

The Co-movement of Inflation and the Real Growth of Output*

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In order to find out whether there really are co-movements between inflation and the real growth of output for the post-WWII period, this paper adopts FIML Markov-Switching Model to solve the equation of Phillips Curve and the equation of Okun's Law together. The findings of this paper are as follows: inflation is procyclical in movement with the real growth of output during the Korean War and the two Oil Shock periods; and there is relatively little evidence of co-movement between inflation and real growth of output with the exceptions of the Korean War and the two Oil Shock periods.

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1. INTRODUCTION

The relationship between inflation and the real growth of output has been assumed to be positive and stable. Mills (1946), Mitchell (1951), Kuznets (1930) found a strong but not perfect conformity in the movements of prices and output both for short term contraction and longer term cycles. For the post-WWII period Lucas (1972) observed that inflation and unemployment rate are negatively correlated, thus indicating a stable Phillips Curve. However, Cooley and Ohanian (1991) argued that with the exceptions of the two world wars, particularly during the period of the Great Depression (1928-1946) and part of the late 19th century, there was relatively little evidence of procyclical prices over the last century and a half including the post-WWII period. Cooley and Ohanian's (1991) findings are quite different from Mankiw (1989) who claimed that in the absence of identifiable real shocks such as OPEC oil price changes, inflation tends to rise during booms and fall during recessions. Cooley and Ohanian's findings are also different from Den Haan (2000) who found that the co-movement between output and prices is positive in the short run and negative in the long run using a VAR model. Thus the purpose of this paper is to find out whether there really are co-movements between inflation and real growth of output for the post-WWII period.

To establish the relationship between inflation and the real growth of output, we adopt FIML Markov-Switching Model suggested by Yoon (2004) and Spagnolo, Psaradakis, and Sola (2005) to solve the equation of Phillips Curve and the equation of Okun's Law together. The equation of Phillips Curve provides an empirical evaluation of the relationship between inflation and unemployment rate. The equation of Okun's Law, which is defined as a relationship between changes in the unemployment rate and real growth of output, provide the relationship between inflation and the real growth of output.¹⁾

¹⁾ Please refer pp. 18-19 in the textbook of *Macroeconomics* (5th edition) by Dornbusch and Fisher (1990).

The merit of the FIML Markov-Switching Model is that we can deal with the problem of simultaneous equations based on the Hamilton filter (1989). So, we can easily find co-movements periods of inflation and the real growth of output.

The findings of this paper are as follows: inflation is procyclical in movement with the real growth of output during the Korean War and the two Oil Shock periods for the post-WWII period; and there was relatively little evidence of co-movement between inflation and the real growth of output with the exceptions of the Korean War and the two Oil Shock periods.

The paper has been divided in 4 sections. Section 2 presents FIML Markov-Switching Model. Section 3 summarizes the empirical results. Section 4 concludes this paper.

2. FIML MARKOV-SWITCHING MODEL

In order to get the consistent estimation of the parameters of the Markov-switching model in the simultaneous equations, we consider the following FIML Markov-Switching Model.

$$YBs_t + Z\Gamma s_t = U_{s_t}, \quad U_{s_t} \sim i.i.d. N(0, \Sigma_{s_t} \otimes I_T), \quad (1)$$

where

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1M} \\ Y_{21} & Y_{22} & \cdots & Y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{T1} & Y_{T2} & \cdots & Y_{TM} \end{bmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{pmatrix},$$

$$B_{S_t} = \begin{bmatrix} \beta_{11,S1t} & \beta_{12,S2t} & \cdots & \beta_{1M,SMt} \\ \beta_{21,S1t} & \beta_{22,S2t} & \cdots & \beta_{2M,SMt} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{M1,S1t} & \beta_{M2,S2t} & \cdots & \beta_{MM,SMt} \end{bmatrix},$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1K} \\ Z_{21} & Z_{22} & \cdots & Z_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{T1} & Z_{T2} & \cdots & Z_{TK} \end{bmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_T \end{pmatrix},$$

$$\Gamma_{S_t} = \begin{bmatrix} \gamma_{11,S1t} & \gamma_{12,S2t} & \cdots & \gamma_{1M,SMt} \\ \gamma_{21,S1t} & \gamma_{22,S2t} & \cdots & \gamma_{2M,SMt} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{K1,S1t} & \gamma_{K2,S2t} & \cdots & \gamma_{KM,SMt} \end{bmatrix},$$

