Liquidity and Currency Competition under a Single Currency: Differences in the Transmission of Shocks[*]

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This paper is a heuristic calibration that helps in the understanding of the changes occurring after the introduction of a single currency system. The shapes of impulse response to shocks are compared with those in the national currency system. A unified-currency open economy model shows that currency unification changes the transmission mechanism of a real shock in one country to another country. Under the single currency arrangements, by currency competition, a country with favorable real shock absorbs money that is an important factor in a cash-in-advance constrained world, causing negative impact to the other country. However, this does not happen in the national currency regime.

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1. INTRODUCTION

The launch of a single currency in Europe was one of the biggest monetary events at the end of the previous century.   European politicians expected the currency unification would boost the long-slumped European economy. However, whether the switch of the nominal currency system would really stimulate real activity has been controversial.   This question can be answered by understanding how nominal money is related to the real activity.

The link between money and real activity is an important issue in monetary economics.   Many researchers have tried to find the exact route that monetary policy affects the real activity.   One of the recent works is Fuerst (1992).   He shows that monetary injection causes asymmetry among economic sectors and it causes the interest rate to fall and boost real activity.

Under a unified currency system, if any shock disturbs the wealth distribution among the participating countries, it may give rise to real effect different from that in the national currency system. If changes in nominal money supply affect real variables in an open economy setup, the exchange rate movement must have some influence on the liquidity effect. Schlagenhauf and Wrase (1995) show that positive innovations in the U.S. monetary policy are associated not only with nominal but also with the real depreciation of the dollar.   The switch of the exchange rate system to a unified currency would cause some changes in the magnitude or persistence of the liquidity effect.

This paper focuses on how the economy responds to a nominal or real shock differently in the single currency system.   For the analysis, this study uses an international version of Fuerst (1992) model, which is close to Ho (1993) and Schlagenhauf and Wrase (1995).   As in Schlagenhauf and Wrase (1995), this study uses numerical calibration to analyze the effect of the change in exchange regime.   The shapes of the impulse responses for nominal and real shocks are compared with those in the national currency system.

This paper is to understand the differences between the two exchange rate

regimes and does not take the route of standard real business cycle literatures such as Yoon (2006). The usual convention of those articles is the following: first, set-up a theoretical model, fit the model with data using some typical statistics, and then derives some implications on the economy using the model. However, this paper chooses one model that can incorporate the two different exchange rate systems that then compare the differences in the dynamic behavior of the economy using impulse responses. The results must be model-dependent and this study uses a model similar to Schlagenhauf and Wrase (1995). Their model does not perfectly fit the real data, but they claim that the model reasonably fits the two-country national currency economy. To analyze the differences between two currency regimes, there is a need to use a model that can encompass both currency systems. Therefore, the single currency economy is analyzed by introducing into the same model a common currency that can be used in both countries. Real aspects of the economy are the same as those of Schlagenhauf and Wrase (1995). However, the foreign exchange market does not exist in single currency regime.

In a single currency system, the transmission pattern of the monetary shock is not much different from that in the national currency system, and the shock leads to the same impulse responses for both countries. However, the currency unification changes the real transmission mechanism across countries. In the national currency system, a real shock in one country is somewhat absorbed by the variation of the exchange rate. However, the unification of currencies removes this firewall. Any real shock in one country is negative and temporarily transmitted to the other country. A favorable productivity shock in one country gives more income and creates a large investment demand for that country. Increases in money demand are followed and more money flows into that country. In the other country, less money is left and investment and output drops. This is called 'currency competition'. As the shock is sufficiently transmitted to the other countries later, the investment and labor demand of those countries is restored.

This paper is organized as follows. In section 2, an international version

of Fuerst (1992) model is briefly described. Section 3 provides some implications derived from the solution of the model. The model is parameterized and calibrated in section 4. Section 5 concludes the article.

2. DESCRIPTION OF THE MODEL

The basic model is an international version of Fuerst (1992). Two countries have two good economy models that are used where each country has a representative household. Countries are called 'home' and 'foreign.' Each country has access to a stochastic production technology. Home country produces good 1 and foreign country good 2. Most of the descriptions below are focused on the home country. However, situations in the foreign country are easily stated by adding an asterisk (*) on appropriate functions or variables.

There are two kinds of assets in this economy: physical capital and money. Initially, the home (foreign) household is endowed with $K_0(K_0^*)$ of capital and $M_0(M_0^*)$ of money stock.

Money is introduced by the cash-in-advance constraint. Each agent has to have enough liquidity in all kinds of transactions. This study considers two exchange regimes: a national currency system and a unified single currency system.[1]

Each country uses local currency in the national currency regime. Home money supply is denoted by $\overline{M_t}$ and foreign by $\overline{M_t^*}$. Assuming that each agent holds all nominal assets in the form of domestic currency, the beginning of each period money holding is equal to the national money supply of the previous period. Following Stockman (1981), it is assumed that the overseas purchase is cleared by the currency of the seller. Consumers and firms have to go to the foreign exchange market to obtain the required currency to import goods from the other country. The exchange rate is determined by the level that equalizes the demand and supply of both

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[1] In the single currency regime, it is assumed that the labor and capital are not mobile.

currencies. In the single currency regime, no foreign exchange market is needed. At the beginning of period t, home country household has M_t and foreign household has M_t^* where the world money supply is $\tilde{M}_t = M_t + M_t^*$.

