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Real Exchange Rate Dynamics under Alternative Approaches to Expectations*

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This paper revisits the model of Mark (2009) where real exchange rate is determined by expected inflation and aggregate economic activity gap with a special emphasis on changing monetary policy rules and learning. By expanding sample periods and countries, this paper documents some basic features of real exchange rate data and significant differences in time-series properties across regimes. Simulation results suggest that the model with real-time learning generally outperforms the model under rational expectations in its ability to account for generating volatile real returns and for why the UIP fundamentals predict real exchange rate returns in the right direction in the great recession era, while there is the possibility that the RE assumption might be more pertinent for a certain sample period or country.

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1. INTRODUCTION

This paper reviews some recent empirical progress in accounting for exchange rate dynamics that has been an arduous research challenge ever since Meese and Rogoff (1983) documented a standard model for exchange rate does not perform better than a simple random walk model. By utilizing an alternative set of macroeconomic fundamentals guided by the Taylor rule, we scrutinize the dynamic behavior of exchange rate in relation to the fundamentals. To assess the importance of the Taylor rule fundamental and the specification of expectations, we obtain a set of measurements and list some salient features of real exchange rate and revisit the real exchange rate model of Mark (2009) in which real exchange rate is determined mainly by expected inflation and aggregate economic activity gap of the home and foreign countries. The introduction of adaptive real-time learning agent is motivated by the fact that, in practice, all market participants do not completely understand the true parameters involved in the model structure. In addition, as in vast number of studies, this paper also suggests changing monetary policy rules play significant role in explaining real exchange rate dynamics.

To understand forces driving exchange rate dynamics, some promising attempts have been provided. One line of research is to propose using alternative macroeconomic fundamentals. For example, Engel and West (2006) and Molodtsova and Papell (2009) utilize the Taylor-rule fundamentals involving relative expected inflation and real activity gap.[1] Benigno (2004) also places a greater weight on endogenous monetary policy and suggests that the Taylor-rule approach offers a potential solution to the purchasing power parity puzzle. The second strand of literature tests the effects of macroeconomic news. Anderson, Bollerslev, Diebold, and Vega (2003) and Clarida and Waldman (2008), among others, examine the role of news on macroeconomic fundamentals to account for exchange rate dynamics and found that the impact of macroeconomic announcement on the exchange rate

[1] Engel and West (2006) successfully reproduce some key characteristics of exchange rate such as high volatility and persistence under rational expectations, but fail to explain correlation between model-based exchange rate and the actual exchange rate.

can be asymmetric depending on the type of the news. The third line of studies places a greater weight on endogenous monetary policy. The third line of studies introduces adaptive learning rules to an exchange rate model that is standard in all respects except that economic agents do not completely understand the true economic structure. The findings from Chakraborty and Evans (2008), Kim (2009), and Mark (2009) underscore the role of learning in accounting for exchange rate dynamics such as excess volatility and substantial persistence of exchange rates.

Following these strands of argument, we reconsider the empirical performance of Mark (2009) modelling real exchange rate determined by expected inflation and real activity gap to show that the model under a least squares learning has better ability to capture the volatility and the major swings in the real deutschemark/euro-dollar exchange rate from 1976 to 2007 than the rational expectations model. This is motivated by some well-known aspects of exchange rate models with rational expectations assumption. Many Taylor-rule approaches to exchange rate under rational expectations exhibit poor performance in fitting actual data with both out-of-sample and in-sample estimation. Therefore, it is worth departing from the full rationality to examine how well a model with learning improves the fit of the data. One reasonable starting point would be the introduction of parameter uncertainty while maintaining the assumption that economic agents are all understand the economic environment except for true parameters. Under this learning environment, the public acts like applied econometrician.[2] Another important reason why Mark (2009) is revisited is that this model explicitly considers changing monetary policy rules, which are apparent in historical data. For instance, Clarida, Gali, and Gertler (2000) show that the Fed less responded to expected inflation in pre-1979 period. However, since the Fed aggressively reacted to changes in expected inflation by excessively raising nominal interest rate to fight against high inflation, an increase in expected inflation led a rise in real interest rate during the Volcker era. Therefore, it is crucial to take

[2] An excellent survey of alternative approaches to the specification of the expectations of economic decisionmakers in dynamic macroeconomic model, see Woodford (2013).

monetary policy changes into account to better understand the behavior of real exchange rate.

Our findings from empirical and numerical applications suggest some important conclusions. By expanding sample period and countries, this paper presents somewhat different stories from Mark (2009) focusing on the real deutschemark/euro-dollar exchange rate from 1976 to 2007. We employ both monthly and quarterly observations covering from 1974 to 2018 and consider four foreign countries, Germany, Japan, Switzerland, and the United Kingdom. First, we document some salient features of real exchange rate data in relation with the macroeconomic fundamentals with a special emphasis on the differences in time-series properties of the real exchange rates across regimes. It appears to be dramatic changes in the dynamic behavior of real exchange rate between the pre- and post-global financial crisis, particularly in terms of persistence and predictability of the exchange rate. Second, our simulation results suggest that the model with real-time learning generally outperforms the model under rational expectations in its ability to account for generating volatile real returns and for why the UIP fundamentals predict real exchange rate returns in the right direction in the great recession era, but not in the pre-crisis period. However, this paper also suggests the possibility that the rational expectations assumption might be more pertinent for a certain sample period or currency to account for real exchange rate dynamics as the relative empirical performance between the models with the alternative specifications for expectations is not fully conclusive.