$$U_{S_t} = \begin{bmatrix} u_{11,S1t} & u_{12,S2t} & \cdots & u_{1M,SMt} \\ u_{21,S1t} & u_{22,S2t} & \cdots & u_{2M,SMt} \\ \vdots & \vdots & \ddots & \vdots \\ u_{T1,S1t} & u_{T2,S2t} & \cdots & u_{TM,SMt} \end{bmatrix} = (u_{S1t} \quad u_{S2t} \quad \cdots \quad u_{SMt}),$$

$$\begin{aligned}
 E(U_{St}' U_{St}) &= E \left(\begin{array}{c} \left(\begin{array}{c} u_{S1t} \\ u_{S2t} \\ \vdots \\ u_{SMt} \end{array} \right) \\ \left(u_{S1t} \quad u_{S2t} \quad \cdots \quad u_{SMt} \right) \end{array} \right) \\
 &= \begin{pmatrix} \sigma_{S1t,S1t} I_T & \sigma_{S1t,S2t} I_T & \cdots & \sigma_{S1t,SMt} I_T \\ \sigma_{S2t,S1t} I_T & \sigma_{S2t,S2t} I_T & \cdots & \sigma_{S2t,SMt} I_T \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{SMt,S1t} I_T & \sigma_{SMt,S2t} I_T & \cdots & \sigma_{SMt,SMt} I_T \end{pmatrix} = \Sigma_{St} \otimes I_T.
 \end{aligned}$$

Y is the T by M matrix of jointly dependent variables, B_{St} is an M by M matrix and nonsingular. Z is the T by K matrix of predetermined variables, Γ_{St} is K by M matrix and $rank(Z) = K$. U_{St} is T by M matrix of the structural disturbances of the system. Thus, the model has M equations and T observations. The structural errors are assumed as a nonsingular M -variate normal (Gaussian) distribution. σ is the covariance of the error terms. Σ_{St} is an M by M matrix and positive definite and no restrictions are placed on it. It is assumed that all equations satisfy the rank condition for identification. Also if lagged endogenous variables are included as predetermined variables, the system is assumed to be stable. An orthogonality assumption, $E(Z'U_{St}) = 0$, between the predetermined variables and structural errors is required and, we assume the presence of contemporaneous correlation but no intertemporal correlation in (1). If we assume that the single Markov-switching variable S_t has an N -state, first-order Markov process, then we can write the transition probability matrix in the following way

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{pmatrix},$$

where $p_{ij} = \Pr(S_t = j | S_{t-1} = i)$ with $\sum_{j=1}^N p_{ij} = 1$ for all i .

To include different first order Markov-switching variables $S_{1t}, S_{2t}, S_{3t}, \dots$, in the proposed model, we assume that the dynamics of an unobserved two-state, first order Markov-switching variables, $S_{1t}, S_{2t}, S_{3t}, \dots$ are independent and can be represented by a single Markov-switching variable, S_t .

For example, if our model involves only two unobserved two-state first order Markov-switching variables such as S_{1t} and S_{2t} . The dynamics of Markov-switching variables can be represented by a single Markov-switching variable S_t in the following manner

$$S_t = 1 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 0,$$

$$S_t = 2 \text{ if } S_{1t} = 0 \text{ and } S_{2t} = 1,$$

$$S_t = 3 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 0,$$

$$S_t = 4 \text{ if } S_{1t} = 1 \text{ and } S_{2t} = 1,$$

with $p_{ij} = \Pr(S_t = j | S_{t-1} = i)$, $\sum_{j=1}^4 p_{ij} = 1$.

To derive the FIML Markov-Switching Model in the simultaneous equations, we can obtain $\Pr(S_t = j | \psi_t)$ by applying a Hamilton filter (1989) as follows

Step 1:

At the beginning of the t th iteration, $\Pr(S_{t-1} = i | \psi_{t-1})$, $i = 0, 1, \dots, N$ is given. And, we calculate

$$\begin{aligned} \Pr(S_t = j | \psi_{t-1}) &= \sum_{i=1}^N \Pr(S_{t-1} = i, S_t = j | \psi_{t-1}) \\ &= \sum_{i=1}^N \Pr(S_t = j | S_{t-1} = i) \Pr(S_{t-1} = i | \psi_{t-1}), \end{aligned}$$

where $\Pr(S_t = j | S_{t-1} = i)$, $i = 0, 1, \dots, N$, $j = 0, 1, \dots, N$ are the transition probabilities.

