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Stock-Based Compensation and Insider Trading with Liquidity Signal*

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We study a stock-based executive (CEO) compensation contract for the case where the insider receives a signal about the demand of liquidity traders in the stock market. The single-period model of Kyle (1985) is adopted for deriving the stock market equilibrium. Based on the equilibrium stock price, the optimal linear contract for executive compensation is obtained by applying a standard principal-agent model. Surprisingly, it is found that the firm's liquidation value is not used in the optimal executive compensation contract in both our model and the benchmark model (where there is no signal about liquidity). We also make comparative statics for the equilibrium stock price and the optimal executive compensation contract with respect to exogenous parameters.

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1. INTRODUCTION

By analyzing the data during the period 1993-2003 in the United States, Bebchuk and Grinstein (2005) show that executive compensation has grown far faster than could be explained by changes in firm size and performance, which provokes the debates about the optimal executive compensation.[1] As is well known, there is a moral hazard problem between the owner (as a representative of the equityholders) and a hired manager. That is, while the owner (she) wants her manager (he) to exert an effort for maximizing the equity value (or firm's liquidation value), the manager may pursue his own private interests, because the owner cannot perfectly monitor the manger's behavior. Executive compensation models based on firm's liquidation value have been extensively discussed since Mirrlees (1976) and Holmström (1979). Adopting Hayek's (1945) idea on the role of prices of conveying information about the markets. Holmström and

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1) For a recent survey on executive compensation, see Edmans and Gavaix (2015).

Tirole (1993) and Baiman and Verrecchia (1995) study the effects of stock prices on the executive (CEO) compensation scheme between the owner and her manager.[2)]

We study a stock-based executive compensation contract for the case where the insider receives a signal about the demand of liquidity (noisy) traders (in short, liquidity signal) in the stock market. To do this, we incorporate the standard principal-agent model (e.g., Holmström, 1979) into Kyle's (1985) stock market model. In the single-period stock market of Kyle (1985) we adopt, a risk-neutral insider and liquidity traders submit their market orders for the stock to competitive market makers, who set stock prices à la Bertrand based on the aggregate order flows. Based on the equilibrium stock price, the optimal linear contract for executive compensation is obtained by applying a standard principal-agent model (e.g., Holmström, 1979). In our model, one risk-neutral owner, who owns a large portion of the firm's equity, hires one risk-averse manager to run the firm by making a compensation contract. In the compensation contract that we consider, in addition to a fixed payment, the manager can earn a bonus payment that depends on the firm's liquidation value and stock price. After accepting the contract, the manager exerts unobservable efforts at his own cost that will affect the firm's liquidation value.

It is found that the firm's liquidation value is not used in the optimal executive compensation contract in both our model and the benchmark model (where there is no liquidity signal). This result is quite surprising in that the existing literature, including Holmström and Tirole (1993), Baiman and Verrecchia (1995), and Calcagno and Heider (2021) among others, argue that both the stock price and the firm's liquidation value are crucial in executive compensation contracts. We also make comparative statics for the equilibrium stock price and the optimal executive compensation contract with respect to exogenous parameters. In particular, we shall examine the effects of the volatility of noisy trades (in short, liquidity volatility) and the volatility of the liquidation value (in short, value volatility) on the equilibrium trading strategy and price in the stock market the stock market and the optimal executive compensation contract. Our model is most closely related with Holmström and Tirole (1993) because they adopt Kyle (1985) for the stock market and investigate the value of the stock price in executive compensation contracts.[3)] However, instead of taking into account the insider of Kyle (1985), they consider a speculator who receives a signal about the firm's liquidation value, whereas the insider in our model has the complete information about the liquidation value as in Kyle (1985) but in addition he receives a liquidity signal. Holmström and Tirole (1993) consider performance incentives based on the stock price, the short-term performance, and the liquidation value. However, we simplify the executive compensation contract scheme by excluding incentives for the short-term performance

2) See also Calcagno and Heider (2021) among others.
3) See also Kang and Liu (2010) for using Kyle's (1985) model. It is noted that Baiman and Verrecchia (1995), Hahn and Kwon (2018), and Calcagno and Heider (2021) adopt the rational expectations model for the stock market.

for more tractable analysis.[4] Furthermore, a costly information acquisition of non-fundamental information is not considered either.