The same methodology as Fuerst (1992) and Ho (1993) is used to model the liquidity effect. Monetary injection is made through a financial intermediary. Since a new money supply is distributed only to the agents in the bank, i.e., depositors, it distorts the wealth distribution across the agents within the economy. However, as mentioned in Fuerst (1992), to keep track of the evolving wealth redistribution over time is almost an impossible task. Following Lucas (1990), they circumvent this complication by grouping all sectors of an economy into one household that is the basic consumption unit. Each members of a household (representing a sector of the economy) behaves independently (sometimes competitively) while they are separated. However, they pool current resources when gathered and act as a single consumption unit. Therefore, the wealth distribution is asymmetric across the sectors, but symmetric across the households.

The infinitely lived household maximizes the expected lifetime utility given by,

$$E_0[\textstyle\sum_{t=0}^{\infty} \beta^t U(C_{1t},\ C_{2t},\ 1-l_t)], \tag{1}$$

where E_0 is the conditional expectation at time 0, time preference β is less than 1, C_{1t} denotes the consumption of good i at time t, and l_t work effort.

The household consisted of four members: a shopper, a worker, a firm, and a financial intermediary. At time t, domestic household enters the period with M_t of home currency and K_t of capital stock. Before the state of the world is revealed, household makes N_t of deposit to the financial intermediary. Because of the prohibitive transaction cost, the money deposited cannot be withdrawn for the purchase of current period consumption. After the deposit is made, the household members separate.

Once they are separated, the foreign exchange market opens. As soon as the foreign exchange market closes, the state of the world is revealed. In this economy, the state of the world is a vector of uncertain technology shock (θ_t) and money injection (X_t). It is denoted by $s_t = (\theta_t, X_t)$.

Under the national currency regime, the shopper has to go to the foreign exchange market to get the foreign currency necessary for the purchase of foreign goods. Since the state of the world is not revealed, they have to rely on the information to decide how much foreign currency to get. The home shopper is subject to the following constraints,

$$M_t - N_t \geq M_{1t} + e_t^s M_{2t}, \tag{2}$$

$$M_{1t} \geq P_{1t} C_{1t}, \tag{3}$$

$$M_{2t} \geq P_{2t} C_{2t}, \tag{4}$$

where M_{2t} denotes the acquired foreign currency, M_{1t} the remaining home currency, P_{it} the price of good i in unit of the currency of the seller and e_t^s is the price of foreign currency in units of the home currency at time t. After the currency conversion is made, the state is revealed and the shopper goes to the goods market to purchases good 1 and good 2 for consumption.

In the single currency regime, the shopper can change the consumption basket after the revelation of shocks, and faces one cash constraint,

$$M_t - N_t \geq P_{1t} C_{1t} + P_{2t} C_{2t}. \tag{5}$$

The home firm is initially endowed with capital stock of K_0. At time t, the firm hires H_t of labor to produce $F(K_t, H_t, \theta_t)$ of good 1 where θ_t is a random technology shock. At the end of the period, it purchases I_{1t} of good 1 and I_{2t} of good 2 to make $Q(I_{1t}, I_{2t})$ of new capital. Since there are cash-in-advance constraints in all transactions in this economy, the firm

has to finance the wage bill and investment.

The firm borrows B_{1t} of home money and B_{2t} of foreign money from the intermediaries and then travel to the labor market to hire workers with wage W_t. Therefore, the firm faces the following cash constraints under two currency regime,

$$B_{it} \geq W_t H_t + P_{1t} I_{1t}, \tag{6}$$

$$B_{2t} \geq P_{2t} I_{2t}. \tag{7}$$

At the end of period, the firm repays the loan. The exchange market reopens for the repayment of foreign loans. To distinguish the two exchange rates the beginning of period exchange rate is denoted by e_t^s and exchange rate determined at this end-of-period market is denoted by e_t^f.

Under the single currency system, home firm borrows B_t from an intermediary and uses it to pay the wage bill and purchase both investment goods. As the shopper does, the firm faces a single constraint,

$$B_{1t} \geq W_t H_t + P_{1t} I_{1t} + P_{2t} I_{2t}. \tag{8}$$

After the production, the firm ships the products to the goods market. The cash earned from the sale of the current production cannot be used until the next period. Then the firm travels to the goods market to purchase capital goods. The cash remained after the payment of wage bill is used to pay for capital goods. The worker travels to the labor market and supplies l_t of labor. At the end of period, the worker returns with $W_t l_t$ of wage income.

Financial intermediaries receive deposits and make loans to the firms. They work as a channel through which new currency is injected into the economy. The financial intermediary has two sources of cash to lend out: deposit receipt (N_t) and new currency injection (X_t). The interest rate is determined so that all available resources are lent out. At the end of each

period, loans are repaid and the financial intermediary returns deposits with interest.   Home financial intermediary in national currency system faces the following budget constraint[2]

$$N_t + X_t = B_{1t} + B_{1t}^*,$$

where B_{1t}^* is home currency borrowed by foreign firm.

