Strategic Delegation with Network Externality and Product Compatibility*

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We investigate the effects of network externality and product compatibility on strategic delegation under price competition in duopoly. We extend the Hoernig (2012) to the case in which product compatibility is endogenously determined and is built into network effects. When the spillover effect for network size depends on the rival’s choice of product compatibility, perfect compatibility is a dominant strategy for each manager. Thus, endogenizing product compatibility excludes the equilibrium of Hoernig (2012) who assumes no compatibility between products. In equilibrium, firm owners require their managers to be more aggressive than profit maximizer if network externality is sufficiently strong but not too strong.

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1. INTRODUCTION

In the context of the separation of ownership and management, Fershtman and Judd (1987) and Sklivas (1987) show that firm owners can increase their profits by strategic delegation, i.e., by providing their managers with incentive contracts to maximize a specific objective other than profits. The objective is implemented by putting different weights on profits and revenues. In duopoly equilibrium under price competition, a higher weight on profits induces a manager to act less aggressively than a profit maximizer by increasing prices. This result is reversed by Hoernig (2012), who demonstrates that firm owners prefer their managers to be more aggressive than profit maximizers under price competition if network externality is sufficiently strong.

As information and communication technologies develop, the associated industries exhibit network externality. In the presence of network externality in oligopoly markets, product compatibility is often involved and is a very important factor in strategic decision-making for firms, because increasing product compatibility enables firms to increase the size of the network that they capture. Product compatibility between products may be given by technological constraint, but can be also a strategic instrument of a firm to attract consumers.

The goal of the paper is to examine the robustness of the results of Hoernig (2012) in the case in which firms can strategically choose product compatibility. By introducing product compatibility as a strategic variable in the model of Hoernig (2012), we show that the managers choose perfect product compatibility for every level of delegation when product compatibility is chosen by the rival firm. This is not the case in Hoernig (2012) in which no product compatibility is assumed. Our main result is that firm owners prefer their managers to be more aggressive if network externality is sufficiently strong but not too strong. This observation implies that the

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1) See Economides (1996) and Chen and Chen (2011) for this mode of choosing product compatibility.
corresponding result of Hoernig (2012) does not hold if network externality is too strong, when managers can choose product compatibility.

In our model, whole game in the market consists of three stages. First, the owner of each firm chooses weights on the profit and revenue. Second, each firm’s manager decides the degree of product compatibility. In the last stage, managers set prices to maximize their utilities. For the convenience of analysis, we assume that each firm has the same marginal cost.

Without strategic delegation, Chen and Chen (2011) and Toshimitsu (2014) investigate the effects of endogenous product compatibility on a firm’s behavior under Cournot competition. Chen and Chen (2011) show that each firm will choose no product compatibility when product compatibility is chosen by the rival firm. Toshimitsu (2014) shows that this result is reversed under the assumption that each firm will perfect product compatibility when it can determine its own product compatibility as in Shy (1995). However, our model implies that the result of Chen and Chen (2011) does not hold under price competition.2)

2. THE MODEL

Two firms 1 and 2 produce differentiated products (or systems) with constant marginal cost $c$.3) The products have network effects. Besides, each system can be made partially or fully compatible with the other, but not necessarily symmetrically. The network effects for firm $i = 1, 2$ is measured by $f(S_i^c)$ where $S_i^c$ denotes the expected network size for firm $i$. We assume that firm $i$ faces demand function:

$$x_i = \alpha - p_i + \beta p_j + f(S_i^c), \quad \forall i, j = 1, 2; \quad j \neq i,$$

2) We also perform robustness test where each firm chooses its own compatibility as in Toshimitsu (2014) and find that our result still holds under the price competition. See Remark 3.1 at the end of subsection 3.1.

3) Here, we ignore the fixed cost of the firms.
where $\alpha > 0$ is the potential market size, $p_i$ and $p_j$ denote the prices set by firm $i$ and its rival firm $j$, and $\beta \in (0, 1)$ denotes product substitutability. We assume that $f(S^*_{ij}) = nS^*_{ij}$ where $n \in (0, 1)$ is the common degree of network externality for network size of firm $i$.