The insider may acquire the information about the noisy traders from market specialists. Following literature, let us refer to the information about the noisy traders as the non-fundamental information, and the information about the liquidation value as the fundamental information. There are several articles that examine the effects of acquiring non-fundamental information on the insider's trading strategy and profits. In a continuous version of the Kyle's (1985) model, Back (1992) finds that it is not advantageous for the insider to observe the noise trades before he submits his market order. Rochet and Vila (1994) confirm that Back's result holds true in the single-period model, i.e., the insider's observing the noise trades has no impact on his expected profits. In a multi-round trading model, Yu (1999) even argues that acquiring non-fundamental information only does harm to the expected profits of the insider. Recently, in the single-period model, Park (2010) investigates in detail the effects of the insider's acquiring non-fundamental information in the perspective of the insider's welfare. Introducing into Kyle's (1985) model "non-insider" speculators who observe non-fundamental information to make inferences about the insider's fundamental information, Madrigal (1996) studies how their presence affects stock prices and market liquidity.[5]

The rest of the paper is organized as follows. In section 2, we introduce the model of the linear executive compensation contract and the stock market. The equilibrium trading strategy and price in the stock market are derived in section 3. In section 4, we obtain and characterize the optimal compensation contract between the owner and the manager. Finally, concluding remarks are made in section 5.

2. THE MODEL

In our model, there are three periods t=0,1,2. There is a publicly traded firm and a financial market of its stocks. In the firm, one owner hires a manager to work for the firm. In the stock market, there are two traders: one insider and many liquidity traders, while one competitive market maker sets efficient stock prices.

2.1. Owner and Manager of the Firm

In the firm, there is an owner, as a principal, as a principal and the representative of shareholders, who hires a manager (CEO) that will work for the firm to make earnings. The owner of the firm has the share δ of the equity of the firm. She is assumed to be risk-neutral and try to maximize

4) For this simplification, see also Baiman and Verrecchia (1995), Hahn and Kwon (2018), and Calcagno and Heider (2021) among others.

5) See also Yang and Zhu (2016).

the expected value of the liquidation value $(\delta v - S)$ of her share of the firm net of a salary payment S to the CEO. The firm's liquidation value is realized and the compensation to the CEO is paid at time $t = 2$.

The CEO of the firm exerts an effort e for the firm at time $t = 0$, which costs him in the quadratic fashion: $c(e) = ke^2/2$. He is assumed to be risk-averse and have a CARA utility with absolute risk-aversion coefficient ρ:

$$U(w) = -\exp(-\rho w).$$

His effort affects the fundamental value θ. For simplicity, we assume that $\theta = e$ as in the literature of principal-agent models (e.g., Holmström and Milgrom, 1987). The owner of the firm does not know the effort level e of the CEO.

However, the liquidation value of the firm is determined by the fundamental value θ and a random factor ε, that is,[6)]

$$v = \theta + \varepsilon = e + \varepsilon.$$

Here the random factor ε is beyond the control of the CEO and assumed to be normal with zero mean: $\tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2)$. Then it is noted that $\tilde{v} \sim N(e, \sigma_\varepsilon^2)$. The owner of the firm does not know the realized liquidation value v of the firm until time $t = 2$. At time $t = 0$, when contracting with the CEO based on the liquidation value v and the stock price p in the market, the owner of the firm sets a linear compensation rule S:

$$S(v, p) = a_0 + a_1 v + a_2 p.$$

Given the linear compensation rule $S(v, p)$, the CEO chooses the optimal effort to solve the following problem.[7)]

$$\max_{e \in [0, \bar{e}]} \mathrm{E}\left[-\exp\left(-\rho[\tilde{S} - c(e)]\right)\right],$$

or its certainty equivalent form:

$$\max_{e \in [0, \bar{e}]} \mathrm{E}[\tilde{S}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}] - c(e').$$

6) The specification $v = \theta + \varepsilon$ is suggested by Grossman and Stiglitz (1980).

7) We define $\tilde{S} = S(\tilde{v}, \tilde{p})$.

Taking into account the optimizing behavior of the CEO as the incentive compatibility condition, the owner chooses the optimal contract to solve her problem:

$$\max_{(a_0,a_1,a_2),e} \mathrm{E}[\delta\tilde{v}-\tilde{S}],$$

$$\text{s.t. } \mathrm{E}[\tilde{S}]-\frac{\rho}{2}\mathrm{Var}[\tilde{S}^*]-c(e)\geq 0,$$

$$e\in \operatorname{argmax}_{e'\in[0,\bar{e}]} \mathrm{E}[\tilde{S}]-\frac{\rho}{2}\mathrm{Var}[\tilde{S}]-c(e'),$$

where the first constraint is the individual rationality and the second condition is the incentive compatibility. If we can identify the coefficient vector $a=(a_0,a_1,a_2)$ in S with the compensation rule itself, we also refer to (a,e) as an *incentive linear contract*.