Under the single currency regime, there needs to be only one intermediary in the world.   It is assumed that this intermediary is co-owned by both home and foreign households.   The share of the home country is denoted by π.[3] The profit of the intermediary is allocated according to the share of ownership.   The financial intermediary is faced with the following constraint

$$N_t + N_t^* + X_t = B_t + B_t^*. \tag{9}$$

At the end of period, household members gather and they pool the cash receipts of all members and enter the next period.   The end-of-period cash balance of home household under national currency system is given by

$$M_{t+1} = [M_t - N_t - P_{1t}C_{1t} - e_t P_{2t}C_{2t}] + [P_{1t}Y_{1t} - R_t B_{1t} - e_t^f R_t^* B_{2t}]$$
$$+ [W_t l_t] + [R_t X_t] + [R_t N_t]. \tag{10}$$

where $R_t = 1 + r_t$ denotes the gross interest rate.   The first bracket is leftover by the shopper and the second is the contribution of the firm.   The third is wage income, and the fourth profit of financial intermediary.   The last term denotes the deposit with interest.

The cash balance in the single currency case is

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[2] If the interest rate on loan r_t is positive, the intermediary will loan out all the available fund.   However, if the interest rate is zero, the financial intermediary is indifferent between keeping the cash or loaning it out.   As in Fuerst (1992), it is assumed that the financial intermediary always opts to loan out all available cash.

[3] It is assumed that the share of financial intermediary is fixed for simplicity.

$$M_{t+1} = [M_t - N_t - P_{1t}C_{1t} - P_{2t}C_{2t}] + [P_{1t}Y_{1t} - R_tB_t]$$
$$+ [W_tl_t] + [R_tX_t] + [R_tN_t]. \tag{11}$$

Next period capital stock is given by

$$K_{t+1} = (1-\delta)K_t + Q(I_{1t},\ I_{2t}). \tag{12}$$

3. SOLUTION OF THE MODEL

The home household maximizes the expected lifetime utility given by (1), subject to cash-in-advance constraints and two transition equations. Let $J(M_t,\ K_t,\ s_{t-1})$ represent the value function at the beginning of period t given cash balances, capital stock, and the previous state of the world. Then J satisfies the following relation

$$J(M_t,\ K_t,\ s_{t-1}) = \max_{N_t} \int \max_{C_{1t},C_{2t},l_t,B_t,H_t,I_{1t},I_{2t}} \{U(C_{1t},\ C_{2t},\ 1-l_t)$$
$$+\ \beta J(M_{t+1},\ K_{t+1},\ s_t)\}d\Phi(s_t|s_{t-1}).$$

The constraints for this maximization in the national currency system are (2)-(4), (6), (7), (10), (12), while in the single currency case, the maximization is subject to (5), (8), (11), (12). Even though the constraints are different in the two regimes, optimality conditions are very similar because most of the conditions are stated in real terms.

The solution to the model provides the same form of interest rate as Fuerst (1992). Interest rate is represented as,

$$1 + r_t = \frac{\lambda_f - \lambda_g + \left(\dfrac{U_1(C_{1t},\ C_{2t},\ 1-l_t)}{P_{1t}}\right)}{\beta E_t\left(\dfrac{U_1(C_{1t+1},\ C_{2t+1},\ 1-l_{t+1})}{P_{1t+1}}\right)},$$

where λ_g and λ_f denote the Lagrange multipliers associated with liquidity constraints of the shopper and of the firm. When the constraints are binding, they are positive and represents the cost (or value) of liquidity. As in Fuerst (1992), interest rate is the sum of the liquidity effect and a real fundamental. Since the value of λ_g is the lost utility and λ_f the lost profit of firm by not holding enough cash, the size of the liquidity effect depends on the relative value of cash in the goods market and financial market. Without the cash-in-advance constraints, the above optimization problem is the same as the saving decision. Then the interest rate becomes the ratio of marginal utility of lost consumption discounted by future marginal utility and is similar to the typical asset pricing equation.

The interest rate in the single currency regime is in the same form as above except that λ_g and λ_f are the multipliers associated with liquidity constraints (5) and (8). However, in the single currency system, there is only one interest rate in the world, i.e.,

$$
1+r_t = \frac{\lambda_f - \lambda_g + \left(\dfrac{U_1(C_{1t},\ C_{2t},\ 1-l_t)}{P_{1t}}\right)}{\beta E_t\left(\dfrac{U_1(C_{1t+1},\ C_{2t+1},\ 1-l_{t+1})}{P_{1t+1}}\right)}
$$
$$
= \frac{\lambda_f^* - \lambda_g^* + \left(\dfrac{U_2^*(C_{1t}^*,\ C_{2t}^*,\ 1-l_t^*)}{P_{1t}}\right)}{\beta^* E_t\left(\dfrac{U_2^*(C_{1t+1}^*,\ C_{2t+1}^*,\ 1-l_{t+1}^*)}{P_{2t+1}}\right)}.
$$