The remainder of the paper is organized as follows. The next section documents some important time-series properties of real exchange rate data in relation with macroeconomic fundamentals. A special emphasis will be placed on the distinctive aspects of the data in the great recession period. Section 3 describes a standard model for real exchange rate by Mark (2009) and introduces alternative approaches to the specification of the expectations of economic decisionmakers in the model. In section 4, we evaluate the models in its ability to account for the stylized facts discussed in section 2. Both the model under rational expectations and the learning path of real exchange rate are assessed. Concluding remarks are contained in section 5.

2. STYLIZED FACTS ABOUT REAL EXCHANGE RATE DYNAMICS

In this section, we document some basic features of real exchange rate data in relation with macroeconomic fundamentals that are popularly used in the literature. By examining real exchange rate data, we suggest some preliminary, but potentially important implications with a special emphasis on structural changes in time-series properties of real exchange rate dynamics over time.

2.1 The Data

Both monthly and quarterly observations covering from 1974 to 2018 are obtained from various sources with exceptions of Japan and Switzerland.[3] In the line of most studies, the United State is referred to the home country. For empirical analysis, four foreign countries, Germany (DEU), Japan (JPN), Switzerland (CHE), and the United Kingdom (GRB), are considered. To construct a set of macroeconomic fundamentals, we utilize short-term interest rate, inflation rate, industrial production index (IP), and unemployment rate. For the aggregate economic activity gap, we employ two popular measures of the output gap and the unemployment gap that are constructed from the Hodrick-Prescott detrended IP and unemployment series, respectively. The rate of inflation is measured by the rate of change in consumer price index (CPI).

We obtain the data on short-term interest rate and federal fund rate from the federal reserve economic data, FRED.[4] The data on unemployment rate is

[3] The data sets for Japan and for Swiss unemployment rate start from 1985:Q3 and 1993:Q1 respectively due to data availability of short-term interest rate.

[4] For Japan, short-term interest rate data is from the IFS.

from OECD Main Economic Indicators (MEI).[5] Finally, the CPI, the IP and nominal exchange rate data are taken from the International Financial Statistics (IFS).[6]

2.2 Real Exchange Rate Dynamics

The real exchange rate, q_t is calculated by

$$q_t = s_t + p_t - p_t^* \tag{1}$$

where s_t is the log nominal exchange rate quoted in American term and p_t and p_t^* denote price level in home and foreign country, respectively. Tables 1, 2, and 3 report some descriptive statistics for quarterly returns of real exchange.[7] Since this paper aims to explore a possible structural change in the real exchange rate dynamics in the relation to macroeconomic fundamentals, we consider three sub-sample periods, regimes 1, 2, and 3, as well as the full sample, 1974:Q1-2018:Q4. This is motivated by numerous studies conducting sub-sample analysis on exchange rate dynamics based on historical episodes.[8] Following Judd and Rudebusch (1998), Kim (2009), Kim, Moon, and Velasco (2017), and Mark (2009), among others, regimes 1 and 2 are the Volker monetary policy regime (1979:Q3-1987:Q4) and the post-Volker era (1988:Q1-2007:Q2), respectively. Finally, global financial crisis and global great recession era, 2007:Q3-2018:Q4, is considered as regime 3 due to unprecedentedly significant changes in monetary policy during the

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[5] For Switzerland and United Kingdom, data on unemployment rate is from the FRED.

[6] Germany introduces the euro as the official currency after 1999. In consistent with the tradition in the previous relevant studies, we employ euro-dollar rate after 1999 and euro adjusted deutschmark-dollar rate before 1999 as the German nominal exchange rate. The data on nominal exchange rate for Germany and IP for Switzerland are from the FRED.

[7] This paper reports time-series properties of quarterly exchange rate returns. Monthly returns offer qualitatively similar results, which is available from the author upon request.

[8] Most studies have documented that the historical episodes are broadly consistent with structural break dates estimated by statistical techniques, for instance, Bernanke and Mihov (1998).

period as shown in Kim and Seol (2016).[9)]

The main points that are instantly drawn from table 1 are as follows. The volatility of real exchange rate returns measured in time-series standard deviation tends to be lower over time, except for GRB in regime 3. Returns for real exchange rates have low first-order serial correlation, and the negative autocorrelations in the returns at a longer quarter indicate the possibility of mean reversion. Interestingly, returns for real exchange rate display somewhat more persistence in regime 3, the global great recession period. In consistent with the literature, the dynamic behavior of real exchange rate is essentially the same as that of nominal exchange rate presented in table 2. The time-series variability and persistence patterns of nominal exchange rate returns is mostly indistinguishable from real exchange rate returns.

Table 1 Time-Series Properties of Real Exchange Rate Returns

		Mean	Std. Dev	Autocorrelations			
				ρ_1	ρ_4	ρ_8	ρ_{16}
CHE	full sample	0.14	6.12	–0.01	0.09	0.02	0.02
	regime 1	0.06	7.68	0.19	0.22	–0.01	–0.05
	regime 2	–0.16	6.10	–0.07	0.08	0.18	–0.03
	regime 3	–0.01	4.62	–0.12	–0.29	–0.11	0.12
DEU	full sample	–0.14	5.64	0.04	0.08	0.03	–0.06
	regime 1	–0.34	6.98	0.26	0.16	0.05	0.04
	regime 2	–0.04	5.75	–0.04	0.16	0.11	–0.10
	regime 3	–0.57	4.86	–0.04	–0.32	–0.13	–0.07
JPN	full sample	0.02	6.04	0.07	0.13	0.03	–0.13
	regime 1	1.08	7.18	0.15	0.06	0.09	–0.45
	regime 2	–0.57	5.92	0.02	0.17	–0.03	–0.13
	regime 3	–0.25	5.95	–0.06	0.11	0.22	–0.10
GRB	full sample	–0.06	5.04	0.15	0.02	–0.01	–0.22
	regime 1	–0.13	6.14	0.32	–0.09	0.15	–0.64
	regime 2	0.07	4.80	0.02	–0.01	0.01	–0.16
	regime 3	–0.93	5.08	0.14	–0.08	0.00	–0.01

Notes: Descriptive statistics are calculated from quarterly observations for percentage real exchange rate returns. ρ_t is autocorrelation coefficient at t quarter(s).