2.2. Participants in Stock Market

The stock market opens at time $t=1$, where the share $(1-\delta)$ of the whole equity is traded. There are three types of participants in the stock market. The stocks of the firm are traded by two types of traders as in Kyle (1985). On the one hand, there is a risk-neutral rational informed trader who is called an insider in that he has the complete information about the realized liquidation value v of the firm at time $t=1$. On the other hand, there are liquidity traders (noisy uninformed traders). The liquidity traders' demand for the stock is represented by a white noise $\tilde{u}$ where:

$$\tilde{u}\sim N(0,\sigma_u^2).$$

Unlike Kyle (1985), the insider acquires the information about the demand of liquidity traders through a signal s_u, while he knows the firm's liquidation value:

$$\tilde{s}_u=\tilde{u}+\tilde{\eta},\text{ where } \tilde{\eta}\sim N(0,\sigma_\eta^2).$$

It is observed that $E[\tilde{s}_u]=0$ and $\mathrm{Var}(\tilde{s}_u)=\sigma_u^2+\sigma_\eta^2$. Against the demand of liquidity traders, the insider submits a linear market order $X(v,s_u)$ based on his information of the liquidation value v and the acquired signal s_u:

$$x=X(v,s_u)=\alpha(v-e)+\beta s_u. \tag{2.1}$$

Then we have

$$\mathrm{E}[\tilde{x}] = 0, \quad \mathrm{Var}(\tilde{x}) = \alpha^2\sigma_\varepsilon^2 + \beta^2(\sigma_u^2 + \sigma_\eta^2).$$

This implies that total order flow is given by

$$\tilde{y} = \tilde{x} + \tilde{u}.$$

The market maker is a third player in the stock market. It sets a linear pricing rule $P(y)$ based on the total order flow y to determine the efficient stock market price p:

$$p = P(y) = E\left[\tilde{v}\middle|\tilde{y} = y\right] = \mu + \lambda y\ . \tag{2.2}$$

Given the linear pricing rule $P(\cdot)$ of the market maker, the insider submits the market order x to the market maker to maximize his expected profit:

$$\max_x \mathrm{E}[(\tilde{v} - \tilde{p})x \mid (\tilde{v}, \tilde{s}_u) = (v, s_u)].$$

The optimal market order x^* should be consistent with the linear order rule above, that is,

$$x^* = X(v, s_u).$$

2.3. Sequence of Events

At $t = 0$, the owner of the firm hires the manager and gives him an incentive contract (S, e), where S is a linear in the firm's liquidation value and the stock price. The manager then chooses an unobservable level of effort e, which affects the firm's fundamental value and thus liquidation value. At $t = 1$, the stock market opens. The insider observes both the firm's liquidation value and liquidity signal s_u about the demand of liquidity traders. All traders submit their orders to the competitive market maker, which sets stock price to make the stock market clear. At $t = 2$, the stock market closes. Then the firm is liquidated at value v. The manager is paid S, after which the owner takes her share $\delta v - S$ net of executive compensation.

2.4. Definition of Equilibrium

Finally, we define the equilibrium in our model as follows.

Definition 2.1: *The equilibrium consists of the optimal CEO compensation contract* (S^*, e^*)*, the optimal, the optimal linear order strategy* X^* *of the insider, and the efficient linear pricing rule* P^**, which satisfy the following.*

1. Given the contract (S^*, e^*)*, the pricing rule* $P^*(y)$ *satisfies.*

$$P^*(y) = \mathrm{E}[\tilde{v} \mid \tilde{y} = y],$$

and the insider maximizes his profit:

$$X^*(v, s_u) \in \mathrm{argmax}_x \ \mathrm{E}[(\tilde{v} - P^*(\tilde{y}))x \mid (\tilde{v}, \tilde{s}_u) = (v, s_u)].$$

2. Given the pricing rule $P^*(y)$ *of the market maker, the owner chooses the contract* (a^*, e^*) *to solve.*

$$\max_{(a_0, a_1, a_2), e} \mathrm{E}[\delta(\tilde{v} - \tilde{S}^*)],$$

$$\text{s.t. } E[\tilde{S}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}^*] - c(e) \geq 0,$$

$$e \in \arg\max_{e' \in [0, \bar{e}]} \mathrm{E}[\tilde{S}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}] - c(e).$$

3. EQUILIBRIUM PRICE IN STOCK MARKET

It is standard to derive

$$\mathrm{E}[\tilde{u} \mid \tilde{s}_u = s_u] = \gamma s_u, \ \mathrm{Var}[\tilde{u} \mid \tilde{s}_u = s_u] = \gamma \sigma_\eta^2, \text{ where } \gamma = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\eta^2}.$$