Let $y_i$ denote the expectation about firm $i$’s equilibrium market share. Let $\gamma_j \in [0, 1]$ be the degree of product compatibility that firm $j$ chooses. If $\gamma_j = 0$, two products are not compatible, if $\gamma_j \in (0, 1)$, they are partially compatible, and if $\gamma_j = 1$, they are perfectly compatible. Then, firm $i$ can obtain extra network benefit of $\gamma_j y_j$ from product compatibility. Here we call $y_i$ the own effect and $\gamma_j y_j$ the spillover effect for the expected network size. Indeed high-tech firms try to generate the spillover effects for the expected network size of the counterpart users by adopting the compatible technology.

In particular, we assume that the expected network size is given by

$$S^*_{ij} = y_i + \gamma_j y_j.$$  

Then firm $i$ faces the following demand function:

$$x_i = \alpha + n(y_i + \gamma_j y_j) - p_i + \beta p_j.$$  

Observe that the case of $\gamma_j = 0$ corresponds to Hoernig’s (2012) model and the case of $\gamma_j = n = 0$ corresponds to Sklivas’ (1987) model under price competition. To ensure that that consumers’ highest valuation $\alpha/(1 - \beta)$ of the good should be above marginal cost $c$, we need the following condition:

$$0 < c < \frac{(1 + \beta)\alpha}{2},$$  

(1)

where $(1 + \beta)\alpha/2 < \alpha/(1 - \beta)$. If condition (1) holds, the demand is higher than zero. For the second-order condition for each owner’s maximization problem to be satisfied (see footnote 10), it is also assumed that the joint effect
of substitutability $\beta$ and network externality $n$ is limited such that

$$(3 + \beta)n < 2. \quad (2)$$

The owner of firm $i$ makes the contract with the manager such that his manager $i$ maximizes the objective function:

$$O_i = \lambda_i \pi_i + (1 - \lambda_i)R_i = (p_i - \lambda_i c)x_i,$$

where $\pi_i = (p_i - c)x_i$ and $R_i = px_i$ are profits and revenue, respectively. If $\lambda_i = 1$, manager $i$ is a profit-maximizer; If $\lambda_i < 1$, manager $i$ is said to be more aggressive (than profit maximizer); If $\lambda_i > 1$, manager $i$ is said to be less aggressive (than profit maximizer).

3. OPTIMAL STRATEGIES OF OWNERS AND MANAGERS

The whole game in the market consists of three stage games. In the first stage game, firm owners choose weights on profits and revenues. In the second stage game, managers chooses the degree of product compatibility. In the third stage game, each manager maximizes his objective function by setting price. The solution concept is subgame-perfect Nash equilibrium. To solve the problem by backward induction, we first consider the third stage game of price setting between the managers.

3.1. Optimal Strategies of Managers

Suppose that the owners’ choices of weights $(\lambda_1, \lambda_2)$, the managers’ choices of product compatibility $(\gamma_1, \gamma_2)$, and firm $j$’s price $p_j$ are given. Manager $i$ maximizes the objective function $O_i$ by choosing own price $p_i$. This results in the best response function of manager $i$, which is given

\[\text{When (1) holds, the equilibrium price is higher than the marginal cost only if (2) is satisfied.}\]
by\(^5\))

\[ B_i(p_j) = \frac{1}{2} [\alpha + \lambda c + \beta p_j + n(y_i + y_j)], \quad \forall i, j = 1, 2, \quad j \neq i. \]

As usual, since \( B_i \) increases in \( p_j \), prices are strategic complements. A Nash equilibrium in the pricing game (the third stage) of managers is given by \((p_1, p_2)\) such that \( p_i = B_i(p_j) \) where \( i, j = 1, 2, j \neq i \). As a consequence, we obtain price \( p_i \) and corresponding quantity \( x_i \):

\[
\begin{align*}
p_i &= \frac{\alpha(2 + \beta) + n(2(y_i + y_j) + (y_j + y_i)\beta) + c(2\lambda_i + \beta\lambda_j)}{4 - \beta^2}, \quad \text{and} \\
x_i &= \frac{\alpha(2 + \beta) + n(2(y_i + y_j) + (y_j + y_i)\beta) - c((2 - \beta^2)\lambda_i - \beta\lambda_j)}{4 - \beta^2}.
\end{align*}
\]