Step 2:

Consider the joint conditional density of y_t and unobserved $S_t = j$ variable, which is the product of the conditional and marginal densities:

$$f(y_t, S_t = j | \psi_{t-1}) = f(y_t | S_t = j, \psi_{t-1}) \Pr(S_t = j | \psi_{t-1}),$$

from which the marginal density of y_t is obtained by

$$\begin{aligned} f(y_t | \psi_{t-1}) &= \sum_{j=1}^N f(y_t, S_t = j | \psi_{t-1}) \\ &= \sum_{j=1}^N f(y_t | S_t = j, \psi_{t-1}) \Pr(S_t = j | \psi_{t-1}), \end{aligned}$$

where the conditional density $f(y_t | S_t = j, \psi_{t-1})$ is obtained from (2)

$$f(y_t | S_t = j, \psi_{t-1}) = (2\pi)^{-M/2} \det(\Sigma_{S_t})^{-1/2} |\det(B_{S_t})| \cdot \exp\left(-\frac{1}{2}(y_t B_{S_t} + z_t \Gamma_{S_t}) \Sigma_{S_t}^{-1} (y_t B_{S_t} + z_t \Gamma_{S_t})'\right), \quad (2)$$

where $\Sigma_{S_t} = \frac{1}{T}(Y B_{S_t} + Z \Gamma_{S_t})'(Y B_{S_t} + Z \Gamma_{S_t})$, y_t is the t th row of the Y matrix. z_t is the t th row of the Z matrix. B_{S_t} and Γ_{S_t} is obtained from (1).

Step 3:

Once y_t is observed at the end of time t , we update the probability terms

$$\begin{aligned} \Pr(S_t = j | \psi_t) &= \Pr(S_t = j | \psi_{t-1}, y_t) \\ &= \frac{f(S_t = j, y_t | \psi_{t-1})}{f(y_t | \psi_{t-1})} \\ &= \frac{f(y_t | S_t = j, \psi_{t-1}) \Pr(S_t = j | \psi_{t-1})}{f(y_t | \psi_{t-1})}. \end{aligned}$$

As a byproduct of the above filter in Step 2, we obtain the log likelihood function

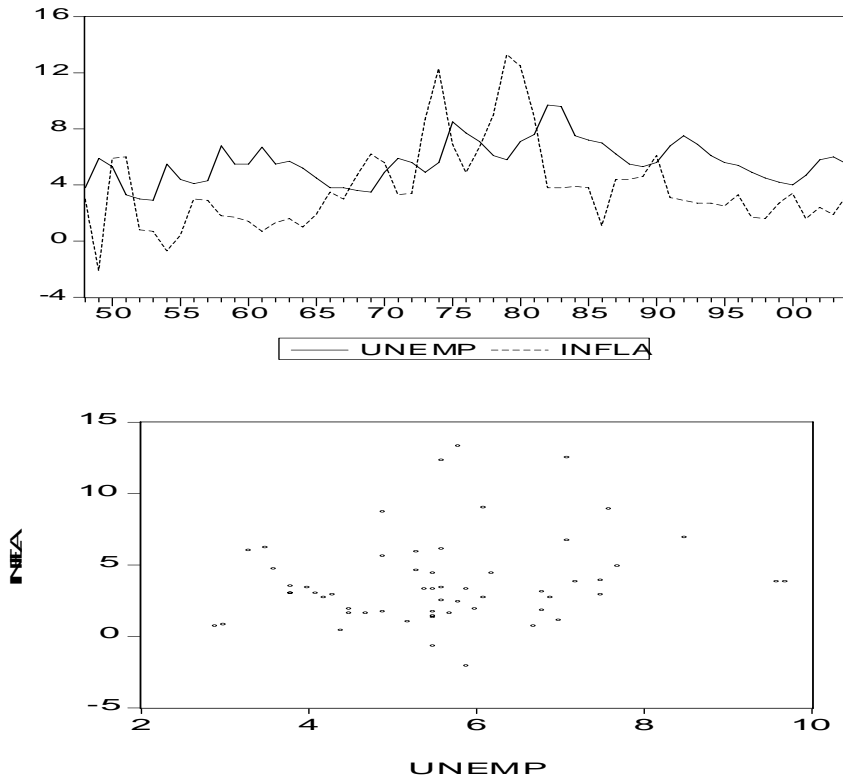
$$\ln L = \sum_{t=1}^T \ln f(y_t | \psi_{t-1}),$$

which can be maximized in respect to the parameters of the model.