When the insider submits a market order x to the market maker, his profit is given by

$$\mathrm{E}[(\tilde{v} - \tilde{p})x \mid (\tilde{v}, \tilde{s}_u) = (v, s_u)] = x\left(\mathrm{E}[\tilde{v} \mid \tilde{v} = v] - \mu - \lambda(x + \mathrm{E}[\tilde{u} \mid \tilde{s}_u = s_u]\right)$$

$$= x\left[(v - \mu - \lambda\gamma s_u) - \lambda x\right].$$

This implies that the optimal order flow x^* of the insider is determined as follows:

$$x^* = \frac{1}{2\lambda}\left[v - \mu - \lambda\gamma s_u\right]$$

and his optimal profit is given by $\pi = \lambda(x^*)^2$. As a consequence, the linear market order $X(v, s_u) = \alpha(v-e) + \beta s_u$ in (2.1) is consistent with the above:

$$\frac{1}{2\lambda}\left[v - \mu - \lambda\gamma s_u\right] = x^* = X\left(v, s_u\right) = \alpha\left(v-e\right) + \beta s_u \,. \tag{3.1}$$

This implies that

$$\frac{1}{2\lambda}(v-\mu) = \alpha(v-e), \quad -\frac{\gamma}{2} = \beta.$$

On the other hand, we observe that the properties of the total order flow y are

$$\mathrm{E}[\tilde{y}] = \mathrm{E}[\tilde{x} + \tilde{u}] = 0\,,$$

$$\mathrm{Var}(\tilde{y}) = \alpha^2\sigma_\varepsilon^2 + \beta^2(\sigma_u^2 + \sigma_\eta^2) + (2\beta+1)\sigma_u^2\,,$$

$$\mathrm{Cov}(\tilde{v}, \tilde{y}) = \mathrm{Cov}(\tilde{v}, \tilde{x}) = \alpha\sigma_\varepsilon^2\,.$$

Then the efficient linear pricing rule P becomes

$$\begin{aligned} P(y) &= \mathrm{E}[\tilde{v} \mid \tilde{y} = y] \\ &= \mathrm{E}[\tilde{v}] + \frac{\mathrm{Cov}(\tilde{v}, \tilde{y})}{\mathrm{Var}(\tilde{y})}\left(y - \mathrm{E}[\tilde{y}]\right) \\ &= e + \left(\frac{\alpha\sigma_\varepsilon^2}{\alpha^2\sigma_\varepsilon^2 + \beta^2(\sigma_u^2 + \sigma_\eta^2) + (2\beta+1)\sigma_u^2}\right) y. \end{aligned}$$

Since $P(y) = \mu + \lambda y$ by (2.2), it follows that

$$\mu = e, \quad \lambda = \frac{\alpha\sigma_\varepsilon^2}{\alpha^2\sigma_\varepsilon^2 + \beta^2(\sigma_u^2 + \sigma_\eta^2) + (2\beta+1)\sigma_u^2}.$$

Because $\mu = e$, in view of (3.1), we have

$$X(v, s_u) = x^* = \alpha(v-e) + \beta s_u = \frac{1}{2\lambda}[(v-e) - \lambda\gamma s_u].$$

and therefore

$$\alpha = \frac{1}{2\lambda}, \quad \beta = -\frac{1}{2}\gamma = -\frac{\sigma_u^2}{2(\sigma_u^2 + \sigma_\eta^2)}.$$

However, this implies that

$$\lambda = \frac{\alpha\sigma_\varepsilon^2}{\alpha^2\sigma_\varepsilon^2 + (\gamma/2)^2(\sigma_u^2 + \sigma_\eta^2) + (1-\gamma)\sigma_u^2} = \frac{1}{2\alpha},$$

which leads to

$$\lambda = \frac{\alpha\sigma_\varepsilon^2}{\alpha^2\sigma_\varepsilon^2 + (\gamma/2)^2(\sigma_u^2 + \sigma_\eta^2) + (1-\gamma)\sigma_u^2} = \frac{1}{2\alpha}.$$

Since

$$\left(\frac{\gamma}{2}\right)^2(\sigma_u^2 + \sigma_\eta^2) = \frac{1}{4}\left(\frac{\sigma_u^4}{\sigma_u^2 + \sigma_\eta^2}\right), \quad (1-\gamma)\sigma_u^2 = \frac{\sigma_u^2\sigma_\eta^2}{\sigma_u^2 + \sigma_\eta^2},$$

it follows that

$$\alpha^2\sigma_\varepsilon^2 = \frac{\sigma_u^2}{4}\left(\frac{\sigma_u^2 + 4\sigma_\eta^2}{\sigma_u^2 + \sigma_\eta^2}\right).$$

As a consequence, we obtain

$$\alpha^* = \frac{\sigma_u}{2\sigma_\varepsilon}\left[\frac{\sigma_u^2 + 4\sigma_\eta^2}{\sigma_u^2 + \sigma_\eta^2}\right]^{\frac{1}{2}}, \quad \beta^* = -\frac{\sigma_u^2}{2(\sigma_u^2 + \sigma_\eta^2)}, \quad \lambda^* = \frac{\sigma_\varepsilon}{\sigma_u}\left[\frac{\sigma_u^2 + 4\sigma_\eta^2}{\sigma_u^2 + \sigma_\eta^2}\right]^{-\frac{1}{2}}.$$

This computation yields the insider' linear order rule X^* and the market maker's linear pricing rule P^* in the stock market equilibrium as stated in the following theorem.