9) Note that the main results presented in this paper are not quite sensitive to the exclusion of global financial crisis period, 2007:Q3-2008:Q4.

Table 2 Time-series Properties of Nominal Exchange Rate Returns

		Mean	Std. Dev	Autocorrelations			
				ρ_1	ρ_4	ρ_8	ρ_{16}
CHE	full sample	0.62	6.10	0.00	0.09	0.02	0.01
	regime 1	0.60	7.53	0.16	0.20	–0.02	–0.12
	regime 2	0.14	6.07	–0.08	0.07	0.18	–0.01
	regime 3	0.38	4.59	–0.08	–0.31	–0.11	0.12
DEU	full sample	0.22	5.64	0.05	0.07	0.03	–0.07
	regime 1	0.29	6.91	0.25	0.16	0.05	–0.01
	regime 2	0.18	5.68	–0.04	0.15	0.11	–0.09
	regime 3	–0.48	4.94	0.00	–0.33	–0.12	–0.09
JPN	full sample	0.51	5.93	0.08	0.14	0.03	–0.13
	regime 1	1.79	7.03	0.11	0.05	0.10	–0.45
	regime 2	0.02	5.80	0.03	0.18	–0.02	–0.14
	regime 3	0.08	5.91	–0.03	0.15	0.21	–0.09
GRB	full sample	–0.35	5.08	0.15	0.03	–0.02	–0.22
	regime 1	–0.49	6.16	0.30	–0.07	0.16	–0.58
	regime 2	0.08	4.71	0.01	–0.01	0.02	–0.16
	regime 3	–1.04	5.27	0.14	–0.09	0.00	0.00

Notes: Descriptive statistics are calculated from quarterly observations for percentage nominal exchange rate rate returns. ρ_t is autocorrelation coefficient at *t* quarter(s).

In contrast to returns to exchange rate, both nominal and real exchange rates exhibit sizeable persistence and much less volatility than the returns as presented in table 3. Autocorrelation coefficients at *t* quarters for *t* = 1, 4, 8, 16 suggest that the exchange rates are highly persistent over a short horizon, but negative serial correlations at 16 confirm mean-reverting behavior of exchange rate that is widely documented in the literature. More interestingly, exchange rates display more persistent to some extent in regime 3 and tends to be nonstationary over the sample period for Germany and the UK because of the positive autocorrelations even at 16 quarters.[10)]

Having reported some important descriptive statistics for exchange rates and dollar returns on the major currency in the foreign exchange market, there is another important aspect of the data worth investigating. Since the UIP condition is regularly regarded as a key building block of open macroeconomic

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10) For the factors driving the movement of the US dollar against most currencies, see, for instance, Fratzscher (2009).

Table 3 Persistence of Exchange Rates

		Autocorrelations			
		ρ_1	ρ_4	ρ_8	ρ_{16}
Panel I: nominal exchange rate					
CHE	regime 1	0.97	0.56	–0.46	–1.30
	regime 2	0.87	0.57	0.19	–0.38
	regime 3	0.72	0.40	0.06	–0.26
DEU	regime 1	0.96	0.52	–0.31	–1.04
	regime 2	0.91	0.66	0.31	–0.30
	regime 3	0.89	0.57	0.47	0.60
JPN	regime 1	1.06	1.01	–0.01	–1.98
	regime 2	0.88	0.59	0.07	–0.23
	regime 3	0.91	0.69	0.31	–0.60
GRB	regime 1	0.93	0.62	0.20	–0.52
	regime 2	0.86	0.51	0.22	–0.45
	regime 3	0.87	0.44	0.34	0.21
Panel II: real exchange rate					
CHE	regime 1	0.91	0.47	–0.28	–0.86
	regime 2	0.87	0.61	0.31	–0.08
	regime 3	0.78	0.50	0.20	–0.43
DEU	regime 1	0.91	0.47	–0.15	–0.68
	regime 2	0.91	0.70	0.41	–0.08
	regime 3	0.91	0.64	0.55	0.72
JPN	regime 1	1.01	0.73	–0.61	–1.76
	regime 2	0.94	0.76	0.38	0.25
	regime 3	0.94	0.81	0.56	–0.25
GRB	regime 1	0.93	0.57	0.10	–0.72
	regime 2	0.82	0.35	–0.02	–0.36
	regime 3	0.86	0.41	0.28	0.05

Note: ρ_t is autocorrelation coefficient at *t* quarter(s).

models, it is imperative to examine whether the UIP fundamental has ability to account for exchange rate movement, and how the predictability changes over time.