Imposing the rational expectations condition \( y_i = x_i \) as in Katz and Shapiro (1985), we have, for each \( i = 1, 2, \)

\[
\begin{align*}
\hat{p}_i &= \frac{\alpha(2 + \beta - (1 - \gamma_j)n)}{(n - 2)^2 - (n\gamma_i + \beta)(n\gamma_j + \beta)} + \frac{c[(1 - \gamma_i) \gamma_j + (\beta - (\gamma_j + \beta)n)\lambda_j]}{(n - 2)^2 - (n\gamma_i + \beta)(n\gamma_j + \beta)}, \quad \text{and} \\
\hat{x}_i &= \frac{\alpha(2 + \beta - (1 - \gamma_j)n) - c[(2 - \beta^2 - (1 + \gamma_i)\beta)\lambda_i - (\beta - (\gamma_j + \beta)n)n\lambda_j]}{(n - 2)^2 - (n\gamma_i + \beta)(n\gamma_j + \beta)}.
\end{align*}
\]

Note that, from (2), we have

\(^5\) Note that the second-order condition for the maximization for manager \( i \) is satisfied: \( \frac{\partial^2}{\partial p_i^2} = -2 < 0. \)
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\[(n - 2)^2 - (n\gamma_i + \beta)(n\gamma_j + \beta) \geq (n - 2)^2 - (n + \beta)^2\]
\[= (2 + \beta)(2 - \beta - 2n)\]
\[> (2 + \beta)\left(2 - \beta - \frac{4}{3 + \beta}\right)\]
\[> 0.\] (3)

In the second stage, manager \(i\) chooses \(\gamma_i\) maximizing \(\hat{O}_i(\gamma_i; \gamma_j) = (\hat{p}_i - \lambda_jc)\hat{x}_i = \hat{x}_i^2\) given by

\[
\hat{O}_i(\gamma_i; \gamma_j) = \left[\frac{\alpha(2 + \beta - (1 - \gamma_j)n)}{(n - 2)^2 - (n\gamma_i + \beta)(n\gamma_j + \beta)} - c(2 - \beta^2 - (1 + \gamma_j\beta)n)\hat{x}_i - (\beta - (\gamma_i + \beta)n)\hat{\lambda}_j}{(n - 2)^2 - (n\gamma_i + \beta)(n\gamma_j + \beta)}\right]^2.
\]

Under conditions (1) and (2), one can easily show that \(\hat{x}_i > 0\). Thus \(\hat{O}_i(\gamma_i; \gamma_j)\) is strictly positive if (1) and (2) hold.6)

**Proposition 3.1:** *In the product compatibility choice game (the second stage), there exists a unique symmetric Nash equilibrium \((\gamma_i^*, \gamma_j^*) = (1, 1)\) for any \((\lambda_i, \lambda_j)\).*

**Proof:** We observe that

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6) As long as (1) and (2) hold, we have \(\hat{x}_i > 0\) for each \(i = 1, 2\) and thus the manager’s objective function \(\hat{O}_i\) is the increasing function of \(\gamma_i\) for each \(i = 1, 2\). It follows that each firm’s manager chooses the perfect compatibility in equilibrium as in Proposition 3.1. Without condition (1), we cannot ensure that \(\hat{x}_i > 0\) for each \(i = 1, 2\). If \(\hat{x}_i = 0\), then \(\hat{O}_i(\gamma_i; \gamma_j) = 0\) and thus the manager \(i\) can choose any value of \(\gamma_i \in [0, 1]\). Therefore, if \(\hat{x}_i = 0\) or \(\hat{x}_j = 0\), there are infinitely many equilibrium compatibility choices. For simplicity, we only consider the case where \(\hat{x}_i > 0\) and \(\hat{x}_j > 0\).
It follows from (1) and (3) that $\frac{\partial \hat{O}_i}{\partial \gamma_i} > 0$. This implies that $\hat{O}_i$ is strictly increasing in $\gamma_i$. Hence $\gamma_i^* = 1$ is the strictly dominant strategy for manager $i$, and thus $(\gamma_1^*, \gamma_2^*) = (1, 1)$ is a unique symmetric Nash equilibrium. □

Proposition 3.1 shows that each manager chooses the perfect compatibility for any his owner’s weights on profits. It is worth noting that the case of Hoernig (2012) where $(\gamma_1^*, \gamma_2^*) = (0, 0)$ is not a Nash equilibrium in our model. In other words, endogenizing product compatibility excludes the equilibrium of Hoernig (2012) who does not consider product compatibility.