Theorem 3.1: *The insider's optimal linear order rule* X^* *and the market maker's linear pricing rule* P^* *in the stock market equilibrium are given by*

$$X^*(v, s_u) = \frac{\sigma_u}{2\sigma_\varepsilon}K(v-e) - \left(\frac{\sigma_u^2}{2(\sigma_u^2 + \sigma_\eta^2)}\right)s_u, \quad P^*(y) = e + \left(\frac{\sigma_\varepsilon}{K\sigma_u}\right)y.$$

Where:

$$K = \left[\frac{\sigma_u^2 + 4\sigma_\eta^2}{\sigma_u^2 + \sigma_\eta^2} \right]^{\frac{1}{2}}.$$

As an aside, we can compute the ex ante profit of the insider in the stock market equilibrium. To obtain it, it is necessary to derive the interim profit of the insider. Recall that the equilibrium strategies (X^*, P^*) yield

$$x^* = \frac{1}{2\lambda}[(v-e) - \lambda\gamma s_u].$$

Therefore, we have

$$\begin{aligned} \mathrm{E}[\pi(X^*, P^*) \mid s_v, s_u] &= x^*[(v-e) - \lambda\gamma s_u - \lambda x^*] \\ &= \frac{1}{4\lambda}\left[(v-e)^2 - 2(v-e)\lambda\gamma s_u + \lambda^2\gamma^2 s_u^2\right] \\ &= \frac{K\sigma_u}{4\sigma_\varepsilon}(v-e)^2 - \frac{\gamma}{2}(v-e)s_u + \frac{\gamma^2\sigma_\varepsilon}{4K\sigma_u}s_u^2 . \end{aligned}$$

Now we can formally write down the interim profit of the insider in the stock market equilibrium as follows.

Proposition 3.1: *With the equilibrium strategies* (X^*, P^*)*, the interim profit of the insider is given by*

$$\mathrm{E}[\pi(X^*, P^*) \mid v, s_u] = \frac{K\sigma_u}{4\sigma_\varepsilon}(v-e)^2 - \frac{\gamma}{2}(v-e)s_u + \frac{\gamma^2\sigma_\varepsilon}{4K\sigma_u}s_u^2.$$

Where:

$$\gamma = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\eta^2}, \quad K = \left[\frac{\sigma_u^2 + 4\sigma_\eta^2}{\sigma_u^2 + \sigma_\eta^2} \right]^{\frac{1}{2}}.$$

By applying the law of iterated expectations, we can compute the ex ante profit of the insider in the stock market equilibrium. That is, the ex ante profit of the insider is obtained by

$$\mathrm{E}[\pi(X^*, P^*)] = \mathrm{E}[\mathrm{E}[\pi(X^*, P^*) \mid s_v, s_u]]$$

$$= \frac{K\sigma_u}{4\sigma_\varepsilon}\sigma_\varepsilon^2 + \frac{\sigma_\varepsilon}{4K\sigma_u}\left(\frac{\sigma_u^2}{\sigma_u^2+\sigma_\eta^2}\right)^2(\sigma_u^2+\sigma_\eta^2)$$
$$= \frac{\sigma_\varepsilon\sigma_u}{4}\left(K+\frac{\gamma}{K}\right).$$

This result is formally stated in the following proposition.

Proposition 3.2: *With the equilibrium strategies* (X^*, P^*), *the ex ante profit of the insider is given by*

$$\mathrm{E}[\pi(X^*,P^*)] = \frac{\sigma_\varepsilon\sigma_u}{4}\left(K+\frac{\gamma}{K}\right), \quad where \quad \gamma = \frac{\sigma_u^2}{\sigma_u^2+\sigma_\eta^2}, \quad K = \left[\frac{\sigma_u^2+4\sigma_\eta^2}{\sigma_u^2+\sigma_\eta^2}\right]^{\frac{1}{2}}.$$

Notice that $K=(4-3\gamma)^{\frac{1}{2}}$. When $\sigma_\eta^2 \to 0$, we see that $K \to 2$ and $\gamma \to 0$. That is, in the limit, the signal gets to include no noise, i.e., $s_u = u$. When $\sigma_\eta^2 \to \infty$, we see that $K \to 1$ and $\gamma \to 1$. That is, in the limit, the signal becomes useless. However, in both cases, the ex ante expected profit converges to that of Kyle (1985). In fact, if $\sigma_\eta^2 \in (0,\infty)$, i.e., $\gamma \in (0,1)$, the ex ante expected profit is lower than that of Kyle (1985). This implies that the insider's acquisition of liquidity signal s_u (non-fundamental information) makes him worse off, which confirms the result of Park (2010). It can be a counterexample to the common sense that the more information, the better.

