F / \tilde{\omega}^G$ is independent of ω_i^F and $\tilde{\omega}^F / \tilde{\omega}^G \geq 1$ if and only if $\tilde{\omega}^F \geq \omega$.

Our goal is to see if we can identify individuals whose *ex ante* college premiums rise or fall due to redistribution by tax-and-subsidy. Observe first that in equation (16), ρ_i^G / ρ_i^F is a decreasing function of ω_i^F . The effect of redistribution on the *ex ante* college premium is more favorable to poor individuals than to rich individuals. If ω_i^F is sufficiently small, then $\rho_i^G / \rho_i^F > 1$. If ω_i^F is sufficiently large, then we have $[(1-\tau)\omega_i^F + \tau\omega] / \omega_i^F < 1$. However, because $\tilde{\omega}^F / \tilde{\omega}^G \geq 1$ if and only if $\tilde{\omega}^F \geq \omega$, it is not obvious from equation (16) to confirm $\rho_i^G / \rho_i^F < 1$ for a sufficiently large value of ω_i^F .

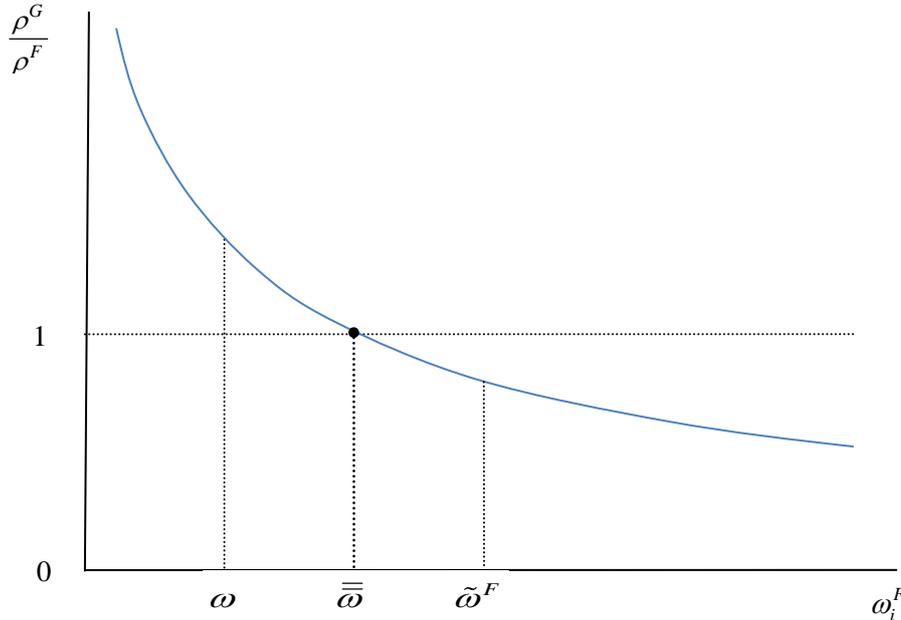
To show that redistribution lowers the *ex ante* college premium for relatively rich individuals, we first consider the obvious case: $\tilde{\omega}^F < \omega$. In this case, $\tilde{\omega}^F / \tilde{\omega}^G < 1$, which, combined with the result $[(1-\tau)\omega_i^F + \tau\omega] / \omega_i^F < 1$, confirms $\rho_i^G / \rho_i^F < 1$ for a sufficiently large value of ω_i^F . Moreover, we know that, for an individual with $\omega_i^F = \tilde{\omega}^F$, $\rho_i^F = 1$ and $\rho_i^G > 1$. This is because the redistribution increases the number of individuals who engage in education competition without affecting the endowment ranking. Thus, we have $\rho_i^G / \rho_i^F > 1$ for $\omega_i^F = \tilde{\omega}^F$. We can easily see that, for an individual with $\omega_i^F = \omega$, $\rho_i^G / \rho_i^F = \sqrt{\tilde{\omega}^F / \tilde{\omega}^G} < 1$. Figure 4 illustrates this case.