3. EMPIRICAL RESULTS

Let's consider the Phillips Curve. An OLS regression for $t=1949$ to 2004 using annual data for inflation (π_t) and unemployment rate u_t in year t is given by equation (3)

Figure 1 Inflation π_t and Unemployment Rate u_t



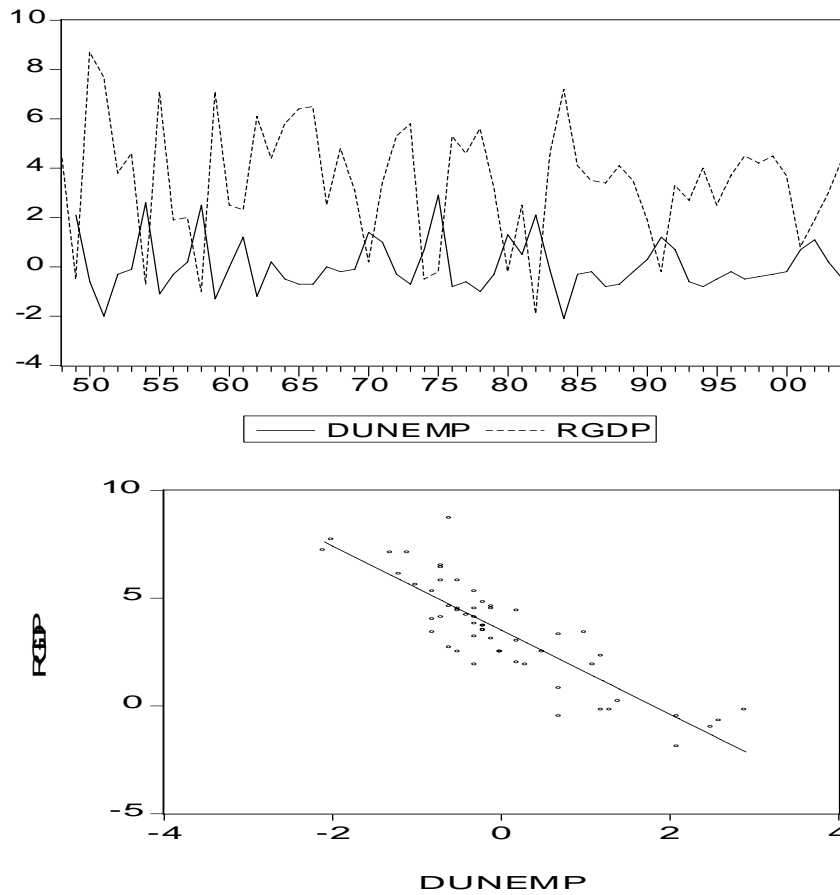
$$\pi_t = 2.55 - 0.28u_t + 0.75\pi_{t-1} + e_t, \tag{3}$$

(1.20) (0.22) (0.11)

where standard errors of the parameters estimates are reported in the parentheses.

The OLS regression (3) reveals statistically insignificant evidence of an inflation-unemployment trade-off because the parameter of unemployment rate u_t in the equation (3) reveals statistically insignificant. Figure 1 depicts the relationship between inflation π_t and unemployment rate u_t .

From the equation (3) and figure 1, we can find that the short run Phillips Curve does not remain stable. These results suggest us to utilize a Markov-

Figure 2 Changes in the Unemployment Rate and Real Growth of GDP

Switching Model for the unstable Phillips Curve for the post WWII period.

To find out the relationship between unemployment and the real growth of output, the equation of Okun's Law for $t=1949$ to 2004 is given by equation (4)

$$\Delta u_t = 1.35 - 0.38y_t + v_t, \quad (4)$$

(0.13) (0.03)

where y_t is the annual real growth of GDP, Δu_t is changes in the unemployment rate in year t .