4. EQUILIBRIUM CONTRACT

The risk-neutral owner of the firm owns fraction δ of the whole share of the equity. At $t=0$, the owner sets a linear compensation rule S when contracting with the CEO:

$$S = a_0 + a_1 v + a_2 p.$$

Given the linear compensation rule S, the CEO chooses the optimal effort to solve the following problem, in the certainty equivalent form,

$$\max_{e\in[0,\bar{e}]} \mathrm{E}[\tilde{S}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}] - c(e). \tag{4.1}$$

On the other hand, the owner solves the following problem:

$$\max_{a_0,a_1,a_2,e} \mathrm{E}[\delta\tilde{v} - \tilde{S}]$$

$$\text{s.t. } E[\tilde{S}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}] - c(e) \geq 0,$$
$$e = \mathrm{argmax}_{e' \in [0,\bar{e}]} \ \mathrm{E}[\tilde{S}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}] - c(e').$$

Since $S = a_0 + a_1 v + a_2 p$ and $p = e + \lambda y$, we see that

$$E[\tilde{S}] = a_0 + a_1 e + a_2 e. \tag{4.2}$$

Furthermore, recalling the fact that

$$\mathrm{Var}[\tilde{y}] = \alpha^2\sigma_\varepsilon^2 + \beta^2(\sigma_u^2 + \sigma_\eta^2) + (1-\gamma)\sigma_u^2 = 2\alpha^2\sigma_\varepsilon^2, \quad \mathrm{Cov}(\tilde{v}, \tilde{y}) = \alpha\sigma_\varepsilon^2, \quad \alpha = \frac{1}{2\lambda}, \tag{4.3}$$

we obtain that

$$\begin{aligned}\mathrm{Var}[\tilde{S}] &= a_1^2\,\mathrm{Var}[\tilde{v}] + 2a_1a_2\lambda\mathrm{Cov}(\tilde{v}, \tilde{y}) + a_2^2\lambda^2\,\mathrm{Var}[\tilde{y}] \\ &= a_1^2\sigma_\varepsilon^2 + +2a_1a_2\lambda\alpha\sigma_\varepsilon^2 + 2a_2^2\lambda^2\alpha^2\sigma_\varepsilon^2 \\ &= \sigma_\varepsilon^2\left(a_1^2 + a_1a_2 + \frac{1}{2}a_2^2\right).\end{aligned}$$

It is observed that $\mathrm{Var}[\tilde{S}]$ does not contain effort e. As a consequence, putting (4.2) and (4.3) into (4.1), the manager's problem reduces to the following problem:

$$\max_e\, (a_0 + a_1 e + a_2 e) - \frac{1}{2}ke^2.$$

Then the first-order condition for the manager's problem yields the optimal effort level of the manager:

$$e^\circ = \frac{a_1 + a_2}{k}.$$

We know that the individual rationality constraint is binding in equilibrium, that is,

$$E[\tilde{S}] = \frac{\rho}{2}\mathrm{Var}[\tilde{S}] + c(e^*).$$

Then, the owner's problem takes the following form:

$$\max_{a_1,a_2} \mathrm{E}[\delta\tilde{v}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}] - c(e^*).$$

Since we know

$$\mathrm{E}[\tilde{v}] = e^* = \frac{a_1 + a_2}{k}, \quad \mathrm{Var}[\tilde{S}] = \sigma_\varepsilon^2\left(a_1^2 + a_1a_2 + \frac{1}{2}a_2^2\right),$$

it follows that

$$\mathrm{E}[\delta\tilde{v}] - \frac{\rho}{2}\mathrm{Var}[\tilde{S}] - c(e^*) = \frac{\delta(a_1 + a_2)}{k} - \frac{\rho\sigma_\varepsilon^2}{2}\left(a_1^2 + a_1a_2 + \frac{1}{2}a_2^2\right) - \frac{(a_1 + a_2)^2}{2k}.$$

Then the first-order conditions for (a_1, a_2) in the owner's problem are

$$0 = \frac{\delta - a_1 - a_2}{k} - \frac{\rho\sigma_\varepsilon^2}{2}(2a_1 + a_2),$$
$$0 = \frac{\delta - a_1 - a_2}{k} - \frac{\rho\sigma_\varepsilon^2}{2}(a_1 + a_2).$$

As a consequence, we obtain

$$a_1^* = 0, \quad a_2^* = \frac{2\delta}{2 + k\rho\sigma_\varepsilon^2},$$

which implies that

$$e^* = \frac{2\delta}{k(2 + k\rho\sigma_\varepsilon^2)}.$$

It is worth noting that the firm's liquidation value is not used in the optimal executive contract (that is, $a_1^* = 0$). This result is quite surprising and in contrast to Holmström and Tirole (1993), where the insider acquires a signal about the firm's liquidation value by paying a quadratic cost and the firm's liquidation value is important in the optimal contract. It is also remarkable that this result holds true even in the benchmark model, as well as in our model.