The UIP as international asset market equilibrium condition is

$$i_t = \mathbb{E}[s_{t+1} - s_t] + i_t^*, \quad (2)$$

Where s is log nominal exchange rate, i and i^* are nominal interest rate on a one-period domestic- and foreign-currency deposits, respectively, and $\mathbb{E}[\cdot]$ is subjective expectations of US dollar depreciation rate.[11] Notice that, as the purpose of this paper is to evaluate empirical performance of a model with alternative expectation specifications, we consider subjective expectations, instead of rational expectations.[12] The UIP condition can alternatively be written as

$$\mathbb{E}[s_{t+1} - s_t] = i_t - i_t^*. \tag{3}$$

To test the UIP condition with the data, there are two important restrictions one must impose in advance. Following the majority of studies, we first assume that market participants forecast future exchange rate using rational expectations (RE). Thus, the forecast of s_{t+1} made at time *t* is given by its mathematical expectation conditioned on the information set available at time *t*. With this assumption, the UIP condition is given by a simple regression model,

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) + \eta_{t+k}, \tag{4}$$

where $\eta_t \sim iid\ N(0, \sigma_\eta^2)$. Second, due to apparent empirical evidence on the violation of UIP, it is worth employing alternative fundamentals in the context of UIP condition that are widely recognized in the literature. Many studies proposed to use an alternative set of fundamentals such as relative expected inflation rate and relative output gap.[13] For instance, Engel and West (2006) utilize the Taylor-rule fundamentals to account for the behavior of real exchange rate such as volatility and persistence.

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11) To investigate the predictability of interest rate spread in explaining long-horizon exchange rate return, Alexius (2001), Meredith and Chinn (1998), and MacDonald and Nagayasu (2000) employ *k*-period return regression for $k > 1$ and suggest explanatory power of the UIP fundamentals becomes larger as the return horizon increases.

12) For empirical violation of the full rationality in foreign exchange market, see Bacchetta and van Wincoop (2006), Chakraborty and Evans (2008), and Kim (2009).

13) As shown in Benigno (2004), the Taylor-rule approach also offers a possible solution to the purchasing power parity puzzle (Obstfeld and Rogoff, 2000).

Table 4 UIP Test Results

		Nominal exchange rate		Real exchange rate	
		UIP	Taylor-rule UIP	UIP	Taylor-rule UIP
CHE	regime 1	–0.991	–1.234	–0.985	–1.384
	regime 2	–0.445	–0.344	–0.561	–0.463
	regime 3	0.715	0.403	0.843	0.476
DEU	regime 1	–1.355	0.333	–0.806	0.413
	regime 2	–0.296	–0.324	-0.362	–0.393
	regime 3	0.727	0.515	0.456	0.279
JPN	regime 1	–0.437	–2.939	0.105	–5.069
	regime 2	–0.665	–0.653	–0.561	–0.568
	regime 3	1.433	1.269	1.426	1.279
GRB	regime 1	–1.312	–1.590	–1.181	–1.090
	regime 2	0.112	0.184	0.177	0.233
	regime 3	3.902	2.679	2.063	1.436

Note: Point estimates from the regression in eq. (4) and heteroscedasticity and autocorrelation consistent (HAC) standard errors of Newey and West (1987) are used to test the UIP condition.

Molodtsova and Papell (2009) also consider an exchange rate model in conjunction with Taylor-rule and show the evidence on short-horizon predictability, but not for the long-horizon changes in exchange rates.[14)]

Table 4 reports the estimate of slope coefficient of UIP condition for both actual interest rate differential and interest rate spread estimated by the Taylor-rule. Given the composite null hypothesis of the UIP and rational expectations, there must be a positive relationship between the US dollar (real) depreciation and interest rate spread. The deviations from the UIP are evident in the samples of regimes 1 and 2 as the slope coefficient estimates are significantly different from the unity. The real puzzle raised by numerous studies is that the estimates are consistently negative during the sample periods

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[14)] Another important strand of studies examines the role of macroeconomic news in explaining exchange rate dynamics. Anderson, Bollerslev, Diebold, and Vega (2003) investigate the link between high frequency exchange rate and fundamentals by dealing with macroeconomic news on GDP, employment, CPI, and trade balance. Clarida and Waldman (2008) also examine the reaction of nominal exchange rate to inflation announcement. These studies suggest the possibility of asymmetric response of exchange rates on macroeconomic announcement.

since the spread predicts exchange rate movement in the wrong direction.[15) However, this conventional empirical regularity collapsed in the great recession period, regime 3. The point estimates are now all significantly different from zero for both interest rate differential specifications without any exception, and thus the violation of UIP is no longer be a serious issue under rational expectations.[16) This motivates us to revisit a standard model for exchange rate determination and to evaluate the role of informational assumption underlying a model.

3. THE MODEL OF REAL EXCHANGE RATE DYNAMICS

To examine the role of informational assumption for economic agents forecasting macroeconomic fundamentals in accounting for exchange rate movements, this section describes a standard model for real exchange rate by Mark (2009). This model show how real exchange rate is determined when the UIP condition holds and central banks respond to changes in inflation and output gap according to the Taylor-rule. Given this economic structure, we consider two alternative approaches to the specification of the expectations of economic decisionmakers in the model.

3.1 Exchange Rate and Interest Rate Reaction Function

To model exchange rates, we employ uncovered interest parity,

$$i_t = \mathbb{E}[s_{t+1} - s_t] + i_t^*, \tag{5}$$

15) Several lines of attempts to uncover why the UIP condition id strictly rejected by the data, such as risk premium, peso-problem, and noise trade approaches. For an excellent survey, see Thornton (2019).