(13)

If the money holdings of the two countries are not equal, the values of the liquidities $(\lambda_g,\ \lambda_g^*,\ \lambda_f,\ \lambda_f^*)$ of the two countries are not the same. Since firms in both countries face the same liquidity constraint (9) regardless of the beginning of period money holdings, λ_f is not expected to be greatly different from λ_f^*. But the value of λ_g of the cash-abundant country is smaller than that of cash-short country. Then the difference of liquidity

values $(\lambda_f - \lambda_g)$ of cash-rich country must be larger than that of the other country. For the equation (13) to hold, the marginal utility (U_1 / P_1) of the cash-rich country is smaller, meaning a greater utility. Therefore, the money distribution is directly connected to the welfare distribution.

Under national currency regime, assuming that all cash-in-advance constraints are binding, optimality conditions become

$$E_{t-1}\left[\frac{U_1(C_{1t},\ C_{2t},\ 1-l_t)}{P_{1t}}\right] = E_{t-1}\left[\frac{U_2(C_{1t},\ C_{2t},\ 1-l_t)}{e_t^s P_{2t}}\right], \tag{14}$$

$$U_1(C_{1t},\ C_{2t},\ 1-l_t) = \beta J_M(M_{t+1},\ K_{t+1},\ s_t)W_t, \tag{15}$$

$$P_{1t}F_H(K_t,\ H_t) / R_t = W_t, \tag{16}$$

$$J_M(A_{t+1},\ K_{t+1},\ s_t)R_t P_{1t} = J_K(A_{t+1},\ K_{t+1},\ s_t)Q_1(I_{1t},\ I_{2t}), \tag{17}$$

$$J_M(A_{t+1},\ K_{t+1},\ s_t)e_t^s R_t^* P_{2t} = J_K(A_{t+1},\ K_{t+1},\ s_t)Q_2(I_{1t},\ I_{2t}), \tag{18}$$

$$\int\left\{-\frac{U_1(C_{1t},\ C_{2t},\ 1-l_t)}{P_{1t}} + \beta J_M(M_{t+1},\ K_{t+1},\ s_t)R_t\right\}d\Phi(s_t|s_{t-1}) = 0, \tag{19}$$

$$J_M(M_t,\ K_t,\ s_{t-1}) = \int\frac{U_1(C_{1t},\ C_{2t},\ 1-l_t)}{P_{1t}}d\Phi(s_t|s_{t-1}), \tag{20}$$

$$J_K(A_t,\ M_t,\ s_{t-1}) = \int\{\beta J_M(A_{t+1},\ K_{t+1},\ s_t)P_{1t}F_K(K_t,\ H_t)$$
$$+ \beta J_K(A_{t+1},\ K_{t+1},\ s_t)(1-\delta)\}d\Phi(s_t|s_{t-1}). \tag{21}$$

Optimality conditions under the single currency system are the same except that (14) and (18) are replaced by (14a) and (18a) below.

$$\frac{U_1(C_{1t},\ C_{2t},\ 1-l_t)}{P_{1t}} = \frac{U_2(C_{1t},\ C_{2t},\ 1-l_t)}{P_{2t}}, \tag{14a}$$

$$J_M(A_{t+1},\ K_{t+1},\ s_t)R_tP_{2t} = J_K(A_{t+1},\ K_{t+1},\ s_t)Q_2(I_{1t},\ I_{2t}). \tag{18a}$$

Equations (14) and (14a) are the equality of marginal utilities of two consumption goods in the real term. In the national currency system, they are equal in expected value because the currency exchange must be made before the realization of shocks. However, in the single currency system, they are exactly equal in value because the consumer can adjust the consumption basket after the realization of shocks.

Equations (15) and (16) are the optimality conditions for the labor supply and demand. Left-hand side of (15) is marginal dis-utility and right-hand side is the discounted value of future utility increase from wage income. In (16) the firm equalizes the nominal wage to the discounted value of marginal product of labor because the firm has to borrow cash in advance. Therefore, the increase in interest rate discourages labor demand.

Equations (17) and (18) determine the investment of both goods. In the national currency case, these equations are reduced to

$$\frac{Q_1(I_{1t},\ I_{2t})}{R_tP_{1t}} = \frac{Q_2(I_{1t},\ I_{2t})}{e_t^f R_t^* P_{2t}}.$$

The allocation of both investment goods depends on the exchange rate and the interest rates of both countries. However, in the single currency case, the allocation of investment goods is determined by equations (17) and (18a). Since the interest rates in both sides of the equation above cancel each other out the intra-temporal investment allocation is independent of the interest rate as seen in the following equation,

$$\frac{Q_1(I_{1t},\ I_{2t})}{P_{1t}} = \frac{Q_2(I_{1t},\ I_{2t})}{P_{2t}}.$$

For the inter-temporal optimization of investment, equation (21) changes to the usual Euler equation.