Finally, the remaining coefficient a_0^* can be obtained from the individual rationality condition. Since

$$E[\tilde{S}^*]-\frac{\rho}{2}\mathrm{Var}[\tilde{S}^*]-c(e^*)=a_0^*+a_2^*e^*-\frac{\rho\sigma_\varepsilon^2}{2}\left(\frac{1}{2}(a_2^*)^2\right)-\frac{(a_2^*)^2}{2k}$$
$$=a_0^*+\frac{1}{k}(a_2^*)^2-\frac{\rho\sigma_\varepsilon^2}{4}(a_2^*)^2-\frac{1}{2k}(a_2^*)^2=0,$$

we see that

$$a_0^*=\left(\frac{\rho\sigma_\varepsilon^2}{4}-\frac{1}{2k}\right)(a_2^*)^2=\frac{k\rho\sigma_\varepsilon^2-2}{4k}\left(\frac{2\delta}{2+k\rho\sigma_\varepsilon^2}\right)^2=\frac{(k\rho\sigma_\varepsilon^2-2)\delta^2}{k(2+k\rho\sigma_\varepsilon^2)^2}.$$

To ensure that the fixed compensation is positive, that is, $a_0^*>0$, we need to impose an assumption that $k\rho\sigma_\varepsilon^2>2$. These results are summarized in the following theorem.

Theorem 4.1: *If* $k\rho\sigma_\varepsilon^2>2$, *then the optimal contract* (a^*,e^*) *is given by*

$$S^*(v,p)=\frac{(k\rho\sigma_\varepsilon^2-2)\delta^2}{k(2+k\rho\sigma_\varepsilon^2)^2}+\left(\frac{2\delta}{2+k\rho\sigma_\varepsilon^2}\right)p,\quad e^*=\frac{2\delta}{k(2+k\rho\sigma_\varepsilon^2)}.$$

Hence, from theorem 2.1 and theorem 3.1, we obtain an overall equilibrium as follows.

Corollary 4.1: *The equilibrium is given by* (S^*,e^*,X^*,P^*) *such that*

$$S^*(v,p)=\frac{(k\rho\sigma_\varepsilon^2-2)\delta^2}{k(2+k\rho\sigma_\varepsilon^2)^2}+\left(\frac{2\delta}{2+k\rho\sigma_\varepsilon^2}\right)p,\quad e^*=\frac{2\delta}{k(2+k\rho\sigma_\varepsilon^2)}.$$

$$X^*\left(v,s_u\right)=\frac{\sigma_u}{2\sigma_\varepsilon}K\left(v-e^*\right)-\left(\frac{\sigma_u^2}{2\left(\sigma_u^2+\sigma_\eta^2\right)}\right)s_u,P^*\left(y\right)=e^*+\left(\frac{\sigma_\varepsilon}{K\sigma_u}\right)y.$$

Where:

$$K=\left[\frac{\sigma_u^2+4\sigma_\eta^2}{\sigma_u^2+\sigma_\eta^2}\right]^{\frac{1}{2}}.$$

5. COMPARSION WITH THE BENCHMARK AND COMPARATIVE STATICS

In the benchmark model, the stock market is exactly the same as that of Kyle (1985). Therefore, the equilibrium strategies $(\hat{X}, \hat{P})$ are given by[8)]

$$\hat{X} = \frac{\sigma_u}{\sigma_\varepsilon}(v - \hat{e}), \quad \hat{P} = \hat{e} + \frac{\sigma_\varepsilon}{2\sigma_u} y.$$

The optimal contract is exactly the same as in our model, that is

$$\hat{S} = S^* = \frac{(k\rho\sigma_\varepsilon^2 - 2)\delta^2}{k(2 + k\rho\sigma_\varepsilon^2)^2} + \left(\frac{2\delta}{2 + k\rho\sigma_\varepsilon^2}\right)p, \quad \hat{e} = e^* = \frac{2\delta}{k(2 + k\rho\sigma_\varepsilon^2)} \; .$$

As is shown above, it is surprising that the optimal executive compensation contract does not depend on the firm's liquidation value in both the benchmark model and out model. Only the difference of two models is in that the equilibrium strategies of the insider and the competitive market maker take different shape from those of the benchmark model. As of the insider's equilibrium order rule, in comparison with the benchmark, our model put less weight on the liquidation value since $K < 2$ and a negative weight on the signal s_u that is a new information For the competitive market maker's pricing rule, in comparison with the benchmark, our model is more sensitive to the aggregate order flow because of $K < 2$. In fact, as $\sigma_\eta^2 \to \infty$, i.e., $K \to 2$, the signal is not informative and thus our model collapses into the benchmark model.