**Remark 3.1:** Robustness of Our Assumption about Network Size.

Now we examine whether our equilibrium compatibility choice in Proposition 3.1 holds under the assumption that each firm chooses its own compatibility. The expected network size is given by

$$S_i^* = y_i + \gamma_j y_j$$

as in Toshimitsu (2013). Note that $\gamma_j$ in our original model is replaced by $\gamma_i$. Then the demand function faced by firm $i$ becomes

$$x_i = \alpha + n(y_i + \gamma_j y_j) - p_i + \beta p_j.$$ 

This follows that the best response function of manager $i$ is given by

$$B_i(p_j) = \frac{1}{2}[\alpha + \beta p_j + n(y_j + \gamma_j y_j)], \quad \forall i, j = 1, 2, \quad j \neq i.$$ 

In the pricing game, for $i, j = 1, 2, j \neq i$, the price and corresponding quantity are
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\[ p_i = \frac{\alpha (2 + \beta) + n(2(y_i + \gamma_j y_j) + (y_j + \gamma_j y_j) \beta) + c(2 \lambda_i + \beta \lambda_j)}{4 - \beta^2}, \quad \text{and} \]

\[ x_i = \frac{\alpha (2 + \beta) + n(2(y_i + \gamma_j y_j) + (y_j + \gamma_j y_j) \beta) - c((2 - \beta^2) \lambda_i - \beta \lambda_j)}{4 - \beta^2}, \]

respectively.

Imposing the rational expectations condition \( y_i = x_i \) as in Katz and Shapiro (1985), for each \( i \),

\[ \hat{p}_i = \frac{\alpha (2 + \beta - (1 - \gamma_i)n)}{(n - 2)^2 - (n \gamma_i + \beta)(n \gamma_j + \beta)} + c\left[\frac{((1 - \gamma_i \gamma_j)n^2 - (3 + \gamma_i \beta)n + 2)\lambda_i + (\beta - (\gamma_i + \beta)n)\lambda_j}{(n - 2)^2 - (n \gamma_i + \beta)(n \gamma_j + \beta)}\right], \]

and

\[ \hat{x}_i = \frac{\alpha (2 + \beta - (1 - \gamma_i)n) - c\left[(2 - \beta^2 - (1 + \gamma_i \beta)n)\lambda_i - (\beta - (\gamma_i + \beta)n)\lambda_j\right]}{(n - 2)^2 - (n \gamma_i + \beta)(n \gamma_j + \beta)}. \]

Note that \( \hat{O}_i(\gamma_i; \gamma_j) \equiv (\hat{p}_i - \lambda_i)\hat{x}_i = \hat{x}_i^2 \) still holds.

By (1) and (3), we have

\[ \frac{\partial \hat{O}_i}{\partial \gamma_i} = \frac{2(2 - n)n}{[(n - 2)^2 - (n \gamma_i + \beta)(n \gamma_j + \beta)]} \hat{x}_i \hat{x}_j > 0. \]

Therefore, \( \hat{O}_i \) is still strictly increasing function of \( \gamma_i \) and thus \( (\gamma_i^*, \gamma_j^*) = (1, 1) \) is a unique symmetric Nash equilibrium. Consequently, under price competition, the equilibrium compatibility choice in our original model is the same as under the assumption that each firm chooses its own compatibility as in Toshimitsu (2013). \(^7\)

\(^7\) Since both models have the same equilibrium compatibility choice, the difference in models does not affect the optimal choice of owners.
3.2. Optimal Strategies of Owners

In the first game, owner $i$ anticipating that his choice $\lambda_i$ of weight affect the second stage game. Given rival firm’s weight $\lambda_j$, owner $i$ maximizes his profits by solving

$$\max_{\lambda_i \in \mathbb{R}} \tilde{\pi}_i = (\hat{p}_i - c)\hat{x}_i,$$

where $\hat{p}_i$ and $\hat{x}_i$ is prices and quantity obtained in the last part of the previous subsection. Given $(\gamma_i, \gamma_j) = (1, 1)$, owner $i$’s best response function with respect to owner $j$’s strategy is given by8)

$$B_i(\lambda_j) = \frac{\delta}{\Delta} + \left( \frac{\theta}{\Delta} \right)\lambda_j,$$

where

$$\delta = (2 + \beta)$$

$$\times \left[ \alpha(\beta^2 - 2n) + c\{2(1 + \beta)n^2 - (6 + \beta - 3\beta^2)n + 4 - 2\beta - 2\beta^2 + \beta^3\} \right],$$

$$\theta = 2c(1 + \beta)\left( n - \frac{\beta^2}{2} \right)\left( n - \frac{\beta}{1 + \beta} \right), \quad \text{and}$$

$$\Delta = 2c\left( n - \frac{2}{3 + \beta} \right)\left( n - \frac{2 - \beta^2}{1 + \beta} \right).$$

Observe that $\Delta$ is always positive under condition (2) and that $\theta$ is negative if $n \in (\beta^2/2, \beta/(1 + \beta))$ and is positive if $n \in (0, \beta^2)$ or $n \in (\beta/(1 + \beta), 1)$. As a consequence, we have the following proposition.