The equation of Okun's Law in (4) reveals statistically significant evidence between changes in the unemployment rate and real growth of GDP.

From the equation (4) and figure 2, we can find that the stable Okun's Law for the post-WWII period.

For the estimation of Phillips Curve and Okun's Law together, the proposed FIML Markov-Switching Model was applied, which adopts a simple two-state Markov switching parameters in the simultaneous equations.

$$\pi_t = \alpha_{S_t} + \beta_{S_t} u_t + \gamma \pi_{t-1} + e_{S_t}, \quad (5)$$

$$\Delta u_t = \phi_1 + \phi_2 y_t + v_t,$$

$$u_t = u_{t-1} + \phi_1 + \phi_2 y_t + v_t, \quad (6)$$

where $\alpha_{S_t} = \alpha_1 S_t + \alpha_0 (1 - S_t)$, $\beta_{S_t} = \beta_1 S_t + \beta_0 (1 - S_t)$, $\Pr(S_t = 0 | S_{t-1} = 0) = q$, $\Pr(S_t = 1 | S_{t-1} = 1) = p$,

$$p = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}, \quad \begin{pmatrix} e_{S_t} \\ v_t \end{pmatrix} \sim i.i.d.N(0, \Sigma_{S_t} \otimes I_T).$$

To solve the equation (5) and (6) together, we can rewrite it as follows

$$\begin{bmatrix} \pi_t & u_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\beta_{S_t} & 1 \end{bmatrix} - \begin{bmatrix} \pi_{t-1} & y_t & u_{t-1} \end{bmatrix} \begin{bmatrix} \gamma & 0 \\ 0 & \phi_2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \alpha_{S_t} & \phi_1 \end{bmatrix} = \begin{pmatrix} e_{S_t} \\ v_t \end{pmatrix},$$

where $\begin{pmatrix} e_{S_t} \\ v_t \end{pmatrix} \sim i.i.d.N(0, \Sigma_{S_t} \otimes I_T)$,

$$\hat{\Sigma}_{S_t} = \frac{1}{T} (Y \hat{B}_{S_t} + Z \hat{\Gamma}_{S_t})' (Y \hat{B}_{S_t} + Z \hat{\Gamma}_{S_t}) = \begin{pmatrix} \hat{\sigma}_{S1t, S1t} & 0 \\ 0 & \hat{\sigma}_{2,2} \end{pmatrix},$$

$$\alpha_{S_t} = \alpha_1 S_t + \alpha_0 (1 - S_t), \quad \beta_{S_t} = \beta_1 S_t + \beta_0 (1 - S_t),$$

$$\Pr(S_t = 0 | S_{t-1} = 0) = q, \quad \Pr(S_t = 1 | S_{t-1} = 1) = p,$$

$$p = \begin{pmatrix} p & 1 - q \\ 1 - p & q \end{pmatrix}.$$

Table 1 reports estimation results using annual data for 1949-2004. The coefficient $\beta_1 = -0.95$ in table 1 is negative during regime 1 period. However, $\beta_0 = -0.19$ is negative, statistically insignificant during the regime 0 period.

Table 1 Maximum Likelihood Estimation of The Model: 1949-2004

$$\pi_t = 8.58S_t + 2.00(1 - S_t) + \{-0.95S_t - 0.19(1 - S_t)\}u_t + 0.68\pi_{t-1} + e_{S_t}$$

(3.37) (1.11) (0.57) (0.21) (0.11)

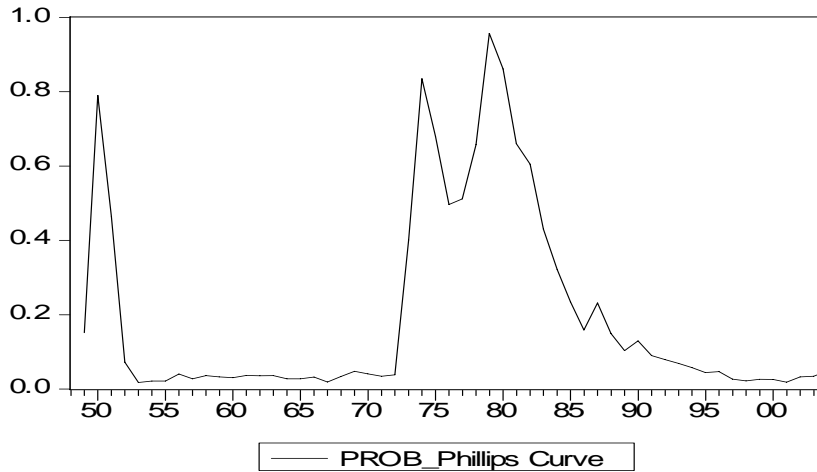
$$u_t = u_{t-1} + 1.35 - 0.381y_t + v_t$$