$$U_1(C_{1t}, C_{2t}, 1-l_t)\frac{P_{1t-1}}{P_{1t}}\frac{R_t}{Q_1(I_{1t}, I_{2t})} =$$

$$E_t\left[\beta U_1(C_{1t+1}, C_{2t+1}, 1-l_{t+1})\frac{P_{1t}}{P_{1t+1}}\left(F_K(K_{t+1}, H_{t+1})+(1-\delta)\frac{R_{t+1}}{Q_1(I_{1t+1}, I_{2t+1})}\right)\right]. \quad (22)$$

Since the deposit decision is made before time t state variables are revealed, equation (19) can be represented by

$$E_{t-1}\left[\frac{U_1(C_{1t}, C_{2t}, 1-l_t)}{P_{1t}}\right] = E_{t-1}\left[\frac{U_1(C_{1t+1}, C_{2t+1}, 1-l_{t+1})}{P_{1t+1}}R_t\right].$$

This is same as typical asset pricing equation. Due to the information structure, the expectation is given time t-1 information. These equations and cash-in-advance constraints with inequality replaced by equality solve for the decision variables of the home household: C_{1t}, C_{2t}, l_t, H_t, I_{1t}, I_{2t}, N_t, M_{t+1}, K_{t+1}.

The optimization of foreign households can be described in the same manner as above. Equilibrium is the state where all markets (goods markets, labor markets, exchange markets, and financial markets) are cleared. It is described by the following market clearing conditions.

$$C_{1t} + C_{1t}^* + I_{1t} + I_{1t}^* = F(K_t, H_t) \qquad C_{2t} + C_{2t}^* + I_{2t} + I_{2t}^* = F^*(K_t^*, H_t^*)$$

$$I_t = H_t \qquad\qquad\qquad I_t^* = H_t^*$$

Exchange rate is determined by balancing demands for the two currencies. In the beginning (end) of period foreign exchange market, $M_{2t}(B_{2t}R_t^*)$ is the demand for the foreign currency of the domestic consumer (firm), and

$M_{1t}^*(B_{1t}^*R_t)$ denotes the home currency demand of the foreign consumer (firm). Equilibrium exchange rates, e_t^s and e_t^f are determined by

$$M_{2t} = e_t^s M_{1t}^*,$$

$$B_{2t}R_t^* = e_t^f B_{1t}^*R_t.$$

In Schlagenhauf and Wrase (1995), rigidities in portfolio adjustment is assumed only for the deposit decision. This paper assumes that the foreign exchange market opens before the shocks are revealed as in Ho (1993). This assumption slows the transmission of shocks to the other country. As for the monetary shock, the timing of foreign exchange does not matter because the supply of liquidity to the financial market is pre-determined by deposits. However, the effect of the technology shock depends on the timing of the foreign exchange. If the foreign exchange market opens before the realization of shock, the outcome of the favorable shock is reaped by the currency-rich consumer. However, in the single currency regime, the shopper does not need to change the currency. The shopper simply adjusts the consumption basket depending on the realized shock level.

Depending on the timing of currency portfolio decision, exchange rates vary and have different meanings. The exchange rate in Schlagenhauf and Wrase (1995) varies according to the realization of the shock. The demand for foreign exchange is purely from transaction motive. However, in Ho (1993) where foreign exchange market opens before the realization of shocks, the exchange rate is determined by the forecast of the shock and the speculative or hedging motive against the exchange risk must be incorporated in the demand. In addition, it is expected to be less volatile.

4. CALIBRATION OF THE MODEL

The model described in the previous section is difficult to solve. There is

a need in the single currency case to keep track of the distribution of money between the home and foreign households. This is difficult even when using a simple functional form and the lumping household methodology of Lucas.

In this section, the model is parameterized and simulated to answer the questions of how the liquidity effects differ under the national currency and the single currency system and how the exchange regime affects the transmission mechanism of unexpected shocks. The effects of monetary and technology shocks are analyzed with the shape of impulse responses of various variables since this study relies on a numerical solution.

The model is solved using the method proposed in Christiano (1991). The procedure is as follows: with equilibrium equations to be solved, first linearize the equations around the steady state. Secondly, assume linear policy functions with coefficients undetermined and plug them back into the linearized equilibrium conditions to get the system of equations of unknown coefficients. Lastly, the study solves the system to get the values of coefficients. Since these equations are usually nonlinear, they are solved with a numerical algorithm.

The same functional form as Schlagenhauf and Wrase (1995) is adopted for the utility and production function,

$$U_1(C_{1t},\ C_{2t},\ 1-l_t) = \frac{1}{\rho}\left\{ C_{1t}^{\Phi(1-\gamma)} C_{2t}^{(1-\Phi)(1-\gamma)} (1-l_t)^\gamma \right\}^\rho,$$

$$F(K_t,\ H_t,\ \theta_t) = K_t^\alpha (\theta_t K_t)^{1-\alpha}.$$

Schlagenhauf and Wrase (1995) assumed only that the home good is purchased by the home firm for investment.   To incorporate the fact that the share of investment goods in world trade is high, it is assumed that both goods are combined to produce capital. This capital good producing technology is assumed deterministic without any stochastic shock and a Cobb-Douglas function is used.