16) This result is consistent with Kim and Seol (2016) that present the empirical validity of UIP condition for euro-dollar exchange rate when monetary policy regime shifts are properly introduced.

where $\mathbb{E}[\cdot]$ is subjective expectations, s_t is time-*t* log spot exchange rate and i_t and i_t^* are time-*t* nominal interest rate on a one-period domestic-currency deposit and foreign-currency deposit, respectively. The UIP condition together with expected inflation differential yields real exchange rate,

$$q_t = \mathbb{E}q_{t+1} - (i_t - i_t^*) + \mathbb{E}[\pi_{t+1} - \pi_{t+1}^*]. \tag{6}$$

Following Clarida, Gali, and Gertler (2000), interest rate reaction function is given by the Taylor-rule. That is, central bank adjusts its target interest rate (i_t^T) in response to the desired nominal interest rate ($\bar{i}$), the deviation of expected future inflation rate from the target inflation rate ($\bar{\pi}$), and the activity gap (x_t). In addition to the traditional Taylor-rule fundamentals, the interest rate response function in Mark (2009) and Engel and West (2006) also involves real exchange rate,

$$i_t^T = \bar{i} + \theta\mathbb{E}[\pi_{t+1} - \bar{\pi}] + \mu x_t + \sigma q_t. \tag{7}$$

As this reaction function is somewhat restrictive to describe the target interest rate actually made by central bank (Clarida, Gali, and Gertler, 2000; Rudebusch, 1995), a partial adjustment to the target interest rate is popularly introduced along with the monetary policy rule,

$$i_t = (1-\rho)i_{t-1} + \rho i_t^T + \eta_t, \tag{8}$$

where $\rho \in [0,1)$ and η_t is an i.i.d. interest shock.[17] Putting all together, the interest rate reaction function for home and foreign countries are now given by

$$i_t = \rho(\bar{i} - \theta\bar{\pi}) + (1-\rho)i_{t-1} + \rho(\theta\mathbb{E}\pi_{t+1} + \mu x_t) + \eta_t, \tag{9}$$

[17] η_t can be also interpreted as monetary policy shock since it represents central bank's temporary deviation from its rule (Clarida, Gali, and Gertler, 2000).

$$i_t^* = \rho^*(\bar{i}^* - \theta^*\bar{\pi}^*) + (1-\rho^*)i_{t-1}^* + \rho^*(\theta^*\mathbb{E}\pi_{t+1}^* + \mu^* x_t^*) + \eta_t^* . \tag{10}$$

As the purpose of this paper is to investigate the empirical validity of exceptional assumption in the model of real exchange rate, inflation rate and activity gap are assumed to be exogenously generated by the following vector autoregression (VAR) representation,

$$Y_t = \alpha + \Gamma Y_{t-1} + \nu_t , \tag{11}$$

$$Y_t^* = \alpha^* + \Gamma^* Y_{t-1}^* + \nu_t^* , \tag{12}$$

where $Y_t = (\pi_t, \cdots, \pi_{t-3}, x_t, \cdots, x_{t-3})'$ and $Y_t^* = (\pi_t^*, \cdots, \pi_{t-3}^*, x_t^*, \cdots, x_{t-3}^*)'$ are the vector of the exogenous variables for home country and foreign country, respectively.

3.2 Real Exchange Rate under Rational Expectations

To find minimum state variable (MSV) rational expectations solution, we substitute interest rates implied by the response functions, eqs. (9) and (10), one-step ahead forecasts of inflation rate and activity gap, $\mathbb{E}\pi_{t+1}$, $\mathbb{E}\pi_{t+1}^*$, $\mathbb{E}x_{t+1}$ and $\mathbb{E}x_{t+1}^*$, to the real exchange rate equation, eq. (7). The stochastic difference equation governing equilibrium real exchange rate dynamics under rational expectations is[18)]

$$q_t = \lambda_0 + \lambda_1 i_t + \lambda_2 i_t^* + \lambda_3 Y_t + \lambda_4 Y_t^* . \tag{13}$$

It is worth noting that systematic patterns of interdependence between real exchange rate and inflation differential can be attributable to changes in the response of central banks to inflation as coefficients λ_3 and λ_4 in the MSV solution depend on $\underline{\theta} = (\theta, \theta^*)'$. For instance, when both θ and θ^* are less than one, which is apparently observed in the pre-Volker era, a fall in

18) For the detailed derivation of the MSV solution and the definition of each coefficient, see Mark (2009).

expected inflation differential might result in a real appreciation of home currency since market participants expect a rise in the interest differential. On the other hand, a real depreciation of home currency may also be possible if both θ and θ^* are greater than one as in the post-Volker period.

3.3 Real Exchange Rate under Adaptive Learning

In the case of rational expectations models, agents forecasting future observations of macroeconomic fundamentals fully understand economic environment including functional form of the model and parameters. Although rational expectations assumption is the standard methodology for modeling expectations and serves as an important benchmark case, an increasing number of studies have questioned its empirical validity as many macroeconomic models under rational expectations have failed to explain even some basic features of the data. In line with the attempts relaxing the expectational assumption instead of raising the model complexity, this paper introduces market participants who have knowledge about the functional form of model structure, but not parameters.