**Proposition 3.2:** Firm owners’ weights on profits are strategic substitutes if $n \in (\beta^2/2, \beta/(1 + \beta))$ and otherwise they are strategic complements.

The unique Nash equilibrium at the first stage game is given by $(\lambda_i^*, \lambda_j^*)$

8) Under assumption (2), the second-order condition for owner $i$’s maximization problem is satisfied: $\partial^2 \hat{\pi}_i/\partial \lambda_i^2 < 0.$
such that $\lambda_i^* = B_i(\lambda_j^*), \forall i = 1, 2; j \neq i$, which implies that the Nash equilibrium is symmetric. The unique symmetric Nash equilibrium is given by

$$\lambda_i^* = \lambda_j^* \equiv 1 + \frac{[\alpha - c(1 - \beta)](\beta^2 - 2n)}{cK(n; \beta), \forall i = 1, 2,} \quad (4)$$

where

$$K(n; \beta) = \left[ 2(1 + \beta)n^2 - (8 - \beta - 3\beta^2)n + 4 - 2\beta - \beta^2 \right].$$

**4. MAIN RESULT**

From the results of the previous section, we obtain equilibrium prices and quantities: for each $i = 1, 2$,

$$p_i^* = \frac{c(1 - 2n)[2 - \beta^2 - (1 + \beta)n] + \alpha[2 - n(3 + \beta)]}{K(n; \beta)}.$$

We should consider $n \in (0, 1)$ such that

$$p_i^* - c = \frac{[\alpha - c(1 - \beta)][2 - (3 + \beta)n]}{K(n; \beta)} > 0, \quad (5)$$

and for each $i = 1, 2$,

$$p_i^* - \lambda_i^* c = x_i^* = \frac{[\alpha - c(1 - \beta)][2 - \beta^2 - (1 + \beta)n]}{K(n; \beta)} > 0. \quad (6)$$

**Lemma 4.1:** Under conditions (1) and (2), the inequalities in (5) and (6) are satisfied if and only if $n \in (0, n^*)$ holds where
\[ n^* = \frac{8 - \beta - 3\beta^2}{4(1 + \beta)} - \frac{1 - \beta}{1 + \beta} \sqrt{2(1 + \beta) + \frac{9}{16}\beta^2}. \]

**Proof:** Clearly, under conditions (1) and (2), the inequality in (5) is satisfied if and only if \( K(n; \beta) > 0 \), which is equivalent to

\[ n < n^* = \frac{8 - \beta - 3\beta^2}{4(1 + \beta)} - \frac{1 - \beta}{1 + \beta} \sqrt{2(1 + \beta) + \frac{9}{16}\beta^2}. \quad (7) \]

It is noted that \( n^* \in (0, 1) \) for all \( \beta \in (0, 1) \). Observe that, under conditions (1) and (7), the inequality in (6) holds if and only if \( 2 - \beta^2 - (1 + \beta)n > 0 \), which is equivalent to the condition:

\[ n < \hat{n} = \frac{2 - \beta^2}{1 + \beta}. \]

One can easily check that \( n^* < \hat{n} \) for all \( \beta \in (0, 1) \). Consequently, under conditions (1) and (2), the inequalities in (5) and (6) are satisfied if and only if \( n < n^* \) holds. \( \square \)

In (4), we observe that under condition (1), whether \( \lambda^* \) is greater than or less than one depends on the signs of \( \beta^2 - 2n \) and \( K(n; \beta) \). As a main result, we find that \( \lambda^* \) is less than one under some condition for the network externality and product substitutability.

**Theorem:** Under conditions (1) and (2), if \( n \in (\beta^2/2, n^*) \) holds, then, in equilibrium, firm owners want their managers to be more aggressive, i.e., \( \lambda^* < 1 \).