Figure 4 Relative College Premium with Respect to ω_i^F When $\tilde{\omega} < \omega$



We now consider the other case: $\tilde{\omega}^F > \omega$. In this case, we know that $\tilde{\omega}^F / \tilde{\omega}^G < 1$. In addition, for an individual with $\omega_i^F = \tilde{\omega}^F$, $\rho_i^F = 1$ and $\rho_i^G < 1$. This is because the redistribution decreases the number of individuals who engage in education competition without affecting the endowment ranking. Thus, we have $\rho_i^G / \rho_i^F < 1$ for $\omega_i^F = \tilde{\omega}^F$. We can also easily see that, for an individual with $\omega_i^F = \omega$, $\rho_i^G / \rho_i^F = \sqrt{\tilde{\omega}^F / \tilde{\omega}^G} > 1$. Figure 5 illustrates this case.

Using continuity and monotonicity of ρ_i^G / ρ_i^F , we now have the following result: (a) Suppose $\tilde{\omega}^F < \omega$. Then there exists a level of initial wealth $\bar{\omega} \in (\tilde{\omega}_F, \omega)$ such that $\rho_i^G / \rho_i^F \geq 1$ if and only if $\omega_i^F \leq \bar{\omega}$. (b) Suppose $\tilde{\omega}^F > \omega$. Then there exists a level of initial wealth $\bar{\omega} \in (\omega, \tilde{\omega}_F)$ such that $\rho_i^G / \rho_i^F \geq 1$ if and only if $\omega_i^F \leq \bar{\omega}$. This result implies that redistribution by tax-and-subsidy reduces the relative *ex ante* college premium for rich individuals and thereby reduces the variation in the distribution of the *ex ante* college premium.

Figure 5 Relative College Premium with Respect to ω_i^F When $\tilde{\omega} > \omega$ 

6. SUMMARY

This paper develops a general equilibrium model where returns to human capital investment are not guaranteed. The success of human capital investment depends on education competition. Individuals with varying levels of endowment choose the level of resource allocation to the unproductive education competition as well as to human capital investment. If an individual chooses not to engage in education competition, then he or she can use the endowment to produce consumables using a conventional technology with diminishing marginal product. If an individual chooses to engage in education competition, then he or she can expect returns linear in the amount of human capital investment.

In equilibrium, the population is partitioned into two groups according to

their choices. The equilibrium partition depends on the initial distribution of wealth endowments and the relative productivity of the conventional technology and human capital investment. In particular, we find a threshold level of initial wealth such that individuals with larger endowments participate in the education competition while individuals with smaller endowments do not. The existence and uniqueness of the equilibrium are also established.

We conduct a comparative analysis to show that a distributional shock may increase or decrease the intensity of education competition depending on the nature of the shock: If the distributional shock increases the fraction of individuals with larger wealth than the initial threshold level, then the intensity of education competition rises. For example, a mean-rank preserving redistribution policy such as the tax-and-subsidy program increases the intensity of education competition if the initial threshold level of wealth was below the average wealth. In this case, the tax-and-subsidy program increases the level of wealth for individuals whose initial level of wealth is below the average. On the other hand, if the threshold level of wealth was initially higher than the average wealth, then the equilibrium intensity of education competition decreases due to redistribution. Thus, the effect of redistribution on the intensity of education competition depends on the initial equilibrium configuration. Currently, more than 80% of high school graduates enter colleges and universities in Korea, which suggests that the threshold level of wealth is below the average. In this situation, our analysis implies that redistribution would result in an increase in the intensity of education competition.

We also constructed a measure of *ex ante* college premium and showed that it is positively related to individual wealth endowment and that redistribution works to reduce variation in the distribution of the *ex ante* college premium. In other words, redistribution raises the *ex ante* college premium for relatively poor people and decreases the *ex ante* college premium for relatively rich people.

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