Since the market participants understand the basic model structure and utilize it as the perceived law of motion (PLM) describing how they form their expectations. In real time learning, it is reasonable to assume that the market participants at time t can access the data up to t-1. Thus, the PLMs for this model economy are summarized as

$$i_t = b_{0,t-1} + b_{1,t-1} i_{t-1} + b_{2,t-1} \mathbb{E}\pi_{t+1} + b_{3,t-1} x_t + \eta_t \,, \tag{14}$$

$$i_t^* = b_{0,t-1}^* + b_{1,t-1}^* i_{t-1}^* + b_{2,t-1}^* \mathbb{E}\pi_{t+1}^* + b_{3,t-1}^* x_t^* + \eta_t^* \,, \tag{15}$$

$$Y_t = \alpha_{t-1} + \Gamma_{t-1} Y_{t-1} + \nu_t \,, \tag{16}$$

$$Y_t^* = \alpha_{t-1}^* + \Gamma^* Y_{t-1}^* + \nu_t^* \,, \tag{17}$$

$$q_t = \lambda_{0,t-1} + \lambda_{1,t-1} i_t + \lambda_{2,t-1} i_t^* + \lambda_{3,t-1} Y_t + \lambda_{4,t-1} Y_t^* \,. \tag{18}$$

Let the selection vectors be $e_1 = (1,0,0,0,0,0,0,0)$ and

$e_1 = (0,0,0,0,1,0,0,0)$. The market participants form interest rate and inflation rate forecasts based on the PLM described above, and thus the actual law of motion (ALM) for real exchange rate is now given by

$$\begin{aligned} q_t = & (\lambda_{1,t-1} b_{1,t-1} - 1) i_t + (1 + \lambda_{2,t-1} b^*_{1,t-1}) i^*_t \\ & + (e_1 + \lambda_{1,t-1} b_{3,t-1} e_2 + \lambda_{3,t-1} + \lambda_{1,t-1} b_{2,t-1} e_1 \Gamma_{t-1}) \Gamma_{t-1} Y_t \\ & + (\lambda_{2,t-1} b^*_{2,t-1} e_2 + \lambda_{4,t-1} + \lambda_{2,t-1} b^*_{2,t-1} e_1 \Gamma^*_{t-1} - e_1) \Gamma^*_{t-1} Y^*_t + \gamma_{t-1} \end{aligned} \tag{19}$$

where

$$\begin{aligned} \gamma_{t-1} = & \lambda_{0,t-1} + \lambda_{1,t-1} (b_{0,t-1} + b_{2,t-1} e_1 \lambda_{1,t-1} + b_{2,t-1} e_1 \Gamma_{t-1} + b_{3,t-1} e_2 \lambda_{1,t-1}) \\ & + \lambda_{2,t-1} (b^*_{0,t-1} + b^*_{2,t-1} e_1 \lambda_{2,t-1} + b_{2,t-1} e_1 \Gamma^*_{t-1} + b^*_{3,t-1} e_2 \lambda_{2,t-1}) \\ & + e_1 \alpha_{t-1} - e_1 e_1 \alpha^*_{t-1} + \lambda_{3,t-1} \alpha_{t-1} + \lambda_{4,t-1} \alpha^*_{t-1}. \end{aligned}$$

It is important to note that the real exchange rate is not determined entirely by the Taylor-rule fundamentals that are observed in each period. The implied learning path of real exchange rate and the coefficients of ALM are generated by the observations of π_t, x_t, and i_t, but not by actual exchange rate data. Learning agents are updating the coefficients or covariance matrix and this process can be presented by the following recursive least square (RLS) solution for each country,

$$R_t = R_{t-1} + g(Z_{t-1} Z'_{t-1} - R_{t-1}), \tag{20}$$

$$\beta_t = \beta_{t-1} + g R_t^{-1} Z_{t-1} (y_t - Z'_{t-1} \beta_{t-1}), \tag{21}$$

where R is the second order moment matrix of explanatory variables, Z, β is a set of coefficients, and y is the dependent variable in the RLS projection. g is a gain sequence, and we consider a constant gain in this paper. To conserve on the space, we do not demonstrate how the coefficient estimates for each endogenous variable are updated in this learning environment in this paper. However, the parameter estimates for inflation rate, activity gap, and interest rate reaction functions are updated in the similar fashion as eqs. (20) and (21). While the observations of i_t, i_t^*, π_t, π_t^*, x_t, and x_t^* are directly used from the actual data in the real-time learning process, the real exchange rate q_t is given by the ALM.

4. NUMERICAL ANALYSIS

Since the main goal of this paper is to scrutinize the role of expectations specifications in explaining the dynamic behavior of real exchange rate, this section evaluates the model in its ability to account for some salient features of real exchange rate. That is, through the numerical analysis, we will decide whether the generated time-series implied by the models under alternative expectations formations look like the observations from the actual data. As a benchmark, the performance of the model under full-information rational expectations is assessed. Next, due to macroeconomic frictions, when decisionmakers are assumed to have incomplete knowledge about economic structure regarding future inflation and the output gap or the unemployment gap and utilize adaptive learning rules to learn about the true economy, we also evaluate the model with this specification of expectations. In particular, since the dynamic behavior of real exchange rate is dramatically different across regimes as documented in section 2, we explore how the relative empirical validity of the expectations specifications changes over time.