As in the benchmark, it is easy to see from theorem 4.1 that all of the a_0^*, a_2^*, and e^* decease in the effort cost coefficient k, (liquidation) value volatility σ_ε, and risk-aversion coefficient ρ. It is noted that they do not depend on liquidity signal s_u, its noise volatility σ_η, or liquidity volatility σ_u. In our model, as value volatility σ_ε increases, the insider's order becomes less sensitive to the liquidation value while the price becomes more sensitive to the aggregate order flow. The liquidity volatility σ_u of noise trades has the reverse effect and, in addition, makes the insider's order (negatively) more sensitive to non-fundamental information s_u because γ increases in σ_u. Finally, as the noise volatility σ_η of signal s_u increases, it only makes t the insider order less sensitive to non-fundamental information. We summarize these results in the following theorem.

Theorem 5.1: *The following hold.*

(1) All of the a_0^*, a_2^*, and e^* decrease in the effort cost coefficient k, value volatility σ_ε, and risk-aversion coefficient ρ.

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8) We denote the equilibrium in the benchmark model by using "hat."

(2) As value volatility σ_ε *increases, the insider's order becomes less sensitive to the liquidation value while the price becomes more sensitive to the aggregate order flow.*

(3) As liquidity volatility σ_u *increases, the price becomes less sensitive to the aggregate order flow. Moreover, the insider's order becomes more sensitive to the liquidation value and (negatively) more sensitive to liquidity signal* s_u*.*

(4) As noise volatility σ_η *of signal* s_u *increases, the insider becomes more sensitive to liquidation value but less sensitive to liquidity signal* s_u*, and the price becomes less sensitive to the aggregate order flow.*

Now we can provide intuitions behind theorem 5.1. In the following, it is noted that $K\sigma_u$ increases in liquidity volatility σ_u and K increases in noise volatility σ_η of signal s_u.

(1) The intuition for (1) is quite similar to the standard executive compensation contract that is not stock-based. The greater the uncertainty of liquidation value, the greater the effort cost, or the more risk averse the CEO, he shares some of the increased risk with the owner and the price is less informative, and therefore a_2^* becomes smaller and the insider exerts less efforts.

(2) As value volatility σ_ε increases, the liquidation value gives less precise information about the firm's fundamental. Consequently, the insider puts less weight on liquidation value in his market order and does not change his sensitivity to signal s_u. Moreover, market liquidity $(\lambda^*)^{-1}$ decreases, which makes the price become more sensitive to the aggregate order flow.

(3) As liquidity volatility σ_u increases, market liquidity $(\lambda^*)^{-1}$ increases so that the price becomes less sensitive to the aggregate order flow. A horizontal shift of supply curve to the right induces the insider to trade more without changing prices. As σ_u increases, liquidity signal s_u is more informative, due to which the insider becomes (negatively) more sensitive to liquidity signal s_u.

(4) As noise volatility σ_η of liquidity signal s_u increases, liquidity signal s_u is less informative. This implies that the insider puts more weight on liquidation value v and less weight on liquidity signal s_u. Because of larger market liquidity $(\lambda^*)^{-1}$, the price becomes less sensitive to the aggregate order flow.

6. CONCLUDING REMARKS

This paper investigates how the stock prices affect the market-based optimal executive compensation contract when the insider acquires a signal about the demand of liquidity. Traders in the stock market. This is done by incorporating the standard principal? Gent problem à la Holmström (1979) into Kyle's (1985) asset market model. We examine how the optimal executive compensation contract, the insider's trading strategy, and the equilibrium price are affected by exogenous parameters such as the manager's risk aversion and effort cost, the volatility of noisy trades, and the volatility of liquidation value. Remarkably, we find the firm's liquidation value is not used in the optimal executive contract. This result is quite surprising and in contrast to Holmström and Tirole (1993), where the insider acquires a signal about the firm's liquidation value by paying a quadratic cost and the firm's liquidation value is important in the optimal executive compensation contract. It is also remarkable that this result holds true even in the benchmark model where the insider does not acquire a signal about liquidity traders, as well as in our model. It would be very interesting to investigate how the optimal executive compensation contract and insider's trading strategy change when the insider acquires two signals, i.e., a signal about the firm's liquidation value and a signal about liquidity traders.

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