(0.13) (0.030)

$$\Pr(S_t = 0 | S_{t-1} = 0) = 0.96, \quad \Pr(S_t = 1 | S_{t-1} = 1) = 0.85$$

(0.04) (0.14)

Log Likelihood -165.02

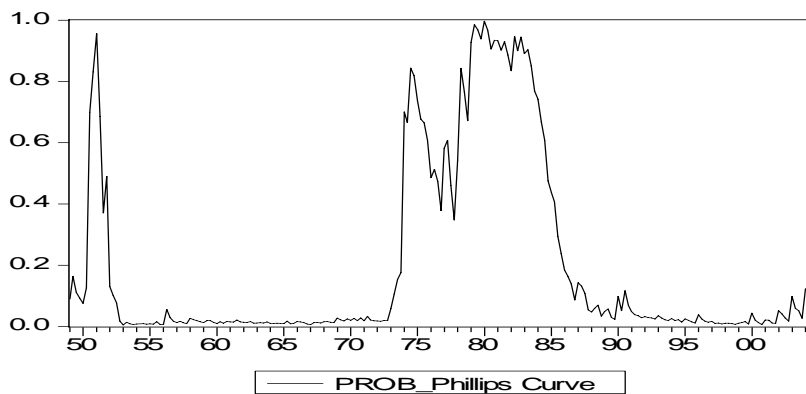
Figure 3 Probabilities of Regime 1 $\Pr(S_t = 1 | S_{t-1} = 1) = p$ for 1949-2004

From figure 3, the inferred probabilities $\Pr(S_t = 1 | S_{t-1} = 1) = p$ accord quite well with the Korean War (1950) and the two Oil Shock periods (1974-1975, 1978-1982).

Although the results of β_1 in table 1 seems to be statistically meaningful during regime 1 period using annual data, we estimate the model again with quarterly data because quarterly data has more precise information than annual data. We can identify regime switching probabilities more concisely with quarterly data, which may be missed by annual data for the unstable Phillips Curve.

We obtained seasonally adjusted quarterly data of the unemployment rate and the consumer price index for the U.S. from the Bureau of Labor Statistics, and seasonally adjusted quarterly GDP percent change based on the chained 2000 dollars from the Bureau of Economic Analysis. Inflation rates are calculated from the log differenced consumer price index. The sample period is from 1949:I to 2004:IV. Table 2 reports estimation results using quarterly data for 1949:I-2004:IV.

Figure 4 Probabilities of Regime 1 $\Pr(S_t = 1 | S_{t-1} = 1) = p$ for 1949:I~2004:IV



The findings of this paper are quite different from Cooley and Ohanian's (1991) who suggested that with the exceptions of the two world wars, particularly in the period of the Great Depression (1928-1946) and part of the late 19th century, as they found relatively little evidence of procyclical prices over the last century and a half including the post-WWII period.

The findings of this paper are also different from Den Haan (2000) who suggested that the co-movement between output and prices is positive in the short run, when demand shocks dominate and negative in the long run.

4. CONCLUSIONS

As the equations of the Phillips Curve with Okun's Law was applied to the FIML Markov-Switching Model for the post-WWII period, The findings of this paper are as follows: inflation is procyclical in movement with the real growth of output during the Korean War (1950:III-1951:II) and the two Oil Shock periods (1974:I-1975:IV, 1978:II-1984:III) for the post-WWII period; and there was relatively little evidence of co-movement between inflation

and the real growth of output with the exceptions of the Korean War and the two Oil Shock periods.

These results suggest another explanation that when there are extremely large shocks such as big wars or oil shocks, inflation is procyclical in movement with the real growth of output, and the procyclical movement occurs in conjunction with the big shocks, exists not only pre-WWII period but also post-WWII period, which is the different result from Cooley and Ohanian's findings.

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