The model-implied paths of real deutschmark/euro-dollar exchange rate together with the actual real exchange rate are presented in Panel (a) of figure 1.[19] The implied learning path largely captures the behavior of actual real exchange rate. The implied learning path tends to dominate the path implied by rational expectations model in pre-great moderation period, 1976:Q1-1984:Q4. During the great moderation era, 1985:Q1-2007:Q2, although both implied real exchange rate paths well capture major swing found in the data,

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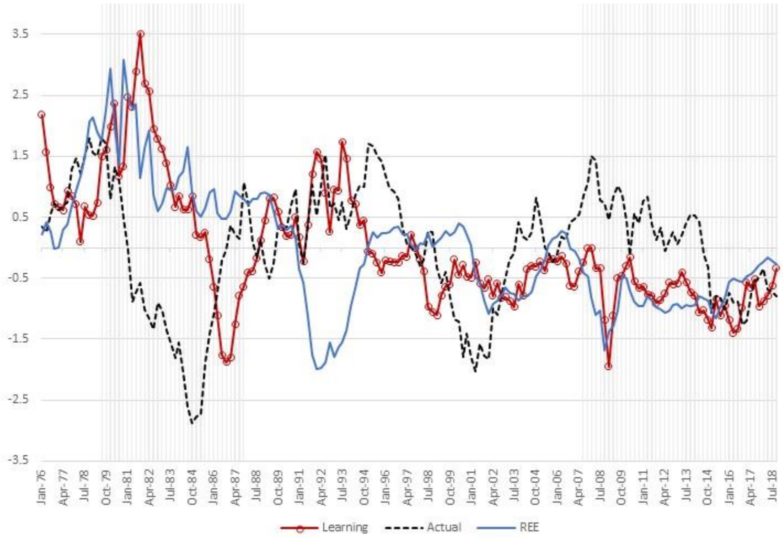
[19] To conserve on space, we did not report simulation results for other countries. These are available from the author upon request.

Figure 1 Actual and Implied Path of Real Exchange Rate

(a) Real Deuschmark/Euro-Dollar Exchange rate



(b) Real Dollar Return on the Deutschemark/Euro

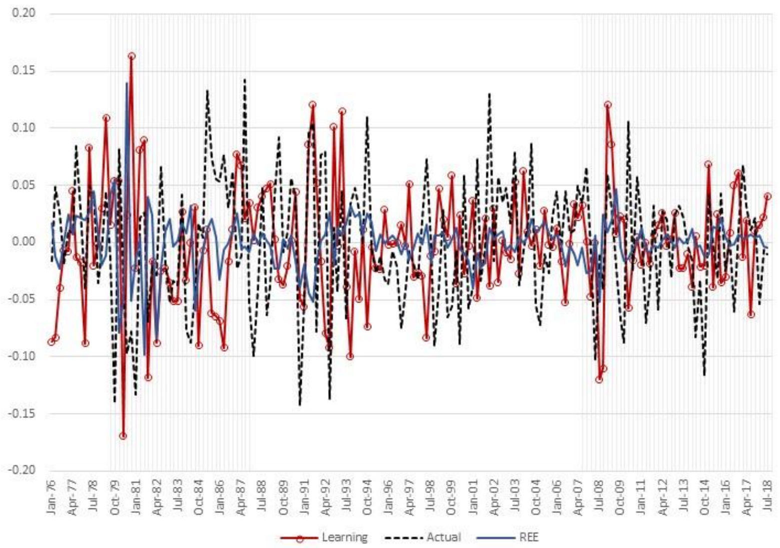


Table 5 Simulation Results: Full-sample Case

	RMSPE				Correlation			
	Output gap		Unemployment gap		Output gap		Unemployment gap	
	AL	RE	AL	RE	AL	RE	AL	RE
CHE	1.477	1.765	1.486	1.830	–0.098	–0.568	–0.118	–0.697
DEU	1.360	1.467	1.398	1.565	0.070	–0.082	0.018	–0.232
JPN	1.087	1.071	1.781	1.223	0.403	0.420	–0.604	0.244
GBR	1.522	1.364	1.435	1.543	–0.165	0.064	–0.036	–0.197

the correlation between actual and implied real exchange rates becomes weaker as the model-implied paths do not pick up turning points of the swings. In regime 3, the great recession era including global financial crisis, the model under both rational expectations and adaptive learning has successfully capture the dynamics of real exchange rate. Regarding real US dollar returns on the deutschmark/euro, the model under adaptive learning performs better than the RE model to account for why real exchange rate returns are so volatile as clearly seen in Panel (b) of figure 1.

While these findings are mostly consistent with Mark (2009), in what follows, our simulation results for other countries and regimes suggest somewhat different stories. First, in addition to gauge overall goodness-of-fit strength measured in RMSPE, we calculate correlation coefficient between actual and model-implied real exchange rates to assess whether each model specification can capture real exchange rate movement. Table 5 presents the RMSPEs and correlation coefficients for output gap and unemployment gap. During the full sample, both measures of real activity gap have qualitatively the same result, except for the Japanese yen. Overall, the implied learning path of real exchange rate performs better in explaining real exchange rate movement for the Swiss franc, deutschmark/euro, and the UK pound. However, the RE model has better ability to account for short-run patterns of real exchange rate except for Japanese yen as correlation coefficients are consistently greater than the model under adaptive learning. This may be because of either gradual adjustment of parameter estimates due to constant gain or the inclusion of great recession era in our full sample period, which evidently motivates sub-sample analysis.