$$Q(I_{1t}, I_{2t}) = I_{1t}^{\zeta} I_{2t}^{(1-\zeta)}.$$

Two types of stochastic processes govern the evolution of the state in this economy: productivity shock and monetary shock. Productivity shock processes are assumed to follow a bivariate AR(1) process

$$\begin{bmatrix} \theta_t \\ \theta_t^* \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} \theta_{t-1} \\ \theta_{t-1}^* \end{bmatrix} + \begin{bmatrix} \eta_t \\ \eta_t^* \end{bmatrix}.$$

The stochastic processes of money growth rate are also assumed to follow an AR(1) process.

$$\begin{bmatrix} x_t \\ x_t^* \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}.$$

In the single currency regime, x_t follows an AR(1) process with only one innovation.

$$x_t = m_1 x_{t-1} + \varepsilon_t.$$

Innovations in all processes $(\eta_t, \eta_t^*, \varepsilon_t, \varepsilon_t^*)$ are serially independent. Since the study is interested only in impulse responses, the exact description on these innovations is not necessary. A steady state equilibrium is calculated when these innovations take on the mean values of zeros.

For the calibration of the model, there is a need to specify parameter values. Most of the parameters are borrowed from Schlagenhauf and Wrase (1995).

Discount rate β and β^* are set to 0.99, which is equivalent to the steady state real interest rate of 1 percent per quarter. The curvature of the utility function (ρ) is equalized to –1. The home good share (ϕ) is assumed to be 0.7. It is derived from the fact that non-tradable good consumption is about 40 percent of GDP (Stockman and Tesar, 1995).

Assuming that half of tradable goods are consumed by home household, the total home good share is regarded as 70 percent. To keep two countries symmetric, 0.3 is used for the foreign household home good share (ϕ^*). With the symmetry assumption, $\pi = 0.5$ is used. Capital shares for both countries (α, α^*) are assumed to be 0.35. Assuming a quarter of non-sleep hours is devoted to work, 0.75 is given as leisure share (γ, γ^*). Capital depreciation rate (δ) is set to 0.02, which implies 8 percent of depreciation annually. The share of home good in capital producing function (ζ) is assumed to be 0.8.

AR parameters in stochastic processes are

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} 0.900 & 0.002 \\ 0.002 & 0.900 \end{bmatrix},$$

and

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} 0.45 & 0.09 \\ 0.09 & 0.45 \end{bmatrix}.$$

These figures are borrowed from Schlagenhauf and Wrase (1995), but are slightly modified. Here, there is an assumption of symmetry between the two countries. Since the matrices are not symmetric, off-diagonal elements are assigned the same value. Values in diagonal elements are adjusted likewise.

In the single currency case, AR parameter in money shock process (m_1) is set to the same value as m_{11}.[4] Trend growth rates of technology shock in both countries are set to 0.0041 as in Schlagenhauf and Wrase (1995).[5]

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[4] For AR(1) shock processes of states, there is a need to only specify AR parameters. This study is not interested in fitting variances and covariances of variables as in real business cycle literatures, so the specifications of variance and covariance structures of innovations are not necessary. The focus is to understand the differences in dynamic adjustments that are understood through impulse responses.

[5] In calibration, all real variables are rescaled using this trend growth rate.