**Proof:** It is obvious that, under conditions (1) and (7), inequality \( n > \beta^2 / 2 \) holds if and only if managers are more aggressive, i.e.,

\[ \lambda^*_i = \lambda^* \equiv 1 + \frac{[\alpha - c(1 - \beta)](\beta^2 - 2n)}{cK(n; \beta)} < 1, \quad (8) \]
clearly, we have $0 < \beta^2 / 2 < n^*$ for all $\beta \in (0, 1)$. Hence, under conditions (1) and (2), the condition that $n \in (\beta^2 / 2, n^*)$ holds if and only if we have a unique equilibrium in which $\lambda^* < 1$, i.e., firm owners want their managers to be more aggressive. □

Roughly speaking, in equilibrium, the managers become more aggressive if network externality is sufficiently strong but is not too strong such that $n \in (\beta^2 / 2, n^*)$ holds. This result is different from that of Hoernig (2012), where the managers are more aggressive if $n \in (1 - \sqrt{1 - \beta^2}, 1)$. Since $\beta^2 / 2 < 1 - \sqrt{1 - \beta^2} < n^* < 1$, the interval of network externality in which the managers to be more aggressive in our model partially overlaps the left side of that in Hoernig (2012). Since the spillover effect due to product compatibility builds a larger network size, smaller degrees of network externality are sufficient to induce the managers to be more aggressive. It is worth noting that, under conditions (1) and (2), firm owners do not prefer their managers to be more aggressive when network externality is too strong such that $n \in (n^*, 1)$ and thus $K(n; \beta) < 0$ (as long as equilibrium is well defined),\(^9\) where, on the contrary, the managers are more aggressive in Hoernig (2012).\(^10\)

Another difference is related to the strategic complementarity or substitutability of firm owners’ weights on profits. In the equilibrium of Hoernig (2012), if network effects are strong enough such that $n \in (1 - \sqrt{1 - \beta^2}, 1)$, then owners’ weights on profits are strategic substitutes: competition between owners then leads to a downward spiral where both managers receive strong incentives to fight for market share. In our equilibrium with $\lambda^* < 1$, firm owners’ weights on profits are strategic substitutes if $n \in (\beta^2 / 2, \beta / (1 + \beta))$ while they are strategic complements if\(^9\) Recall that there is no equilibrium when $n \in (n^*, \hat{n})$ because $p^*$ and $x^*$ diverge to infinity if $n = n^*$, and $p^* < c$ and $x^* \leq 0$ if $n \in (n^*, \hat{n})$.

\(^10\) Note that $\beta > (1 + \sqrt{5}) / 2$ if and only if $\hat{n} < 1$. In this case, if $n \in (\hat{n}, 1)$, firm owners’ weights on profits are strategic complements and thus they want their managers to be less aggressive, i.e., $\lambda^* > 1$. 

In other words, if we allow for the endogeneity of product compatibility in addition to network externality, firm owners want their managers to be more aggressive even when their weights on profits are strategic complements.

**Remark 4.1:** Checking the second-order condition for firm owner’s maximization problems, we know that condition (2) is not necessary (see footnote 4). Indeed, condition \( n > \hat{n} \) works for the second-order condition. This implies that \( K(n; \beta) < 0 \). However, for (8) to hold, it must be that \( n < \beta^2 \). Since \( \beta^2 / 2 < \hat{n} \), there is no solution for \( n \). Therefore, condition (2) is justified in our model. \( \Box \)

## 5. CONCLUDING REMARKS

In this study, we analyze the effects of network externality and product compatibility on strategic delegation under price competition in duopoly. By introducing product compatibility as a strategic variable in the model of Hoernig (2012), we show that the managers always choose perfect product compatibility when product compatibility is chosen by the rival firm. Thus, endogenizing product compatibility excludes the equilibrium of Hoernig (2012) who assumes no compatibility between products. In equilibrium, firm owners require their managers to be more aggressive than profit maximizer if network externality is sufficiently strong but not too strong. Consequently, firm owners require their managers to be less aggressive than profit maximizer if network externality is sufficiently close to one, in which case Hoernig (2012) obtains the reverse result. It is because perfect product compatibility has spillover effects in addition to network externality. Moreover, unlike Hoernig (2012), our equilibrium may exhibit the strategic complementarity of firm owners’ weights on profits.
REFERENCES