To judge how well the model with alternative approaches to expectations in each of the three apparent regimes based on historical events, great inflation, great moderation, and great recession, table 6 presents both RMSPEs and correlation coefficients.[20] On the whole, the implied learning path distinctly dominates the RE path in the great inflation period. This does not come to a surprise since a number of studies have shown that the dynamic behavior of inflation, which is one of key ingredients of exchange rate models, in the great inflation period is predominantly explained by adaptive learning agents (Bullard and Cho, 2005; Gaspar, Smets, and Vestin, 2010; Sargent, 1999). Unlike the full sample analysis, the relative performance of the models in the other regimes is somewhat mixed, and a greater number of cases suggests the possibility that the RE model outperforms the model under adaptive learning. In particular, in addition to the model for expectations, the choice of aggregate economic activity gap measure, output gap and unemployment gap, becomes crucial. Nonetheless, even in the post-great inflation era, the learning path behaves more closely to actual real exchange rate data.[21]

Table 6: Simulation Result: Sub-sample Case

		RMSPE				Correlation			
		Output gap		Unemployment gap		Output gap		Unemployment gap	
		AL	RE	AL	RE	AL	RE	AL	RE
Great inflation	CHE	3.443	4.962	–	–	–0.686	0.168	–	–
	DEU	2.181	1.976	2.169	2.050	–0.185	0.125	–0.367	–0.338
	JPN	–	–	–	–	–	–	–	–
	GBR	2.270	1.907	1.900	1.757	–0.143	0.500	–0.036	–0.279
Great moderation	CHE	1.532	1.433	1.496	2.191	–0.164	–0.196	–0.305	–0.336
	DEU	1.037	1.326	1.097	1.453	0.253	–0.200	0.220	–0.164
	JPN	1.242	0.990	1.709	1.222	–0.130	0.416	–0.616	0.061
	GBR	1.153	1.223	1.095	1.238	–0.011	–0.080	0.245	0.034
Great recession	CHE	1.006	1.822	1.479	1.520	–0.038	–0.440	–0.171	–0.488
	DEU	1.044	1.245	1.126	1.321	0.416	–0.419	0.102	–0.601
	JPN	0.842	1.168	1.871	1.223	0.689	–0.098	–0.431	–0.229
	GBR	1.408	1.039	1.613	1.909	0.087	–0.376	0.002	–0.620

[20] Note that, due to the data availability, the Swiss franc and the Japanese yen are not included for the great inflation period.

[21] In many cases for the Swiss franc, the RE model tends to work better. This may be due to the fact that Switzerland has never engaged in an international monetary agreement (Mark, 1995).

As presented in section 2, the behavior of US dollar real exchange rate returns to the foreign currencies changes considerably across regimes. Table 7 indicates how properly each model accounts for the dynamic aspects of real deutschmark/euro-dollar exchange rate in terms of volatility measured by time-series standard deviation and predictability of the quarterly returns, the slope coefficient estimate of eq. (4).[22] Overall, the model under adaptive learning with constant gain outperforms the RE model in its ability to account for generating volatile real returns and for why the UIP fundamentals predict real exchange rate returns in the right direction in the great recession era, but not in regimes 1 and 2. Finally, it is worth noting that the relative empirical performance between the two models is not entirely conclusive in regime 2, the great moderation period, and this suggests the possibility that the RE assumption might be more pertinent for a certain sample period or currency to account for real exchange rate dynamics.

Table 7 Simulation Results: Volatility and Predictability of Returns

	Historical Data	AL	RE	
			one-regime case	three-regime case
Panel I: Volatility				
Regime 1	0.036	0.029	1.959	0.107
Regime 2	0.029	0.020	0.701	0.205
Regime 3	0.030	0.035	0.849	0.058
Panel II: Predictability				
Regime 1	–0.215	–0.220	–38.278	–2.683
Regime 2	–0.131	0.084	–1.949	–1.923
Regime 3	0.188	0.133	–2.629	0.167

[22] For the model under rational expectations, we consider two specifications, one-regime case and three-regime case, depending on whether true economic structure undergoes a structural change or not. On the other hand, in this real-time learning experiment, we do not explicitly impose a restriction on a change in parameters.

5. CONCLUDING REMARKS

In this paper, we revisit the real exchange rate model of Mark (2009) in which real exchange rate is determined mainly by expected inflation and aggregate economic activity gap of the home and foreign countries with a special emphasis on the Taylor-rule fundamentals. In addition to the standard rational expectations model, this model also introduces an alternative approach to expectations, adaptive real-time learning, since the true parameters involved in the model structure are not completely known to all market participants and the economic environment even undergo unanticipated changes.

Some important implications directly emerge from the results of our empirical and numerical analysis. By expanding sample period and countries, we found somewhat different stories from Mark (2009) who focuses on the real deutschemark/euro-dollar exchange rate from 1976 to 2007. This paper documents some salient features of real exchange rate data in relation with macroeconomic fundamentals and evident differences in time-series properties of the real exchange rate and its quarterly returns across regimes, the great inflation, the great moderation, and the great recession triggered by the global financial crisis. It appears to be important to differentiate the dynamic behavior of real exchange rate in the post-crisis from that in the pre-crisis era, particularly in terms of persistence and predictability of the exchange rate. Next, our simulation results suggest that the model with real-time learning generally outperforms the model under rational expectations in its ability to account for generating volatile real returns and for why the UIP fundamentals predict real exchange rate returns in the right direction in the great recession era, but not in the pre-crisis period, which is consistent with Mark (2009)'s finding. However, there is also the possibility that the RE assumption might be more pertinent for a certain sample period or currency to account for real exchange rate dynamics as the relative empirical performance between the models with the alternative specifications for expectations is not fully conclusive.

Fruitful further research agenda from our findings can be suggested in a number of directions. One of the most promising approach must be to introduce model uncertainty. In this paper, we consider a simple learning environment in the sense that the only source of uncertainty is that the public do not completely know the true parameters in central bank's response function of the Taylor rule. The introduction of model uncertainty as in Branch and Evans (2007) and Lewis and Whiteman (2007), among others, to the standard model of real exchange rate helps to better understand the data in the foreign exchange market.

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