**Figure 1   Impulse Response for one Standard Deviation Increase of
Domestic (World) Money Supply**

Note: 'national' and 'single' denote
national currency system
and single currency system.
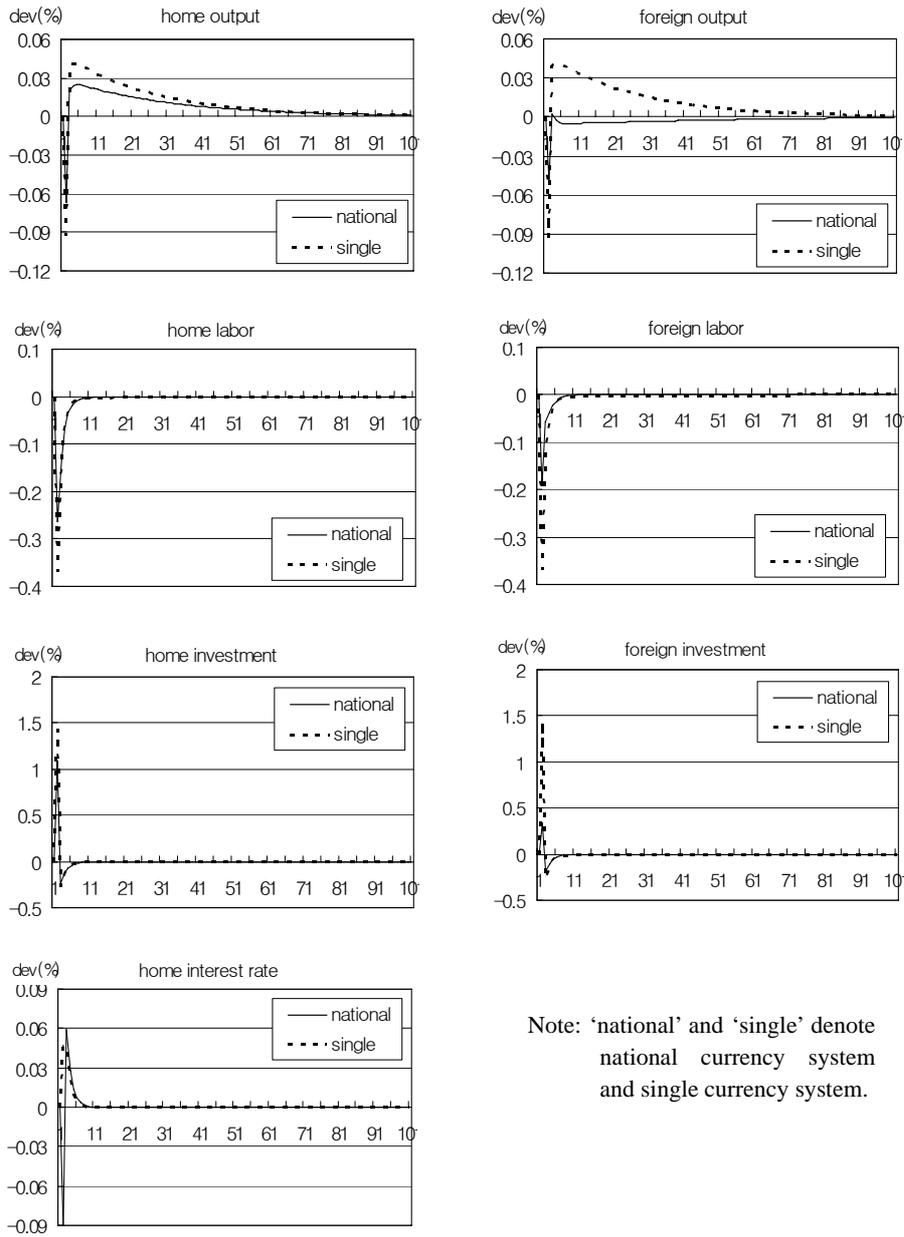
Mean money growth rates in both countries and under the single currency system are equally set to 0.018.

Figure 1 shows the impulse responses of variables to one standard deviation of domestic money growth in national currency case, and of world money supply in the single currency case in period 2. Response patterns of domestic and foreign variables are not much different. The increase of the money supply decreases contemporaneous employment and output. As in Schlagenhauf and Wrase (1995), liquidity effect is dominated by the anticipated inflation effect and nominal interest rate increases from the third period on. Higher financing cost lowers the labor demand and decreased employment reduces output.[6] However, from the next period, money injection has a positive liquidity effect and the interest rate slowly drops.[7] As investment and capital increases, output increases. Though not reported, money injection reduces deposit because positive serial correlation implies that the future liquidity will be abundant. From the positive cross-country spillover effect, foreigners behave in a similar manner.

The single currency regime does not make much difference in liquidity effect and transmission. Money injection has positive effect on real output from the third period on and reduces the saving of all agents. Since the monetary shock affects the home and foreign economy in the same manner, the responses of domestic and foreign households are the same.

However, in figure 2 the impacts of productivity shock are different in two regimes. In the national currency regime the positive productivity shock on home good increases the employment and investment of domestic household.

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[6] These unappealing phenomena appear in most of the other literature employing Fuerst (1991) type liquidity model such as Christiano (1991), Christiano and Eichenbaum (1992) and Schlagenhauf and Wrase (1995). In those articles, some kinds of sluggishness are incorporated to have enough liquidity effect. Sluggish household saving or inflexible investment is usually incorporated and warrants some improvement in fitting real data. There is no pursuit of this modification of the model since an empirical fit is not the focus of this paper.

[7] Graphs for foreign interest rate movements are not provided. In the single currency system, interest rate for foreign country is exactly the same as that of the domestic country, which is the one world interest rate. The movement of foreign interest rate in the national currency case is very close to that of the domestic interest rate except that the magnitudes are much smaller.

Output increases as the two production factors increase. Saving also increases because current income increase dominates the expected increase of future income.

Foreigners show the same behavior as the domestic household even though the magnitude is much smaller. This is because of the positive cross-country spillover of productivity shock.

However, in the single currency regime, foreigners show a different behavior. Favorable productivity shock in domestic country reduces employment and investment of foreign country for a while. This opposite response patterns are from the competition in the financial market (loan market). Under a national currency system, increased labor productivity increases the loan demand of a firm to hire and invest more. However, it does not compete for money with foreign firm. The possible competition is only from the increase of the foreign currency demand of home firm to buy more foreign capital goods. However, under the single currency regime, foreign firms must be crowded out from the financial market by the increased demand of the home firm. Since the deposit decision is made before the revelation of the shock, the left hand side of equation (10) is fixed when firms decide the amount of borrowings. Therefore, for the increase of the borrowing of the home firm that is willing to afford higher interest rate due to the favorable productivity shock, foreign firms should reduce borrowing. This is called *currency competition*. By the liquidity constraint of the firm, foreign firm should cut the level of employment and of investment. The investment and output of foreign country catches up as the productivity shock is sufficiently transmitted across countries later.

From the two figures, it is concluded that even though the currency unification is a nominal phenomenon, it does not matter much in the nominal side. It causes real differences by the increase of competition for the single currency which is one of the important factors for both production and consumption.

Figure 2   Impulse Response for one Standard Deviation Increase of Domestic Labor Productivity

Note: 'national' and 'single' denote national currency system and single currency system.
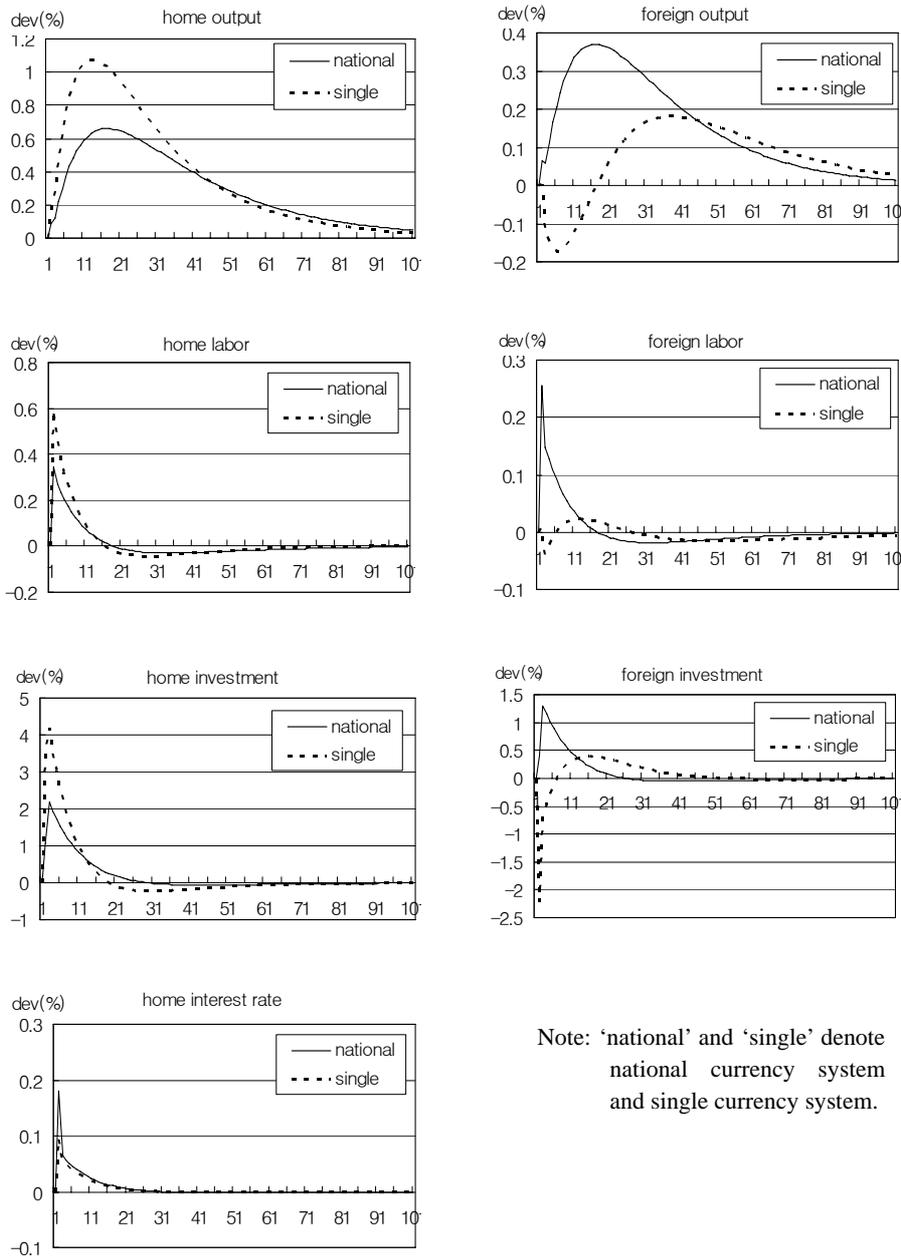
5. CONCLUSION

This study examined the differences in the liquidity effects caused by the switch of the currency regime.   Impulse responses to monetary and productivity shocks under the national currency and under the single currency regimes are compared.   Lucas (1990) and Fuerst (1992) liquidity mode are used in an open economy setup.   The following findings are provided with a numerical calibration of the model.

First, liquidity effect in the single currency system is not much different from that under national currency system.   The effect of monetary shock must be similar regardless of the exchange regime.   In the national currency, due to the existence of exchange rate firewall, any monetary shock in one country is transmitted to the other country with a similar effect.   In a single currency regime, any monetary shock affects all the member countries symmetrically.

Second, the currency unification changes the impulse responses of major variables to a real shock and changes the transmission mechanism of the real shock of one country to the other country.   It implies that the business cycle patterns in the single currency regime may not be inferred from the national currency experiences.   This must be compared with impulse response patterns derived with real data from any econometric model such as a vector auto-regression model (VAR).   However, the data from the single currency system are sufficient for extensive econometric works and is another issue for future research.

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